

MAT 205E - Theory of a Complex Variable Functions

Homework 1

- Express the following complex numbers in the polar form.

a) $4 + 3i$	b) $2 - 5i$	c) $-2 - 2i$	d) $-1 + 4i$
e) $\frac{\sqrt{5}}{2+2i}$	f) $\frac{4i}{3+6i}$	g) $\frac{1+i}{i-1}$	h) $(\sqrt{3} + i)^2$
- Express the following complex numbers in the Cartesian form such as $a + ib$.

a) $\sqrt{2}e^{i\pi/4}$	b) $\sqrt{5}e^{-i\pi/3}$	c) $4e^{i\pi}$	d) $2e^i$
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- Calculate following complex numbers.

a) $(2 + i)^2$	b) $(3i - 1)^3$	c) $(4i + 5)^2(-3i - 1)^4$
d) $(i - 1)^5(1 - 2i)^6$	e) $\frac{(1-4i)^2(3-3i)}{(-5i-2)(1+i)^4}$	f) $(3 - 2i)^8(-1 - i)^4$
- Find all values of the following roots.

a) $(3i)^{1/2}$	b) $(-i)^{1/4}$	c) $(-1)^{1/4}$	d) $27^{1/6}$
e) $(-4)^{1/2}$	f) $(64i)^{1/3}$	g) $(1 + i)^{1/2}$	h) $(-1)^{1/12}$
- For the complex number $z = 2 - i$ calculate following functions.

a) z^n	b) $\frac{1}{z}$
c) z^{-n}	d) $z^3 + 2z^2 + 5z + 4 - 2i$
e) $ z ^2$	f) $(\bar{z})^2$
g) $z \cdot \bar{z}$	h) $Arg(z - 2) + Arg(z + i)$
- For the complex numbers z_1, z_2 and z_3 , if $|z_1| = |z_2| = |z_3|$, show that

$$Arg\left(\frac{z_3 - z_2}{z_3 - z_1}\right) = \frac{1}{2} Arg\left(\frac{z_2}{z_1}\right)$$

Discuss the geometrical interpretation of this relation.

- Draw the points $z, -z, \bar{z}, -\bar{z}, z^2$ and $1/z$ in complex plane for the complex numbers $z_1 = 3 - 4i, z_2 = 1 + 3i$ and $z_3 = -5i$.
- Show that $|z - 3| = |\bar{z} - 3|$ for the complex number $z = x + iy$.
- Sketch the following expressions in complex plane.

a) $ z < 1$	b) $ z + 1 < z - 2i $
c) $ z + i\bar{z} \geq 3$	d) $Re\{z\} < Im\{z\} + 3$
e) $ Arg(z) < \frac{\pi}{3}, Re\{z\} < 1$	f) $ z + 2 - 3i \geq 4$
g) $-\frac{\pi}{2} < Arg(z - 1 + 3i) < 0$	h) $0 < (Re\{z\})^2 + (Im\{z\})^2 \leq 3$
i) $-\frac{\pi}{2} < Arg(z) < \frac{\pi}{3}, z > 2$	j) $-\frac{\pi}{2} < Arg(iz) < \pi, z < 2$
- Calculate following complex numbers.

a) $2i^{37} + i^{5078} - 4i^{-77} + i^{-1}$	b) $4(-i)^{-4} + 6(-i)^5 - 3\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^{45}$
c) $i^{0.2} + (-i)^{1.7} + (-2 + 2i)^{-3.3}$	d) $(i^i)^i + i^i + (-1)^{2+i}$
- Find complex number z in following equations.

a) $z = 2z + 1$	b) $z^2 = -9i$	c) $iz = z + 2 - i$
d) $\frac{z}{z+1} = 1 + 5i$	e) $z^2 + 3z + 2 = 0$	f) $z^3 - iz = 0$
- Verify following equalities for $0 \leq r < 1$.

a) $1 + r \cos \phi + r^2 \cos 2\phi + \dots = \frac{1 - r \cos \phi}{r^2 - 2r \cos \phi + 1}$
b) $r \sin \phi + r^2 \sin 2\phi + \dots = \frac{r \sin \phi}{r^2 - 2r \cos \phi + 1}$