HOMEWORK # 5

1. Use seven iterations of the power method to compute the largest eigenvalue and the corresponding eigenvector of the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & 1 & 2 \\ 1 & 3 & 1 \end{bmatrix} \tag{1}$$

Use $x_0 = (1, 1, 1)^\top$ as the initial approximation.

2. The Lorenz equations are the following system of differential equations

$$\frac{dx}{dt} = -10(x - y) \tag{2}$$

$$\frac{dy}{dt} = -xz + 28x - y \tag{3}$$

$$\frac{dz}{dt} = xy - 8z/3 \tag{4}$$

Use the fourth-order Runge-Kutta method with $x_0 = 0.0$, $y_0 = 20.0$, $z_0 = 25.0$ and $dt = 0.001$ to solve the above equations. Plot the solution in $x - y$ and $x - z$ plane.

3. Solve the Laplace equation in a square enclosure with the boundary condition given in the figure below. The Laplace equation is given:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{5}$$

Assume that $\Delta x = \Delta y$.

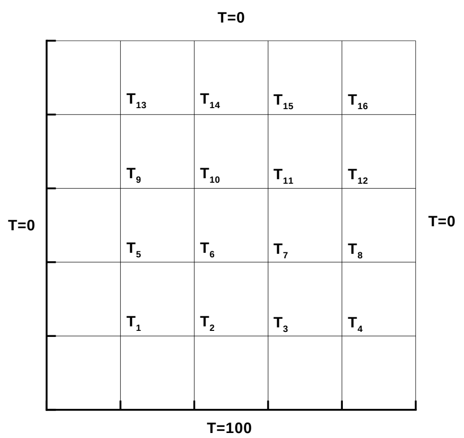

Figure 1: The computational domain with the Dirichlet boundary conditions.

4. Use the shooting method to solve the ordinary differential equation

$$y'' + 3xy' + 7y = cos(2x) \tag{6}$$

with $y(0) = 1$ and $y(\pi) = 0$.