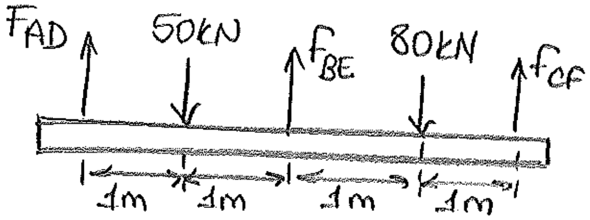
STRENGTH OF MATERIALS I

2nd MIDTERM

QUESTION 1:

Free Body Diagram of the rigid beam.



Equations of Equilibrium:

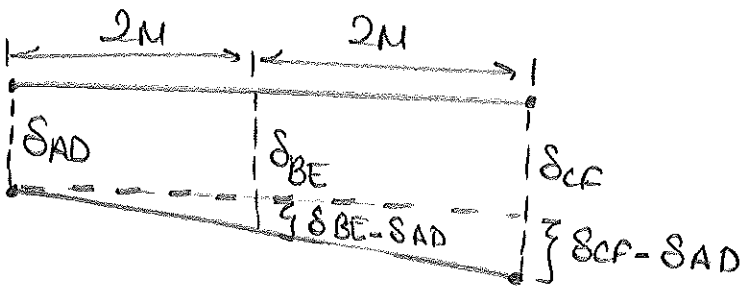
$$+\uparrow \Sigma F_y = 0 \quad F_{AD} + F_{BE} + F_{CF} - 50(10^3)N - 80(10^3)N = 0$$

$$F_{AD} + F_{BE} + F_{CF} = 130(10^3)N \quad (1)$$

$$\circlearrowleft + \Sigma M_D = 0 \quad F_{BE}(2m) + F_{CF}(4M) - 50(10^3)N(1m) - 80(10^3)N(3m) = 0$$

$$2F_{BE} + 4F_{CF} = 290(10^3)N \quad (2)$$

Compatibility Equation



from the triangle similarity

$$\frac{\delta_{BE} - \delta_{AD}}{\delta_{CF} - \delta_{AD}} = \frac{2}{4}$$

$$\delta_{BE} = \frac{1}{2}(\delta_{AD} + \delta_{CF})$$

$$\delta_{BE} = \frac{1}{2}\left(\delta_{AD} + \delta_{CF}\right)$$

$$\frac{F_{BE} \cdot \cancel{L}}{\cancel{AE}} = \frac{1}{2}\left(\frac{F_{AD}\cancel{L}}{\cancel{AE}} + \frac{F_{CF}\cancel{L}}{\cancel{AE}}\right) \Rightarrow 2F_{BE} = F_{AD} + F_{CF} \quad (3)$$

Solving Eq.(1), Eq.(2) and Eq.(3) together gives

$F_{BE} = 43.33$ kN

$F_{AD} = 35.83$ kN

$F_{CF} = 50.83$ kN

Average Normal Stress in Each Bar:

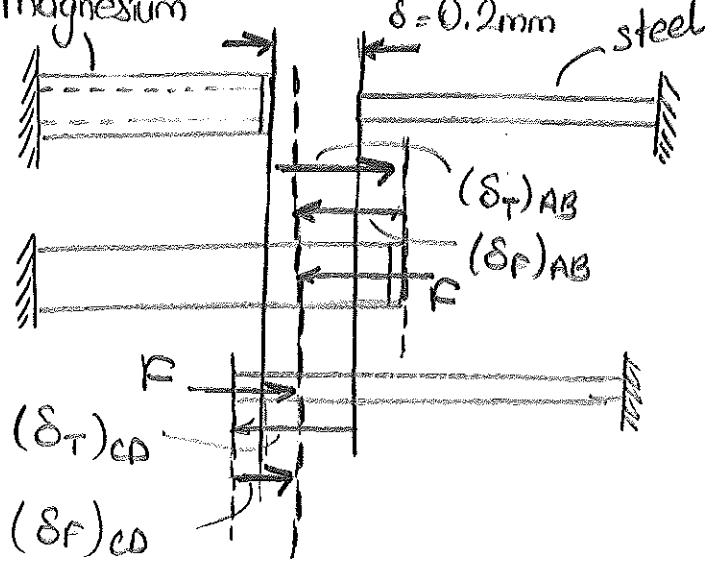
$$\sigma_{BE} = \frac{F_{BE}}{\text{Area}} = \frac{43.33(10^3)\,N}{450\,mm^2} = 96.3\ MPa$$

$$\sigma_{AD} = \frac{F_{AD}}{\text{Area}} = \frac{35.83(10^3)\,N}{450\,mm^2} = 79.6\ MPa$$

$$\sigma_{CF} = \frac{F_{CF}}{\text{Area}} = \frac{50.83(10^3)\,N}{450\,mm^2} = 113\ MPa$$

QUESTION 2:

Compatibility Equation:



$\alpha_{mg} = 26\times10^{-6}/°C$

$E_{mg} = 44.9$ GPa

$\alpha_{st} = 12\times10^{-6}/°C$

$E_{st} = 200$ GPa

$$\delta = \left[(\delta_T)_{AB} - (\delta_F)_{AB}\right] + \left[(\delta_T)_{CD} - (\delta_F)_{CD}\right]$$

$$(\delta_T)_{AB} = \alpha_{mg}(\Delta T)L_{AB} = (26\times10^{-6}/°C)(80°C-30°)(0.3m) = 0.39mm$$

$$(\delta_F)_{AB} = \frac{F L_{AB}}{A_{AB}E_{mg}} = \frac{F(0.3m)}{\pi(0.025^2-0.02^2)m^2(44.9\times10^9\,N/m^2)}$$

$$= (9.45\times10^{-9})F \;M/N$$

$$(\delta_T)_{CD} = \alpha_{st}(\Delta T)L_{CD} = (12\times10^{-6}/°C)(80°C-30°C)(0.45m) = 0.27mm$$

$$(\delta_F)_{CD} = \frac{F L_{CD}}{A_{CD}E_{st}} = \frac{F(0.45m)}{\frac{\pi}{4}(0.025^2 m^2)(200\times10^9 N/m^2)} = (4.6\times10^{-9})F \;M/N$$

$$0.2\,mm = 0.39\,mm - (9.45\times10^{-6})F + 0.27\,mm - (4.6\times10^{-6})F$$

$$(14.05\times10^{-6})F = 0.46$$

$$\underline{\underline{F = 32.7\,kN}}$$

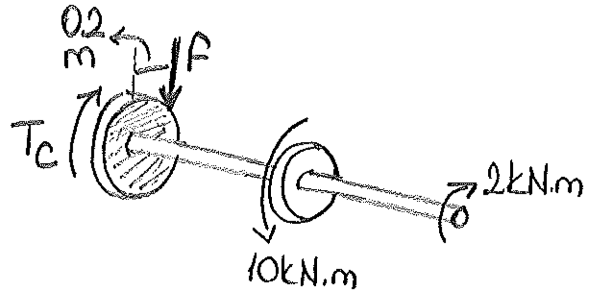
<u>Average Normal Stresses</u>

$$\sigma_{AB} = \frac{F}{(Area)_{AB}} = \frac{32.7(10^3)\,N}{\pi(0.025^2 - 0.02^2)\,m^2} = \underline{\underline{46.3\,MPa}}$$

$$\sigma_{CD} = \frac{F}{(Area)_{CD}} = \frac{32.7(10^3)\,N}{\frac{\pi}{4}(0.025^2)} = \underline{\underline{66.6\,MPa}}$$

QUESTION 3:

Free Body Diagram of the shaft CE:



Moment Equation of Equilibrium,

$$\curvearrowleft^{+}_{x} \sum M_C = 0 \qquad 2kN.M - 10kN.M + T_C = 0$$

$$T_C = 8kN.M$$

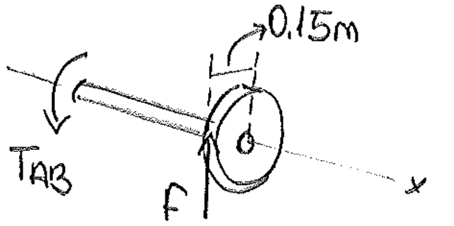
Thus,

$$T_C = F(0.2m)$$

$$8(10^3)\,Nm = (0.2m)F$$

$$\underline{F = 40\,kN}$$

Free Body Diagram of AB



The internal loadings of AB is obtained from the moment eqn. of equilibrium about the x axis

$$\curvearrowleft^{+} \sum M_x = 0 \qquad F(0.15m) - T_{AB} = 0$$

$$T_{AB} = 40(10^3)N\,(0.15m) = \underline{6\,kNM}$$

Maximum Stress on Shaft AB

$$\tau_{max} = \frac{T_{AB}\,c}{J} = \frac{6(10^3)N.m\,(0.04m)}{\frac{\pi}{2}(0.04^4)M^4} = \underline{59.7\,MPa}$$

Twist Angle of End E:

$$\phi_B = \frac{T_{AB} L_{AB}}{G_{st} J} = \frac{6(10^3)\,Nm\,(0.6m)}{75(10^9)\,N/m^2 \,\frac{\pi}{2}(0.04^4)\,m^2} = 0.01194\,rad$$

Using the gear ratio,

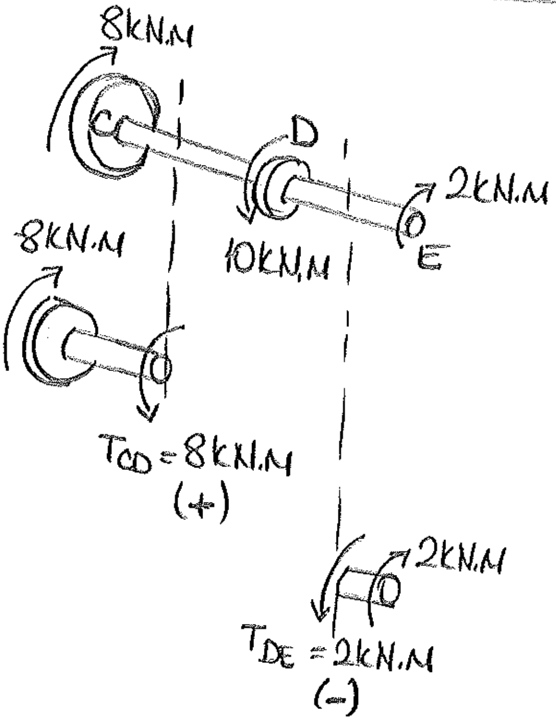
$$\phi_B\,(0.15\,m) = \phi_C\,(0.2m)$$

$$\phi_C = \left(\frac{0.15}{0.2}\right)(0.01194) = 0.008952\,rad$$

The twist of end E with respect to end C,

$$\phi_{E/C} = \sum \frac{T_i L_i}{G_{st} J} = \frac{T_{CD} L_{CD}}{GJ} + \frac{T_{DE} L_{DE}}{GJ}$$

Free Body Diagram of the shaft CE:



$$J = \frac{\pi}{2}(0.04^4)\,m^4 = 4.02 \times 10^{-6}\,m^4$$

$$\phi_{E/C} = \frac{(8\,kN.m)(0.6m)}{75(10^9)\,N/m^2\,(4.02 \times 10^{-6})\,m^4} + \frac{(-2\,kN.m)(0.6m)}{75(10^9)\,N/m^2\,(4.02 \times 10^{-6}\,m^4)}$$

$$\phi_{E/C} = 0.01194\,rad$$

$$\phi_E = \phi_{E/C} + \phi_C = 0.01194\,rad + 0.008952\,rad = 0.02089\,rad$$

$$= 1.20^\circ$$