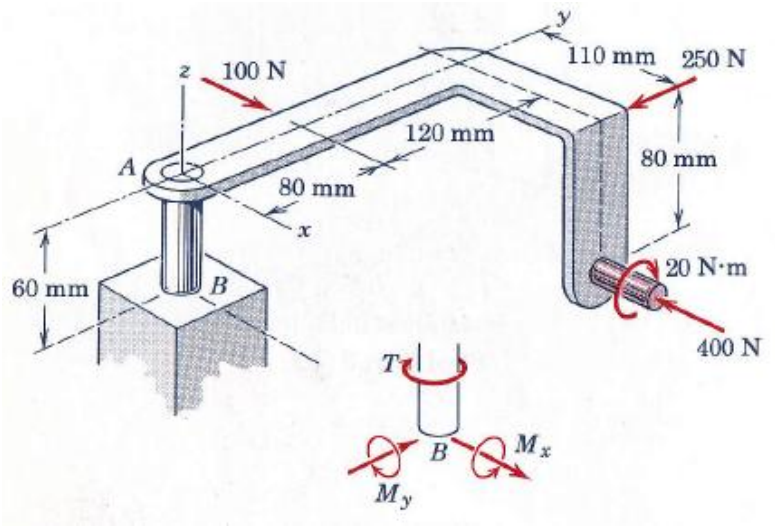
ITU Faculty of Aeronautics and Astronautics
2016-2017 Spring Semester
STA201E Statics

QUIZ – 1 SOLUTION

Problem: The bracket of negligible weight is welded to the shaft at *A*, and the shaft, in turn, is welded to the rigid support at *B*. Compute the torsional moment *T* (moment about the *z*-axis) and the bending moment *M* (moment about an axis normal to the shaft) at *B* as a result of the three forces and one couple applied to the bracket.



Solution:

$M_x = \sum M_{B,x} = (250)(0.06) - 20 \;\Rightarrow\; M_x = -5\ Nm$
$M_y = \sum M_{B,y} = (100)(0.06) + (400)(0.02) \;\Rightarrow\; M_y = 14\ Nm$

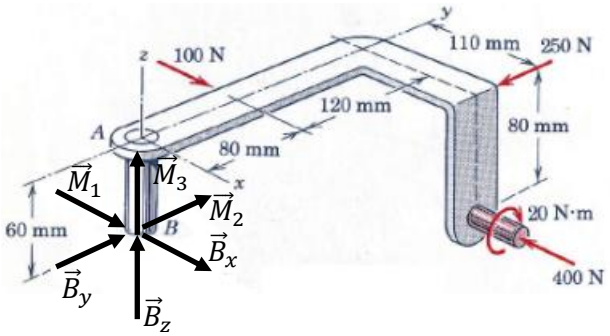
$T = \sum M_{B,z} = -(100)(0.08) - (250)(0.11) + (400)(0.2) \;\Rightarrow\; \boxed{T = 44.5\ Nm}$

$$\boxed{M = \sqrt{(M_x)^2 + (M_y)^2} = \sqrt{(-5)^2 + (14)^2} = 14.87\ Nm}$$

M_x, M_y, and T are the *x*, *y* and *z* components, respectively, of the total moment about *B* of the force system. The reaction at the fixed support (shaft is welded at point *B* therefore no movements are allowed in any direction, neither translation nor rotation) will be the opposite of these quantities.

The lines given below are not part of the solution, rather they're just for showing how the reactions at *B* can be found.

FBD (Free Body Diagram) of the bracket and shaft:



Equations of equilibrium:

$\sum F_x = 0\text{: } B_x + 100 - 400 = 0 \;\Rightarrow\; \boxed{B_x = 300\ N}$

$\sum F_y = 0\text{: } B_y - 250 = 0 \;\Rightarrow\; \boxed{B_y = 250\ N}$

$\sum F_z = 0\text{: } B_z = 0 \;\Rightarrow\; \boxed{B_z = 0}$

$\sum M_{B,x} = 0\text{: } (250)(0.06) - 20 + M_1 = 0 \;\Rightarrow\; \boxed{M_1 = 5\ Nm}$
$\sum M_{B,y} = 0\text{: } (100)(0.06) + (400)(0.02) + M_2 = 0 \;\Rightarrow\; \boxed{M_2 = -14\ Nm}$
$\sum M_{B,z} = 0\text{: } -(100)(0.08) - (250)(0.11) + (400)(0.2) + M_3 = 0 \;\Rightarrow\; \boxed{M_3 = -44.5\ Nm}$

Thus, in summary, T, M_x, and M_y are the components of the total moment about B, which is equivalent to the moment of the force system, whereas B_x, B_y, B_z, M_1, M_2, and M_3 are the reactions at the fixed support B. These reactions at B exist as a result of external loads (three forces and one couple) acting on the shaft and bracket system. The fixed support is capable of opposing six movements (translation and rotation in three directions), that may occur due to external loads. However, in the current case only 5 reaction exist at B, since total external force in z-direction is zero and therefore there's no need for a reaction in z-direction to eliminate the translational effect of forces in this direction.