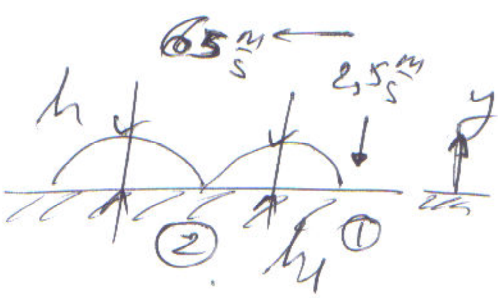
Problem 1: (a) $e = -\dfrac{v_{Af} - v_{Bf}}{v_A - v_B}$



A : plane

B : ground $\longrightarrow$ $v_G = v_{Gf} = 0$, $\underline{v_{x_p} = \text{const.}}$

1. bounce : $y \uparrow$ $\overset{v_p'}{\uparrow} \downarrow 2.5\ m/s - v_p$

① $\qquad -\dfrac{v_p'}{v_p} = e \longrightarrow -v_p' = (-2{,}5)e$

$v_p' = 2{,}5\, e$

2. bounce: $\overset{v_p''}{\uparrow} \downarrow v_p'$ $\uparrow y$

② $\qquad -\dfrac{v_p''}{v_p'} = e \rightarrow v_p'' = 2{,}5e^2$

$$h = \frac{v_p''^2}{2g} \qquad \left(mgh = \tfrac{1}{2} m v^2 \right)$$

$$h = \frac{(2.5e^2)^2}{2 \cdot 9{,}81} = 1.25 \cdot 10^{-3} \longrightarrow \underline{e = 0.25028 \cong 0{,}25}$$

(b) $h_1 = \dfrac{v_p'^2}{2 \cdot g} = \dfrac{(2.5 \cdot 0.25)^2}{2 \cdot 9{,}81} = \underline{0.0199 \cong 0.02\ m}$

(c) $\int \vec{F}\, dt = \Delta(m\vec{v}) = 90.000\left(2.5 \cdot 0.25 - (-2.5)\right)\vec{j}$

$= 281.250\ kgm/s = 2{,}8 \cdot 10^5\, \vec{j}\ Ns$

(d) $\int \vec{F}\, dt = \Delta(m\vec{v}) = 90000\left(2.5 \cdot 0.25^2 - (-2.5 \cdot 0{,}25)\right)$

$= 70312{,}5\ kgm/s = 70{,}3\ kNs\, \vec{j}$

(e) $T_0 = \frac{1}{2} m v_0^2 = \frac{1}{2} 90000 \left(65^2 + 2.5^2\right)$

$= 190406250 \ J \cong 190,4 \ MJ$

$T_1 = \frac{1}{2} m v_1^2 = \frac{1}{2} 90000 \left(65^2 + (2.5 \cdot 0.05)^2\right)$

$= 190142578. \ J \cong 190,14 \ MJ$

$T_2 = \frac{1}{2} m v_2^2 = \frac{1}{2} (90000) \left(65^2 + (2.5 \cdot 0.05^2)^2\right)$

$= 190126098 \ J = 190,12 \ MJ$

$\Delta T_{0 \to 2} = 190.12 - 190.4 = -0.28 \ MJ$

$\% n = -\frac{0.25}{190.4} \times 100 = \% \ 0.147$

$\Delta T_{1 \to 2} = 190.12 - 190.14 = -0.02 \ MJ$

$\% n = -\frac{0.02}{190.14} \times 100 = \% \ 0.0105$

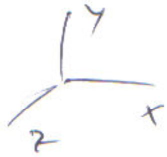
loss in y-direction:

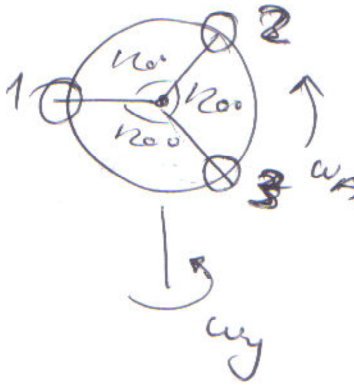
$n = \left[1 - \left(\frac{v_p''}{v_0}\right)^2\right] \times 100 = \left[1 - \left(\frac{2.5 e^2}{2.5}\right)^2\right] \times 100 =$

$n = \frac{v_0^2 - v_p''^2}{v_0^2} \times 100 = \left[1 - \left(\frac{v_p''}{v_0}\right)^2\right] \times 100 = \% \ 99.6$

loss

Problem 2: (a) $\omega_y = 1.5$ rpm





$= 1.5 \frac{2\pi}{60} = 0.157 \frac{rad}{s}$

$\omega_A = 2$ rpm $= 2 \cdot \frac{2\pi}{60} = 0.209$ rad/s

due to ω_A: $v_{1A} = v_{2A} = v_{3A} = \omega r_4$

$= 0.209 \cdot 6 = 1.254$ m/s

due to ω_y: $v_{1y} = 6 \cdot 0.157 = 0.942$ m/s $(+z)$

$v_{2y} = v_{3y} = (6 \sin 30°)\, 0.157 = 0.471$ m/s $(-z)$

$$T = \Sigma \frac{1}{2} m v^2 = \frac{1}{2} 500 \left[(1.254)^2 \cdot 3 + (0.942)^2 + 2 \cdot (0.471)^2 \right]$$

$= 1512{,}1$ J

(b) $\vec{H_0} = (3) \cdot (6)(500)(1.254) \vec{k}$

$+ (6)(500)(0.942) \vec{j}$

$+ 2(6)(\sin 30°)(500)(0.471) \vec{j}$

$= (11286 \vec{k} + 4239 \vec{j})$ kg m²/s

(c) $\Sigma \int_{t_1}^{t_2} M_0 \, dt = \vec{H_2} - \vec{H_1}$

$$\vec{H}_2 = (3)(6)(500)(6\omega_A)\vec{k}$$
$$+ (6)(500)(6\omega_y)\vec{j}$$
$$+ (2)(6)(\sin 30^\circ)(500)(6 \sin 30^\circ)\omega_y \vec{j}$$
$$= 54000\,\omega_A \vec{k} + 27000\,\omega_y \vec{j}$$

Angular impulse : $\sum \int_1^2 M_i\,dt = (-1600\vec{k} - 700\vec{j})\,0.03$
$$= -48\vec{k} - 21\vec{j}$$

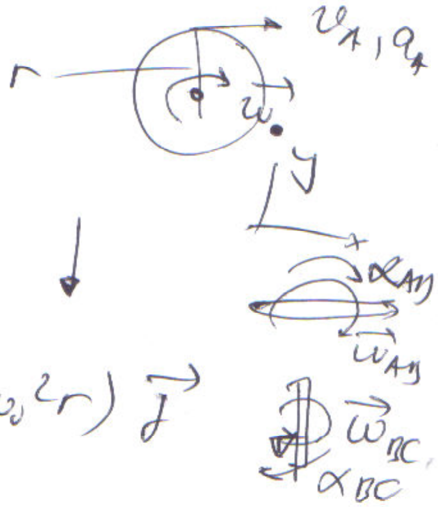
$$-48\vec{k} - 21\vec{j} = 54000\,\omega_A \vec{k} + 27000\,\omega_y \vec{j} - (11286\vec{k} + 4239\vec{j})$$

$$\rightarrow \omega_A = \frac{11286 - 48}{54000} = 0.208 \text{ rad/s}.$$

$$\omega_y = \frac{4239 - 21}{27000} = 0.156 \text{ rad/s}$$

Problem 3 : $v_A = \omega_0 r \longrightarrow$

$$\vec{v}_A = \omega_0 r \vec{i}$$



$$a_{At} = 0, \quad a_{An} = \frac{v_A^2}{r} = \frac{\omega_0^2 r^2}{r} = \omega_0^2 r \downarrow$$

$$\rightarrow a_A = \omega_0^2 r \downarrow \Longrightarrow \vec{a}_A = -(\omega_0^2 r)\vec{j}$$

$\vec{a}_C = 0 \ , \quad \vec{v}_C = 0 .$

$$\vec{v}_B = \vec{\omega}_{BC} \times \vec{r}_{CB} + \vec{v}_C = 0 = -\omega_{BC} \vec{k} \times (2r \vec{j}) = + 2r\omega_{BC} \vec{i}$$

$\vec{v}_A \qquad \vec{v}_B \quad \longrightarrow \quad \omega_{AB} = 0 .$

$$\vec{v}_B = \vec{v}_A + \vec{\omega}_{AB} \times \vec{r}_{AB} = \omega_0 r \vec{i} + 2r\omega_{AB} \vec{j}$$

$$\omega_{AB} = 0$$

$$\omega_0 r = 2r\omega_{BC}$$

$$\omega_{BC} = \frac{\omega_0}{2}$$

$$\vec{a}_B = a_A + \alpha_{AB} \times \vec{r}_{AB} + \underbrace{\vec{\omega}_{AB} \times (\vec{\omega}_{AB} \times \vec{r}_{AB})}_{0}$$

$$= -\omega_0^2 r \vec{j} - (\alpha_{AB} \vec{k}) \times 2r \vec{i}$$

$$= -\omega_0^2 r \vec{j} - \alpha_{AB} 2r \vec{j} = -(\omega_0^2 r + 2\alpha_{AB} r) \vec{j}$$

$$\vec{a}_B = \vec{a}_C + \vec{\alpha}_{BC} \times \vec{r}_{CB} + \vec{\omega}_{BC} \times (\vec{\omega}_{BC} \times \vec{r}_{CB})$$

$$= 0 + \alpha_{BC} \vec{k} \times 2r \vec{j} + \frac{\omega_0}{2} \vec{k} \times \left(-\frac{\omega_0}{2} \vec{k} \times 2r \vec{j}\right)$$

$$= + 2r\alpha_{BC} \vec{i} - \frac{\omega_0^2}{4} \cdot 2r \vec{j}$$

$$\alpha_{BC} = 0 \ , \quad \omega_0^2 r + 2\alpha_{AB} r = \frac{\omega_0^2 r}{2} \rightarrow \alpha_{AB} = -\frac{\omega_0^2}{4}$$