**ITU Faculty of Aeronautics and Astronautics** **06.11.2014**
**2014-2015 Fall Semester**
**10932-10933 DNK201E Dynamics**
**Midterm Exam 1**

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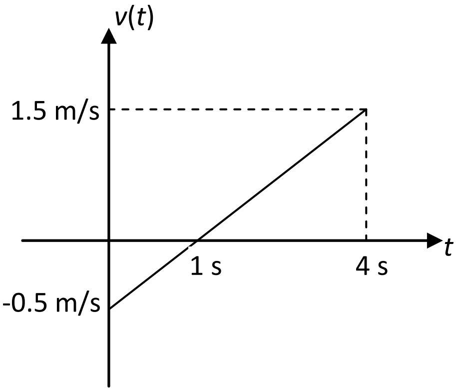
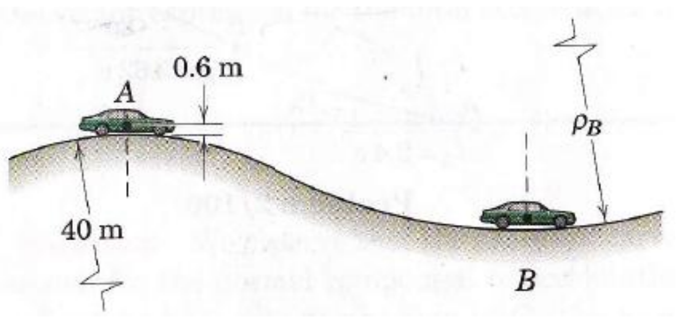
**Problem 1:(15 p)** A particle $P$ moves along a straight line and its velocity varies linearly with time as given in the figure below. At time $t = 0$ the position of the particle is $x_0 = 0$.

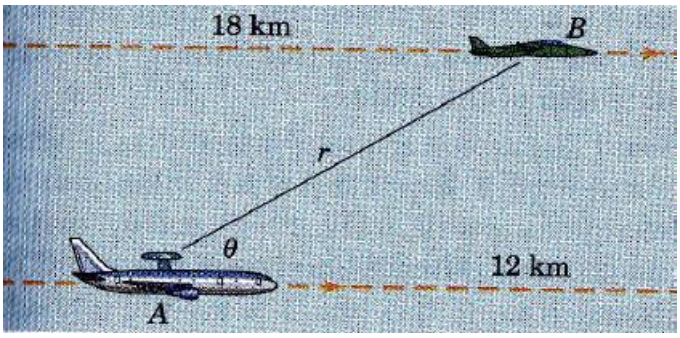
**(a)** Find the functional relationships $a = a(t)$ and $x = x(t)$.
**(b)** Draw the graphs *a*-*t* and *x*-*t* in the interval 0-4s.
**(c)** Calculate the total distance the particle has travelled in the interval 0-4s.



**Problem 2:** **(20 p)** The speed of a car increases uniformly with time from 50 $km/h$ at $A$ to 100 $km/h$ at $B$ during 10 seconds. The radius of curvature of the hump at $A$ is 40 $m$. If the magnitude of the total acceleration of the car's mass center is the same at $B$ as at $A$, compute the radius of curvature $\rho_B$ of the dip in the road at $B$. The mass center of the car is 0.6 $m$ from the road.

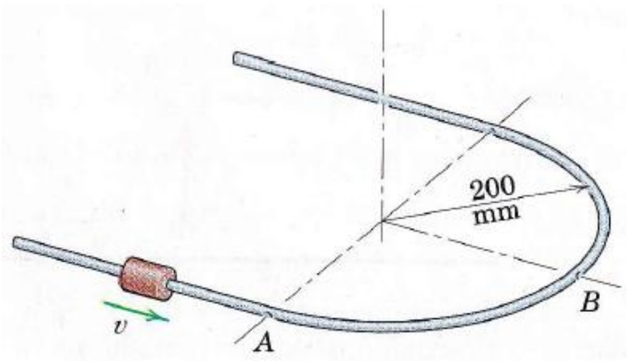


**Problem 3:** **(30 p)** The aircraft $A$ with radar detection equipment is flying horizontally at an altitude of 12 $km$ and is increasing its speed at the rate 1.2 $m/s$ each second. Its radar locks onto an aircraft flying in the same direction and in the same vertical plane at an altitude of 18 $km$. If $A$ has a speed of 1000 $km/h$ at the instant when $\theta = 30°$, determine the values of $\ddot{r}$ and $\ddot{\theta}$ at this same instant if $B$ has a constant speed of 1500 $km/h$.

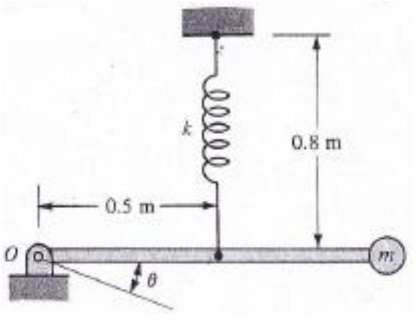


**Problem 4:** **(15 p)** The 120-g slider has a speed $v = 1.4$ $m/s$ as it passes point $A$ of the smooth guide, which lies in a horizontal plane. Determine the magnitude $R$ of the force which the guide exerts on the slider

**(a)** just before it passes point $A$ of the guide,
**(b)** as it passes point $B$.



**Problem 5:** **(20 p)** A 5-kg block is attached to a rigid bar of negligible mass which is pivoted at point $O$. The spring of stifness $k = 700$ $N/m$ is attached to the middle of the bar and is undeformed when the bar is released from rest in the horizontal position. Calculate the speed of the block at $\theta = 30°$.



10933 DNK201E DYNAMICS MIDTERM EXAM 1 - SOLUTION

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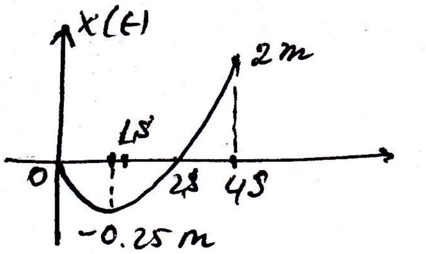
Problem 1: (a) $v(t) = \frac{1{,}5 - (-0.5)}{4-0} t - 0.5 = 0.5(t-1)$ m/s

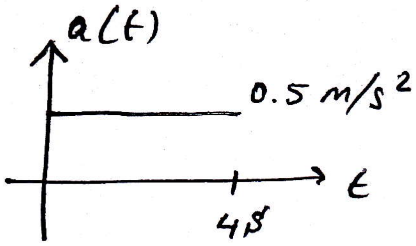
$a = \frac{dv}{dt} = 0.5$ m/s$^2$

$\frac{dx}{dt} = v \longrightarrow \int_{x_0=0}^{x} dx = \int_0^t v dt \Rightarrow x - 0 = \left[0.25t^2 - 0.5t\right]_0^t$

$x = x(t) = 0.25t^2 - 0.5t$

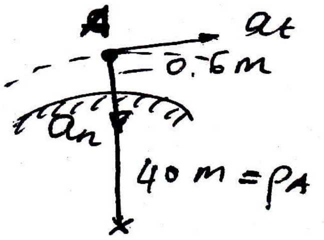
(b)

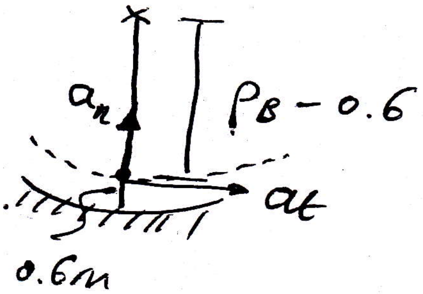




(c) distance travelled : 0.25 + 2.25 = 2.5 m

Problem 2:





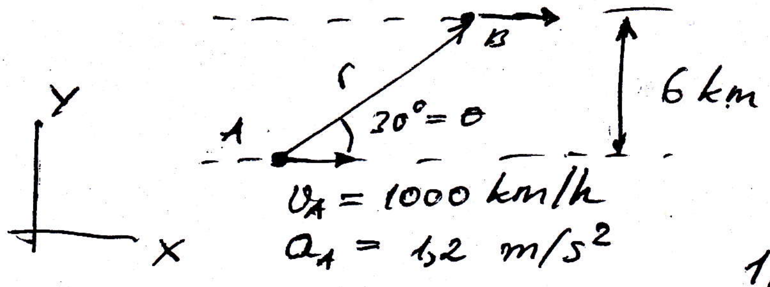
$a_t$ between points A and B: $a_t = \frac{(100-50)/3.6}{10}$
$= 1.38$ m/s$^2$

$a_A^2 = \cancel{a_{tA}^2} + a_{nA}^2 = a_B^2 = \cancel{a_{tB}^2} + a_{nB}^2$

$a_{nA} = a_{nB} \longrightarrow \frac{(50/3.6)^2}{40.6} = \frac{(100/3.6)^2}{\rho_B - 0.6}$

$\longrightarrow \rho_B = 163$ m

Problem 3: $v_B = 1500$ km/h = const, $a_B = 0$

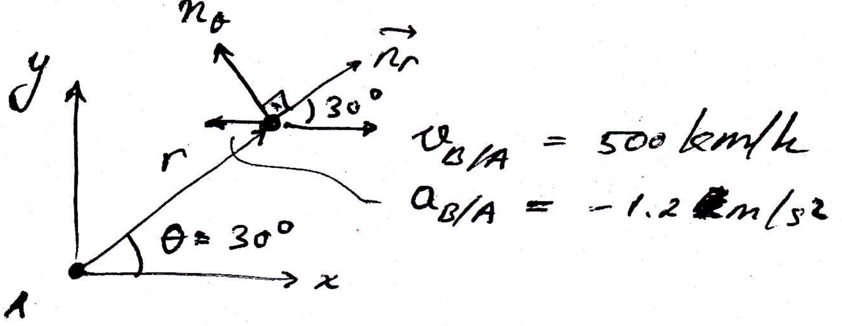


$r = \frac{6}{\sin 30°} = 12$ km

Relative motion of B with respect to A :

$$\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A = 0 - 1.2\vec{i} = -1.2\vec{i} \text{ m/s}^2$$

$$\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A = 1500\vec{i} - 1000\vec{i} = 500\vec{i} \text{ km/h}$$



$$v_r = \frac{500}{3.6}\cos 30° = 120.28 \text{ m/s}$$

$$v_\theta = -\frac{500}{3.6}\sin 30° = -69.44 \text{ m/s}$$

$$\dot{\theta} = \frac{v_\theta}{r} = -\frac{69.44}{12000}$$

$$a_r = -1.2\cos 30° = -1.039 \text{ m/s}^2$$

$$a_\theta = 1.2\sin 30° = 0.6 \text{ m/s}^2$$

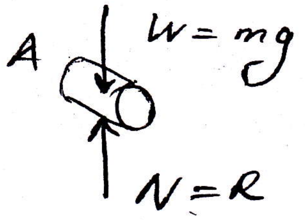
$$a_r = \ddot{r} - r\dot{\theta}^2 = -1.039$$

$$\rightarrow \ddot{r} = -1.039 + 12000\left(-\frac{69.44}{12000}\right)^2 = \underline{-0.637 \text{ m/s}^2}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0.6$$

$$\ddot{\theta} = \frac{0.6 - 2\cdot \cancel{\frac{500}{3.6}}\, 120.28\cdot\left(-\frac{69.44}{12000}\right)}{12000} = \underline{1.66\cdot 10^{-4} \text{ rad/s}^2}$$

Problem 4: (a)

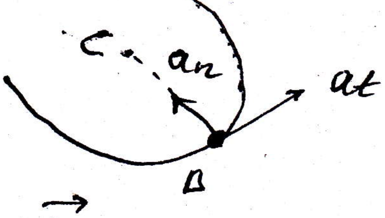


$$\rightarrow \Sigma F_y = ma_y = 0$$

$$N - W = 0 \rightarrow N = W = R$$

$$R = mg = 0{,}120 \cdot 9.81 = 1.1772 \text{ N}$$

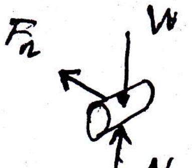
(b)



conservation of mechanical energy:

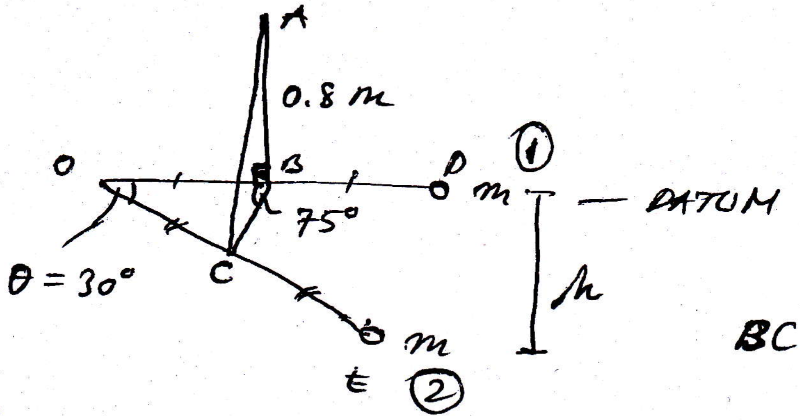
$$v_B = v_A = 1.4 \text{ m/s}$$

$$a_t = 0 \text{ , } a_{n_B} = \frac{v_B^2}{\rho_B} = \frac{1.4^2}{0.2} = 9.8 \text{ m/s}^2$$



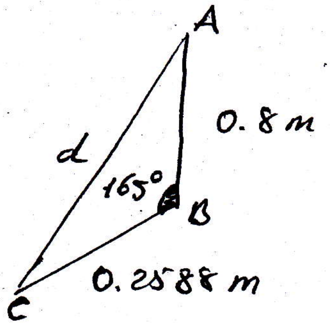
$$\left.\begin{array}{l} F_n = ma_n = 0.12 \cdot 9.8 = 1.176 \text{ N} \\ N = W = 1{,}1772 \text{ N} \end{array}\right\} R = \sqrt{F_n^2 + N^2} \cong 1.664 \text{ N}$$

Problem 5:



OB = BD = OC = CE = 0.5 m

$BC = 2 \cdot 0.5 \cos 75°$
$\cong 0.2588$ m



cosine theorem:

$$d^2 = (0.2588)^2 + (0.8)^2 - (2)(0.8)(0.2588)\cos 165°$$

$$d \cong 1.052 \text{ m}$$

spring deformation $\delta = 1.052 - 0.8 = 0.252$ m

No non-conservative forces present

→ conservation of mechanical energy [illegible]

$$\cancelto{0}{T_1} + \cancelto{0}{V_{1g}} + \cancelto{0}{V_{1,s}} = T_2 + V_{2g} + V_{2,s}$$

$$\frac{1}{2} m v_2^2 - mgh + \frac{1}{2} k \delta^2 = 0$$

$$\rightarrow \frac{1}{2}(5) v_2^2 - (5)(9.81)(1)(\sin 30°) + \frac{1}{2} \cdot 700 \cdot 0.252^2 = 0$$

$v_2 \cong$ [illegible] 0.9588 m/s