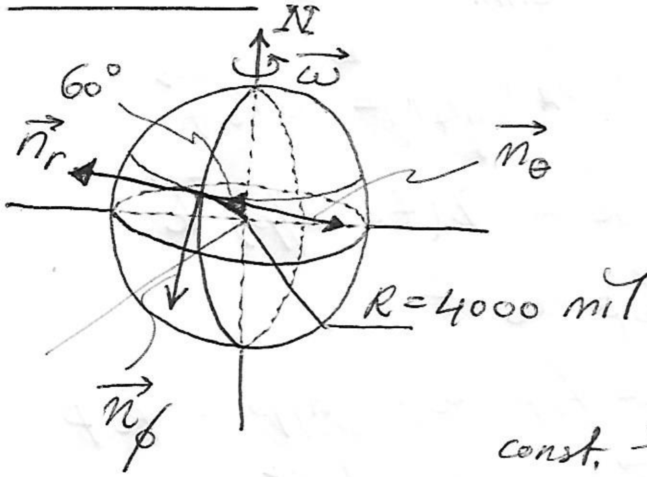
12366 DNK201E DYNAMICS

2016-2017 FALL TERM

ITU FACULTY OF AERONAUTICS & ASTRONAUTICS

# QUIZ-5 SOLUTION

Problem:




$R = 4000$ mil, $\phi = 60°$

$\dot{R} = \ddot{R} = 0$

$v_\phi = 5$ mph $=$ const

$= R\dot{\phi}$

$$\underset{\text{const.}}{\dot{\phi}} = \frac{v_\phi}{R} = \frac{5}{4000\cdot 3600} = 3.47\bar{2}\cdot 10^{-7}\ \frac{\text{rad}}{s}$$

$$\vec{\omega} = \omega \vec{\lambda}_N \rightarrow \omega = \underset{\text{const.}}{\dot{\theta}} = \frac{2\pi}{1\text{ day}} = \frac{2\pi}{24\cdot 3600} \cong 7.27\cdot 10^{-5}\ \frac{\text{rad}}{s}$$

$$a_r = \cancel{\ddot{r}} - r\dot{\phi}^2 - r\dot{\theta}^2\sin^2\phi$$

$$= -4000\cdot 5280\left[(3.47\cdot 10^{-7})^2 + (7.27\cdot 10^{-5})^2\right]$$

$$= -0.0837\ ft/s^2$$

$$a_\phi = r\cancel{\ddot{\phi}} + 2\cancel{\dot{r}}\dot{\phi} - r\dot{\theta}^2\sin\phi\cos\phi$$

$$= -4000\cdot 5280\,(7.27\cdot 10^{-5})^2\sin 60°\cos 60°$$

$$= -0.0483\ ft/s^2$$

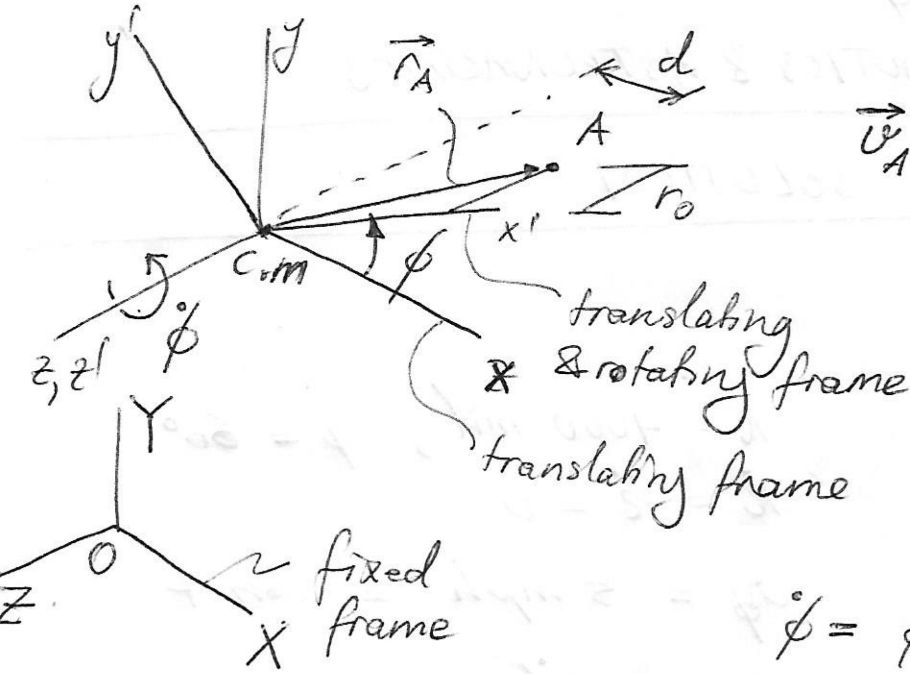
$$a_\theta = r\cancel{\ddot{\theta}} + 2\cancel{\dot{r}}\dot{\theta}\sin\phi + 2r\dot{\phi}\dot{\theta}\cos\phi$$

$$= 2\cdot 4000\cdot 5280\cdot(3.47\cdot 10^{-7})(7.27\cdot 10^{-5})\cos 60°$$

$$= 5.32\cdot 10^{-4}\ ft/s^2$$

$$\boxed{\vec{a} = -0.0837\,\vec{n}_r - 0.0483\,\vec{n}_\phi + 5.32\cdot 10^{-4}\,\vec{n}_\theta\ \ ft/s^2}$$

Problem 2: (a) $\phi(t=2\pi/p) = \phi_0 \sin p \cdot 2\pi/p = 0$




$$\vec{v}_A = \vec{v}_{c.m.} + (\vec{v}_A)_{x'y'z'} + \vec{\Omega} \times \vec{r}_A$$

$$\vec{v}_{c.m.} = v\vec{i}$$

$$(\vec{v}_A)_{x'y'z'} = \omega r_0 \vec{j}$$

$$\vec{\Omega} = \dot{\phi}(t=2\pi/p)\vec{k}$$

$$\dot{\phi} = \phi_0 p \cos pt$$

$$\rightarrow \dot{\phi}(t=2\pi/p) = \phi_0 p \cos p \cdot 2\pi/p = \phi_0 p$$

$$\rightarrow \vec{\Omega} \times \vec{r}_A = \phi_0 p \vec{k} \times (d\vec{i} + r_0\vec{k}) = \phi_0 p d\vec{j}$$

$$\Rightarrow \boxed{\vec{v}_A = v\vec{i} + (\omega r_0 + \phi_0 p d)\vec{j}}$$

(b) $\vec{a}_A = \vec{a}_{c.m.} + (\vec{a}_A)_{x'y'z'} + \dot{\vec{\Omega}} \times \vec{r}_A + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}_A) + 2\vec{\Omega} \times (\vec{v}_A)_{x'y'z'}$

$\vec{a}_{c.m} = a\vec{i}$, $(\vec{a}_A)_{x'y'z'} = \omega^2 r_0 \vec{k}$

$\dot{\vec{\Omega}} = \ddot{\phi}\vec{k}$, $\ddot{\phi} = -\phi_0 p^2 \sin pt$

$$\ddot{\phi}(t=2\pi/p) = -\phi_0 p^2 \sin p \cdot 2\pi/p = 0$$

$$\rightarrow \dot{\vec{\Omega}}(t=2\pi/p) = 0$$

$$\vec{\Omega} \times (\vec{\Omega} \times \vec{r}_A) = \phi_0 p \vec{k} \times (\phi_0 p d \vec{j}) = -\phi_0^2 p^2 d\vec{i}$$

$$2\vec{\Omega} \times (\vec{v}_A)_{x'y'z'} = (2\phi_0 p\vec{k}) \times (\omega r_0 \vec{j}) = -2\phi_0 p \omega r_0 \vec{i}$$

$$\rightarrow \boxed{\vec{a}_A = (a - \phi_0^2 p^2 d - 2\phi_0 p \omega r_0)\vec{i} + \omega^2 r_0 \vec{k}}$$

Unit vectors in all frames are equal at given instant.