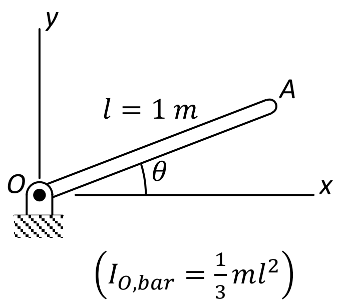
Istanbul Technical University Faculty of Aeronautics and Astronautics **11.12.2017**
2017-2018 Fall Semester
11729 DNK201E Dynamics

Quiz – 4 SOLUTION

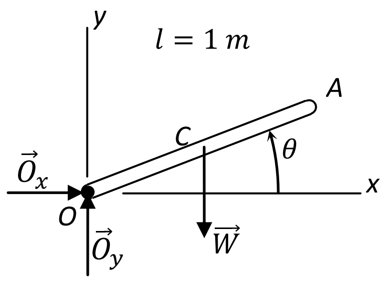
Problem: A uniform 4-kg bar is released from rest at $\theta = 60°$. If the hinge at *O* is frictionless, find the angular velocity of the bar when $\theta = 0$,

(a) using equations of motion, and
(b) using an energy method.



Solution:

(a) FBD of bar is given in the figure below. Forces working on the bar are the reactions at the point O and weight at the center of mass. The motion of the bar will be pure rotation about the origin and the corresponding angular acceleration can be obtained from moment about that point:



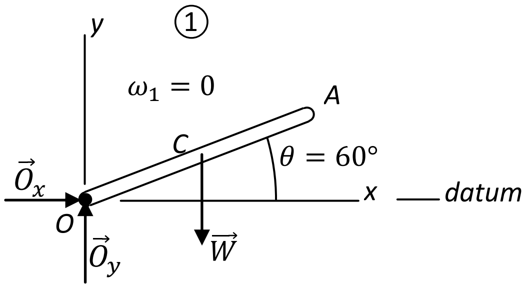
$$\sum M_O = I_O\alpha: \quad -W(l/2)\cos\theta = \frac{1}{3}ml^2\alpha \quad \rightarrow \quad \alpha = -\frac{3}{2}(g/l)\cos\theta$$

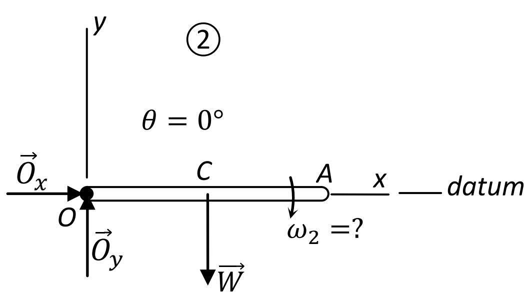
Now the kinematic expression $\alpha d\theta = \omega d\omega$ can be used in order to find the angular velocity of the bar at position $\theta = 0$:

$$\int_{\omega_1=0}^{\omega_2} \omega d\omega = -\int_{\theta=60°}^{\theta=0} \frac{3}{2}\left(\frac{g}{l}\right)\cos\theta\, d\theta \quad \rightarrow \frac{\omega^2}{2} = -\frac{3}{2}\left(\frac{g}{l}\right)[\sin\theta]_{\theta=60°}^{\theta=0}$$

$$\rightarrow \omega = \sqrt{3\left(\frac{g}{l}\right)\sin 60°} = \sqrt{3\left(\frac{9.81}{1}\right)\sin 60°} = 5.048\, rad/s$$

(b) Since the hinge at *O* is frictionless total mechanical energy is conserved during the motion (reactions at the origin do no work):





$$T_1 + V_1 = T_2 + V_2 \quad \rightarrow \quad 0 + mgh_1 = \frac{1}{2}I_O\omega_2^2 + 0 \quad \rightarrow mg(l/2)\sin\theta = \frac{1}{2}\frac{1}{3}ml^2\omega_2^2$$

$$\rightarrow \omega_2^2 = 3\frac{g}{l}\sin\theta \rightarrow \omega_2 = \sqrt{3\left(\frac{9.81}{1}\right)\sin 60°} = 5.048\, rad/s$$