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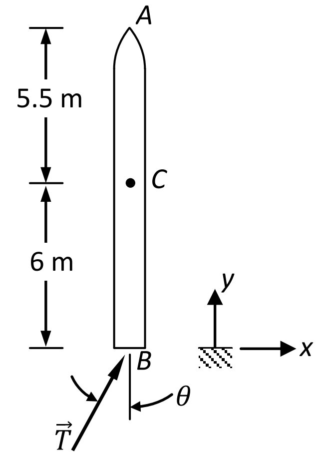
11729 DNK201E Dynamics

Quiz – 3 SOLUTION

Problem: The mass of a rocket is $6000\ kg$, and its radius of gyration about the mass center *C* is $3.5\ m$.

(a) Determine the thrust *T* that can cause an angular acceleration of $0.1\ rad/s^2$ when applied at $\theta = 5°$.
(b) Calculate the absolute acceleration of point *A* by this thrust.
(c) Find the point on the rocket where the magnitude of absolute acceleration is maximum. (Neglect thickness of the rocket.)
(d) Find that point on the rocket where the horizontal component of the absolute acceleration vector is equal to zero. (Neglect thickness of the rocket.)
(e) Is the point found in (d) the instantaneous center of rotation? Explain.



Solution:

(a) $r_g = 3.5\ m,\ \alpha = 0.1\ rad/s^2$

Eqs. of motion:

$$\sum F_x = ma_x:\quad T\sin 5° = 6000 a_{C,x} \qquad (1)$$
$$\sum F_y = ma_y:\quad T\cos 5° - 6000g = 6000 a_{C,y} \qquad (2)$$
$$\sum M_C = I_C\alpha:\quad (T\sin 5°)(6) = (3.5^2)(6000)(0.1) \quad \rightarrow \quad \boxed{T = 14055\ N}$$

(b) The acceleration of a point which is not the center of gravity can be found using kinematic relations. For this first the acceleration must be calculated, which is done by solving (1) and (2) for the acceleration components:

$$\left.\begin{aligned} a_{C,x} &= \frac{14055\sin 5°}{6000} = 0.204\frac{m}{s^2} \\ a_{C,y} &= \frac{14055\cos 5° - 6000g}{6000} = -7.48\frac{m}{s^2} \end{aligned}\right\} \rightarrow \boxed{a_C = 0.204\vec{\imath} - 7.48\vec{\jmath}\ m/s^2}$$

The relation between accelerations of points *C* and *A* is as follows:

$$\vec{a}_A = \vec{a}_C + \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r}) = 0.204\vec{\imath} - 7.48\vec{\jmath} + 0.1\vec{k} \times 5.5\vec{\jmath} = -0.346\vec{\imath} - 7.48\vec{\jmath}\ m/s^2$$

(c) The point *P* on the rocket where the magnitude of absolute acceleration will be maximum must be that point where absolute value of the *x*-component of the absolute acceleration is maximum as the *y*-component is the same for all points on the rocket:

$$\vec{a}_P = 0.204\vec{\imath} - 7.48\vec{\jmath} + 0.1\vec{k} \times (y - 6)\vec{\jmath} = (0.204 - (0.1y - 0.6))\vec{\imath} - 7.48\vec{\jmath}\ m/s^2$$
$$\rightarrow \quad a_{P,x} = 0.804 - 0.1y \qquad (3)$$

The *x*-component is equal to zero at position $= 8.04\ m$. For greater values of y it will become negative, for smaller values it will have a positive value. Its obvious that the point where $|a_{P,x}|$ is maximum must be the point which is the farthest from $y = 8.04\ m$. Indeed, this is point *B*:

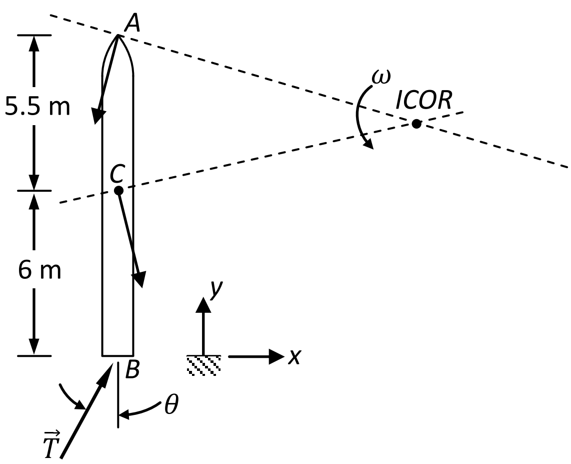
$$\vec{a}_B = \big(0.804 - (0.1)(0)\big)\vec{\imath} - 7.48\vec{\jmath} = 0.804\vec{\imath} - 7.48\vec{\jmath}\ \ m/s^2 \quad \rightarrow \quad |a_B| = 7.52\ m/s^2$$

(d) This was already found in (c), namely the point $y = 8.04\ m$.

(e) There is no information about angular and linear velocities at given instant. Thus, it is not possible to locate the instantaneous center of rotation (instantaneous center of zero velocity). Alternatively, if it assumed that the rocket is motionless initially, which could be the case because the sole force that causes a 0.1 rad/s^2 angular acceleration is the thrust force and aerodynamic effects are not taken into account, then it is still not possible to find the instantaneous center of rotation due to the fact that all points on the rocket have zero velocity. However, the following reasoning will give an idea of where the instantaneous center of rotation will be after motion develops: The thrust force $\vec{T}$ will bring about both translational and rotational motion. Immediately after implementation of the thrust force all parts of the rocket will gain momentum and it may be assumed that the velocity vectors are proportional to the acceleration vectors at this very beginning instant, actually, immediately after that instant. Following the expression

$$\frac{dv_y}{dv_x} = \frac{a_y dt}{a_x dt} = \frac{a_y}{a_x}$$

directions for the velocity vectors can be constructed as shown in the figure below. The intersection of the lines perpendicular to the velocity vectors will then show close to where the ICOR might be floating after time t=0.



Had the *y*-component of acceleration been equal to zero then, indeed, the location $y = 8.04\ m$ would have been found as a position in the vicinity of the instantaneous center of rotation right after the instant t=0. Since this is not the case it must be somewhere to the right of that point. Analytically, the exact location can be determined by finding the intersection of the radii as is shown schematically in the figure above or simply by finding the point where total acceleration is equal to zero:

$$\begin{aligned}\vec{a}_{xy} &= \vec{a}_C + \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r}) = \vec{a}_C + \vec{\alpha} \times (\vec{r}_{xy} - \vec{r}_C)\\ &= 0.204\vec{\imath} - 7.48\vec{\jmath} + 0.1\vec{k} \times (x\vec{\imath} + y\vec{\jmath} - 6\vec{\jmath})\\ &= (0.804 - 0.1y)\vec{\imath} + (0.1x - 7.48)\vec{\jmath} \quad m/s^2\end{aligned}$$

$$\vec{a}_{xy} = 0: \ \ x = 74.8\ m,\ \ y = 8.04\ m \quad \rightarrow \quad \lim_{t\to 0^+} ICOR = Q(74.8, 8.04)$$

Note that the point $Q(74.8,\ 8.04)$ is the point of zero acceleration at the moment motion is impending, thus a discontinuous point on the space centrode of the rocket because the instantaneous center of rotation cannot be defined in a zero velocity field (singularity).