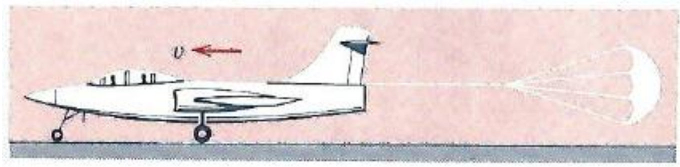
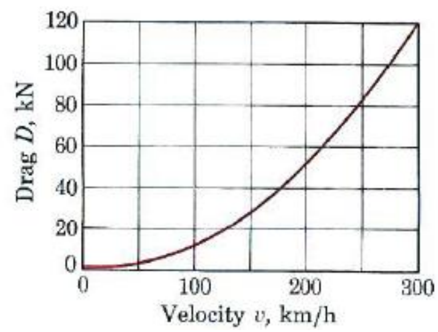
Istanbul Technical University Faculty of Aeronautics and Astronautics **23.10.2017**
2017-2018 Fall Semester
11729 DNK201E Dynamics

Quiz – 2 SOLUTION

Problem: A jet airplane with a mass of 5 Mg has a touchdown speed of 300 km/h, at which instant the braking parachute is deployed and the power shut off. If the total drag on the aircraft varies with velocity as shown in the accompanying graph, calculate the distance x along the runway and time required to reduce the speed to 150 km/h. Approximate the variation of drag by an equation of the form $D = kv^2$, where k is a constant.





Solution:

$\sum F_x = ma_x : -D = ma_x \quad \rightarrow a_x = -\frac{k}{m}v^2$

Value of the constant k is found from the $D - v$ graph:

For $v = 300\ km/h \rightarrow D = 120\ kN$: $k\left(\frac{300}{3.6}\right)^2 = 120 \cdot 10^3 \quad \rightarrow \quad k = 17.28\ \ kg/m$

Distance and time can be found exactly by integration as shown below:

$$a_x dx = vdv \ \rightarrow \ -\frac{k}{m}\int_0^x dx = \int_{v_0}^{v_1}\frac{dv}{v} \quad \rightarrow \ -\frac{k}{m}x = \ln\frac{v_1}{v_0}$$

$$\Rightarrow x = \frac{m}{k}\ln\frac{v_0}{v_1} = \frac{5000}{17.28}\ln\frac{300}{150} = 200.56\, m$$

$$v(x) = v_0 \exp\left(-\frac{kx}{m}\right) = \frac{dx}{dt} \quad \rightarrow v_0\int_0^t dt = \int_0^x e^{kx/m}dx \quad \rightarrow v_0 t = \frac{m}{k}\left[e^{kx/m} - 1\right]$$

$$\Rightarrow \quad t = \frac{5000}{(17.28)(300/3.6)}[2-1] = 3.47\, s$$

Alternative solution:

$$a = \frac{dv}{dt} \quad \rightarrow \quad -\frac{k}{m}v^2 = \frac{dv}{dt} \quad \rightarrow \quad -\frac{k}{m}\int_0^t dt = \int_{v_0}^{v_1} \frac{dv}{v^2} \rightarrow -\frac{k}{m}t = \frac{1}{v_0} - \frac{1}{v_1}$$

$$\Rightarrow \quad t = \frac{5000}{17.28}\left[\frac{1}{(150/3.6)} - \frac{1}{(300/3.6)}\right] = 3.47\bar{2}\ s$$

$$v(t) = \frac{1}{\frac{1}{v_0} + \frac{k}{m}t} = \frac{dx}{dt} \rightarrow \int_0^x dx = \int_0^t \frac{dt}{\frac{1}{v_0} + \frac{k}{m}t} \quad \rightarrow \quad x(t) = \frac{m}{k}\ln\left[1 + \frac{kv_0}{m}t\right]$$

$$\Rightarrow x(3.47\bar{2}\ s) = \frac{5000}{17.28}\ln\left[1 + \frac{(17.28)\left(\frac{300}{3.6}\right)}{5000}(3.472)\right] = \frac{5000}{17.28}\ln 2 = 200.56\ m$$

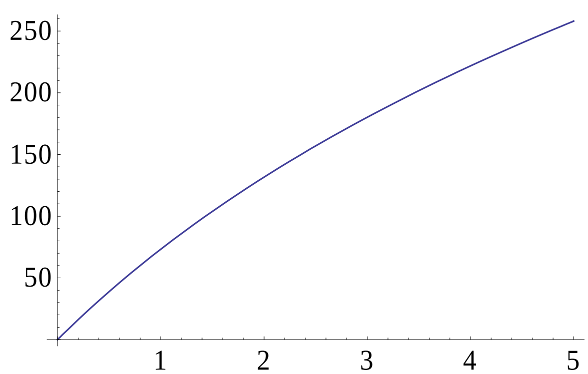
Although not asked series expansions and graphs of position, velocity and acceleration are given below:

$$x(t) = \frac{m}{k}\ln\left[1 + \frac{kv_0}{m}t\right] = \frac{m}{k}\left[\frac{kv_0}{m}t - \frac{1}{2}\left(\frac{kv_0}{m}t\right)^2 + \frac{1}{3}\left(\frac{kv_0}{m}t\right)^3 - \frac{1}{4}\left(\frac{kv_0}{m}t\right)^4 + \cdots\right]$$

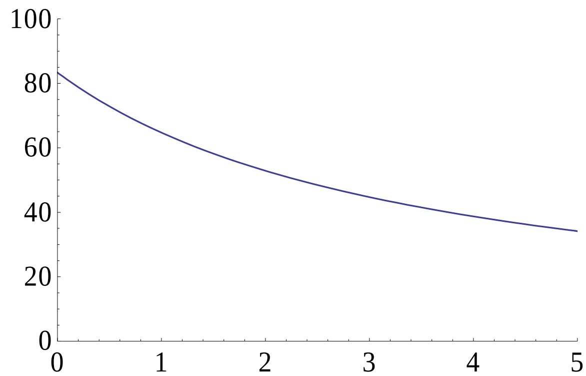
$$v(t) = \frac{1}{\frac{1}{v_0} + \frac{k}{m}t} = \frac{v_0}{1 + \frac{kv_0}{m}t} = v_0\left[1 - \frac{kv_0}{m}t + \left(\frac{kv_0}{m}t\right)^2 - \left(\frac{kv_0}{m}t\right)^3 + \cdots\right]$$

$$a(t) = -\frac{k/m}{\left(\frac{1}{v_0} + \frac{k}{m}t\right)^2} = -\frac{kv_0^2/m}{\left(1 + \frac{kv_0}{m}t\right)^2} = -\frac{kv_0^2}{m}\left[1 - 2\frac{kv_0}{m}t + 3\left(\frac{kv_0}{m}t\right)^2 - 4\left(\frac{kv_0}{m}t\right)^3 + \cdots\right]$$

<u>x-t graph:</u>



v-t graph:



a-t graph: