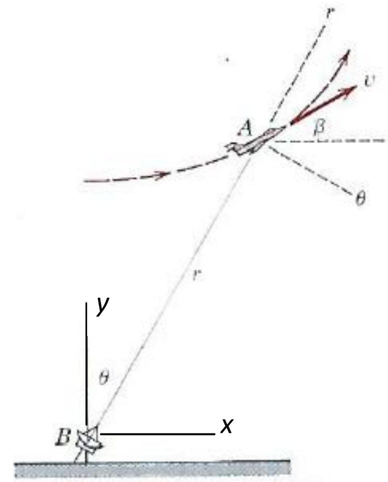
Istanbul Technical University Faculty of Aeronautics and Astronautics **16.10.2017**
2017-2018 Fall Semester
11729 DNK201E Dynamics

Quiz – 1 SOLUTION

Problem: During a portion of a vertical loop, an airplane flies in an arc of radius $\rho = 600\ m$ with a constant speed $v = 400\ km/h$. When the airplane is at A, the angle made by $\vec{v}$ with the horizontal is $\beta = 30°$, and radar tracking gives $r = 800\ m$ and $\theta = 30°$.



(a) Calculate v_r, v_θ, a_r, and $\ddot{\theta}$ for this instant.

Ans. $v_r = 96.2\ m/s$, $v_\theta = 55.6\ m/s$
$a_r = 10.29\ m/s^2$, $\ddot{\theta} = -0.0390\ rad/s^2$

(b) Find the position vector of the airplane at point A in the xy-frame.

(c) Calculate the magnitudes of velocity and acceleration vectors at the point A.

If possible,

(d) find v_x, v_y, a_x, and a_y;

(e) find the components of the velocity and acceleration vectors in spherical coordinates; (r and θ are given in the figure, $\phi =$?)

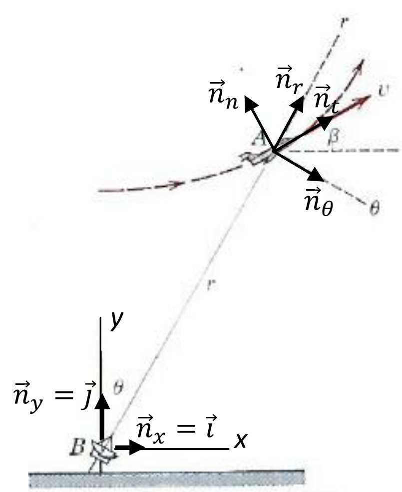
(f) find the tangential and normal components of the resultant acting on the airplane at the point A, if the mass of the airplane is $m = 10.000\ kg$;

(g) find the rate of work done by the resultant at the point A;

(h) find the rate of work done by weight of the airplane at the point A.

If you think it is not possible to find exact values of the quantities given above then explain why.

Solution:



(a) $v_r = \frac{400}{3.6}\cos 30 = 96.2\ m/s$

$v_\theta = \frac{400}{3.6}\sin 30 = 55.6\ m/s$

Acceleration of airplane:

$$\vec{a} = a_n\,\vec{n}_n + a_t\vec{n}_t = \frac{v^2}{\rho}\,\vec{n}_n + \dot{v}\vec{n}_t = \frac{\left(\frac{400}{3.6}\right)^2}{600}\,\vec{n}_n + 0\vec{n}_t$$
$$\Rightarrow \vec{a} = 20.58\,\vec{n}_n \quad m/s^2$$

$$a_r = (20.58)\cos 60 = 10.28\, m/s^2$$
$$a_\theta = -(20.58)\sin 60 = -17.82\frac{m}{s^2} = r\ddot{\theta} + 2\dot{r}\dot{\theta} = r\ddot{\theta} + 2v_r\frac{v_\theta}{r}$$
$$\ddot{\theta} = \frac{\left[-17.82 - \frac{2(96.2)(55.6)}{800}\right]}{800} = -0.0390\, rad/s^2$$

(b) $\vec{r} = 800(\sin 30\ \vec{n}_x + \cos 30\,\vec{n}_y) = 400\vec{\imath} + 692.8\vec{\jmath}\ m$

(c) $|\vec{a}| = |a_n\,\vec{n}_n + a_t\vec{n}_t| = |20.58\,\vec{n}_n\ \ | = 20.58\quad m/s^2$

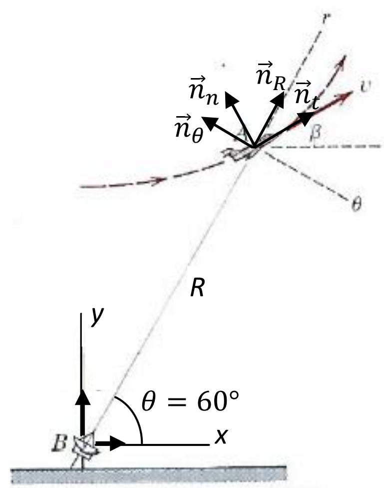
(d) $v_x = \frac{400}{3.6}\cos 30 = 96.2\frac{m}{s},\ \ v_y = \frac{400}{3.6}\sin 30 = 55.6\frac{m}{s}$

$a_x = -(20.58)\sin 30 = -10.28\, m/s^2,\ a_y = -(20.58)\cos 30 = 17.82\frac{m}{s^2}$

(e) $\phi = 90°$ (angle measured from *z*-axis), $\theta = 60°$ (angle of projection of *R* on *Oxy* plane with the *x*-axis; here, *R* lies in the *Oxy* plane)

$$v_R = 96.2\, m/s$$
$$v_\theta = -55.6\, m/s$$
$$v_\phi = 0$$

$$a_R = (20.58)\cos 60 = 10.28\, m/s^2$$
$$a_\theta = +(20.58)\sin 60 = +17.82\frac{m}{s^2}$$
$$a_\phi = 0$$



(f) $\vec{F} = m\vec{a} = m(a_n\,\vec{n}_n + a_t\vec{n}_t) = ma_n\,\vec{n}_n = (10000)(20.58)\,\vec{n}_n = (205.8\, kN)\ \vec{n}_n$

(g) $P_F = \vec{F}\cdot\vec{v} = (F_n\,\vec{n}_n)\cdot(v_n\,\vec{n}_t) = 0$

(h) $P_W = \vec{W}\cdot\vec{v} = [(-W\cos 30)\,\vec{n}_n - (W\sin 30)\ \vec{n}_t]\cdot(v_n\,\vec{n}_t)$
$= -(10000)(9.82\,)(\sin 30)(400/3.6) = -5.45\, MW$

Negative work is done by gravity at point A. In (g) it was found that the rate of work done by resultant is zero. This means that forces acting on the airplane other than gravity do work at a rate of +5.45 MW, which makes the sum zero. These forces are the aerodynamic forces L and D (lift and drag) and T (thrust force produced by the engines of the aircraft. Lift force as part of the total aerodynamic force can be calculated from the normal component of the equation of motion. Also the sum of thrust and drag forces can be found from the tangential component of equation of motion, but not separately. For this, aerodynamic specifications of the aircraft need to be known.