ITU Faculty of Aeronautics and Astronautics Aeronautical Engineering Department
2017-2018 Fall Semester
11729 DNK201E Dynamics

Sample Problems – 1

Problem 1: The acceleration of a projectile shot in the vertical direction with a velocity v_0 is given as $a = -(9.81 + 0.02v^2)\ m/s^2$. If the projectile reaches maximum altitude in 3.5 s calculate,

(a) initial velocity (v_0),
(b) maximum altitude of projectile,
(c) velocity of projectile at t = 2 s.

Solution 1:

(a) $a = dv/dt \quad \Rightarrow \quad -(9.81 + 0.02v^2) = dv/dt$

$$\frac{dv}{9.81 + 0.02v^2} = -dt \quad \Rightarrow \quad \int_{v_0}^{v(y)} \frac{dv}{9.81 + 0.02v^2} = -\int_0^t dt$$

$$\Rightarrow \quad \int_{v_0}^{v(y)} \frac{dv}{9.81 + 0.02v^2} = -\int_0^t dt = -t$$

$$\Rightarrow \quad \frac{1}{\sqrt{0.02}} \int_{v_0}^{v(y)} \frac{d\left(\sqrt{0.02}v\right)}{\left(\sqrt{9.81}\right)^2 + \left(\sqrt{0.02}v\right)^2} = -t$$

$$\left[\frac{1}{\sqrt{9.81}} \arctan \frac{\sqrt{0.02}v}{\sqrt{9.81}}\right]_{v(0)}^{v(y)} = -\sqrt{0.02}t \qquad \Rightarrow \arctan\frac{\sqrt{0.02}}{\sqrt{9.81}}v(y) - \arctan\frac{\sqrt{0.02}}{\sqrt{9.81}}v(0) = -\sqrt{9.81}\sqrt{0.02}t$$

Maximum height when t=3.5, $v(h_{max})$=0:

$$\Rightarrow \frac{\sqrt{0.02}}{\sqrt{9.81}}v(0) = \tan\left[\sqrt{0.02}\sqrt{9.81}(3.5)\right] \quad \boxed{\Rightarrow \quad v(0) = 1080.7\ m/s}$$

(b) $ady = vdv \quad \Rightarrow -(9.81 + 0.02v^2)dy = vdv$

$$\Rightarrow -\int_0^y dy = \int_{v_0}^{v(y)} \frac{vdv}{9.81 + 0.02v^2} \quad \Rightarrow \quad -y = \frac{1}{0.04}\ln\left[\frac{9.81 + 0.02v^2}{9.81 + 0.02v_0^2}\right]$$

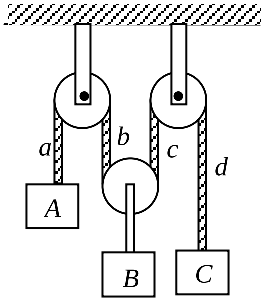
At maximum height v=0:

$$h_{max} = -\frac{1}{0.04}\ln\left[\frac{9.81}{9.81 + 0.02v_0^2}\right] \quad \Rightarrow \quad h_{max} = 194,4\ \ m$$

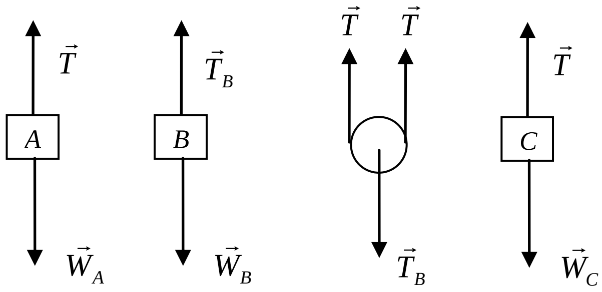
(c) $\Rightarrow \arctan\frac{\sqrt{0.02}}{\sqrt{9.81}}v(2s) - \arctan\frac{\sqrt{0.02}}{\sqrt{9.81}}1080.7 = -\sqrt{9.81}\sqrt{0.02}(2)$

$$\Rightarrow \ v(2s) = 17.35\ m/s \ \boxed{\Rightarrow a(2s) = -(9.81 + 0.02 * 17.35^2) = -15.83\ m/s^2}$$

<u>Problem 2:</u> In the pulley system below m_A=2 kg, m_B=6 kg ve m_C=3 kg. The block are at rest initially and then released. Neglecting friction and mass of pulleys find,
(a) tesion in the rope *abcd* and acceleration of blocks,
(b) the relative velocity and relative position of blocks A and B, if all block are lined up initially,
(c) What is the degrees of freedom of pulley system? Explain.



<u>Solution 2:</u>(a)



Length of string is constant:

$$l = a + b + c + d = const$$

$$\Rightarrow y_A + 2y_B + y_C = const$$
$$\Rightarrow a_A + 2a_B + a_C = 0 \qquad (1)$$

Eq.of motion for block A: $m_A g - T = m_A a_A$ (2)

Eq. of motion of pulley in the middle: $T_B - 2T = 0$ (pulley has no mass) (3)

Eq. of motion for block B: $m_B g - T_B = m_B a_B$ (4)

Eq. of motion for block C: $m_C g - T = m_C a_C$ (5)

(2), (3), (4), (5) → (1):

$$\frac{m_A g - T}{m_A} + 2\frac{m_A g - 2T}{m_B} + \frac{m_C g - T}{m_C} = 0 \qquad \boxed{\Rightarrow T = 26.16\ N}$$

and $a_A = -3.27\ m/s^2\ \uparrow$, $a_B = 1.09\ m/s^2\ \downarrow$, $a_B = 1.09\ m/s^2\ \downarrow$

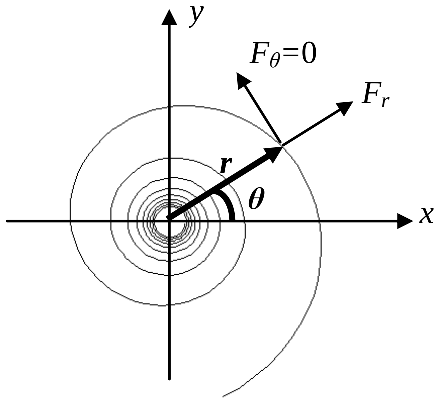
$$y_A(0) = y_B(0),\ v_A(0) = v_B(0) = 0,\quad a_{A/B} = -3{,}27 - 1{,}09 = -4{,}36\ m/s^2$$
$$v_{A/B} = \int a_{A/B} dx = -4{,}36t,\quad y_{A/B} = \int v_{A/B} dx = -2{,}18t^2$$
$$\boxed{v_{A/B}(t = 2s) = -8.72\ m/s}$$
$$\boxed{y_{A/B}(t = 2s) = -8.72\ m}$$

(c) The pulley system has two degress freedom. Since according to the relation $y_A + 2y_B + y_C = const$ only two coordinates can be chosen arbitrarily.

Problem 3: The trajectory of a particle under the action of a central force is given as $r\theta = C = const$. Find the force central $F = F(r,\theta)$ acting on the particle.

Solution 3:

Kinematic expressions:



$a_r = \ddot{r} - r\dot{\theta}^2$, $\quad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0$

$r\theta = C_1 = sbt$ (i)

Taking derivative of (i) two times

$\dot{r}\theta + r\dot{\theta} = 0$ (ii)

$\ddot{r}\theta + 2\dot{r}\dot{\theta} + r\ddot{\theta} = 0$ (iii)

is obtained. But, $r\ddot{\theta} + 2\dot{r}\dot{\theta} = a_\theta = 0$. Therefore,

$\ddot{r}\theta + 2\dot{r}\dot{\theta} + r\ddot{\theta} = \ddot{r}\theta + 0 = 0 \;\Rightarrow\; \ddot{r}\theta = 0 \quad \Rightarrow \theta = 0 \;\vee\; \ddot{r} = 0$ (iv)

Since path of the particle is a spriral $\ddot{r} = 0$ and $\dot{r} = sbt = C_2$

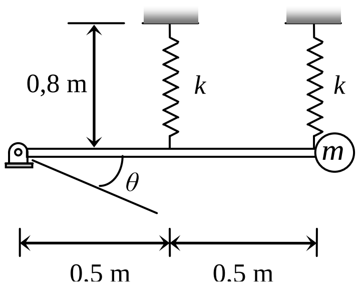
Radial acceleration $a_r = \ddot{r} - r\dot{\theta}^2 = 0 - r\dot{\theta}^2 = -r\dot{\theta}^2$ (v)

From (ii) : $\dot{\theta} = -\dfrac{\dot{r}\theta}{r}$ (vi)

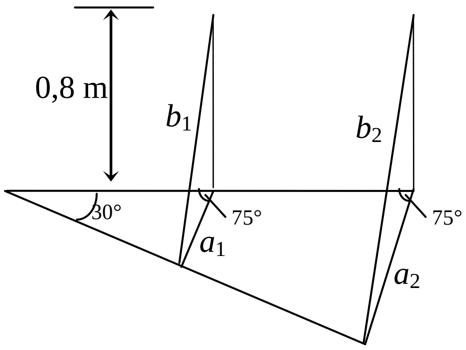
(i), (vi) → (v) : $a_r = -r\dot{\theta}^2 = -r\left(-\dfrac{\dot{r}\theta}{r}\right)^2 = -\dfrac{\dot{r}^2\theta^2}{r} = -\dfrac{C_2^2 C_1^2}{r^3} = -\dfrac{C^*}{r^3}$

$F_r = ma_r = -m\dfrac{C^*}{r^3} = -\dfrac{C}{r^3} \quad \Rightarrow \boxed{F_r = -\dfrac{C}{r^3}}$ (this result can also be found easily by using conservation of angular momentum)

Problem 4: A 30-kg block is attached to a rigid bar of negligible mass which is pivoted at point O. The two springs of stiffness *k*=700 N/m are attached to the middle and eng point of the bar and are undeformed when the bar is released from rest in the horizontal position. Calculate the speed of the block at θ=30°.



Solution 4:



$U = T_1 + V_{1s} + V_{1g} = 0 + 0 + 0 = 0$

Sine rule:

$$\frac{a_1}{\sin 30} = \frac{0.5}{\sin 75} \Rightarrow a_1 = 0.258\ m$$

Similarity: $a_2 = 2 * a_1 = 0.516\ m$

Cosine rule:

$b_1^2 = 0.258^2 + 0.8^2 - 2 * 0.258 * 0.8 * \cos 165 \Rightarrow b_1 = 1.051\ m$

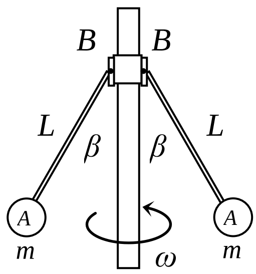
$b_2^2 = 0.516^2 + 0.8^2 - 2 * 0.516 * 0.8 * \cos 165 \Rightarrow b_2 = 1.305\ m$

Deformation of springs:
$\delta_1 = 1.051 - 0.8 = 0.251\ m$, $\delta_2 = 1.305 - 0.8 = 0.505\ m$

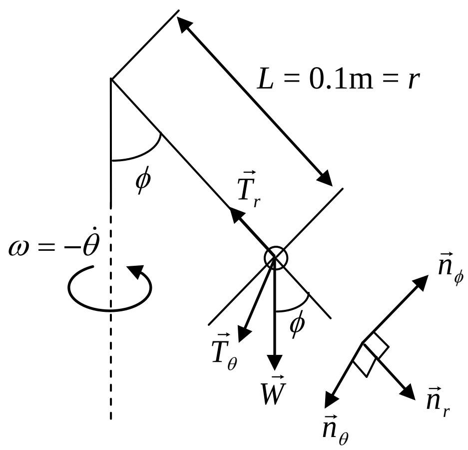
$V_{2s} = 0.5 * 700 * 0.251^2 + 0.5 * 700 * 0.505^2 = 111.3\ Nm$
$V_{2g} = -mgy = -30 * 9.81 * 1.0 * \sin 30 = -147.1\ Nm$
$T_1 + V_{1s} + V_{1g} = T_2 + V_{2s} + V_{2g}$
$\Rightarrow \quad 0 = 0.5 * 30 * v_2^2 + 111.3 - 147.1$

$\boxed{\Rightarrow \quad v_2 = 1.545\ m/s}$

Problem 5: In the speed-governing mechanism *m*=0.25 kg, *L*=12 cm (rods *AB* are weightless), *ω*=550 rpm, θ=45°, and $\dot{\beta} = 0$. Determine $\ddot{\beta}$ at that instant and the tension in the rods.



Solution 5:



m = 0.25 kg, *L*=0.12 m, $\phi = 45°$, $\dot{\phi} = 0$,
$\dot{\theta} = -\omega = -550 * 2\pi / 60 = -57.6\ rad/s$, $\ddot{\theta} = 0 \Rightarrow \quad \ddot{\phi} = ?$
$\vec{a} = (\ddot{r} - r\dot{\phi}^2 - r\dot{\theta}^2 \sin^2 \phi)\vec{n}_r +$
$(r\ddot{\phi} + 2\dot{r}\dot{\phi} - r\dot{\theta}^2 \sin\phi \cos\phi)\vec{n}_\phi +$
$(r\ddot{\theta} \sin\phi + 2\dot{r}\dot{\theta}\sin\phi + 2r\dot{\phi}\dot{\theta}\cos\phi)\vec{n}_\theta$
$\sum F_r = ma_r \quad \Rightarrow \quad -T_r + mg\cos\phi = m(\ddot{r} - r\dot{\phi}^2 - r\dot{\theta}^2 \sin^2 \phi)$

$T_r = 0.25*9.81*\cos 45° - 0.25*(-0.12*(-57.6)^2 \sin^2 45°)$ $\boxed{\Rightarrow T_r = 51.5\ N}$

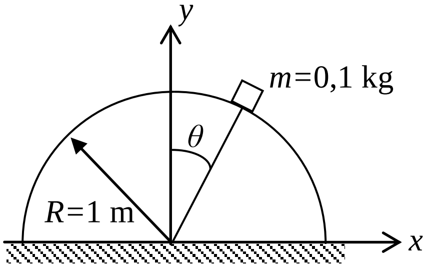
$\sum F_\phi = ma_\phi \quad \Rightarrow \quad -mg\sin\phi = m(r\ddot{\phi} + 2\dot{r}\dot{\phi} - r\dot{\theta}^2 \sin\phi\cos\phi)$

$-0.25*9.81*\sin 45° = 0.25(0.12*\ddot{\phi} - 0.12*(-57.6)^2 \sin 45° \cos 45°)$ $\boxed{\Rightarrow \ddot{\phi} = 1716.7 \quad rad/s^2 = \ddot{\beta}}$

$\sum F_\theta = ma_\theta \quad \Rightarrow \quad T_\theta = m(r\ddot{\theta}\sin\phi + 2\dot{r}\dot{\theta}\sin\phi + 2r\dot{\phi}\dot{\theta}\cos\phi) = 0.25*(0+0-0)$

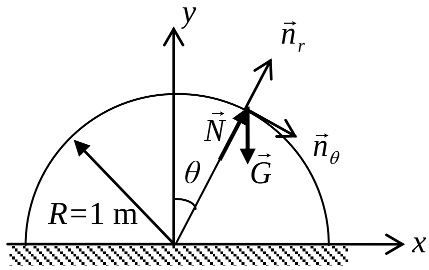
$\boxed{\Rightarrow T_\theta = 0}$

Problem 6: A block of 0.1 kg mass, initially at rest at position $\theta_0 = 10°$, start sliding downward along an arched smooth surface of R=1m curvature, upon release. Find the angle θ where the block will leave the surface. (*Hint*: at point of separation *N*=0)



Solution 6:

Eqs. of motion in polar coordinates:



$\sum F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2) \rightarrow \quad N - mg\cos\theta = m(\ddot{r} - r\dot{\theta}^2)$ (a)

$\sum F_r = ma_r = m(\ddot{r} - r\dot{\theta}^2) \rightarrow mg\sin\theta = m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)$ (b)

$r = const \rightarrow \dot{r} = \ddot{r} = 0$, and therefore (b) becomes

$g\sin\theta = r\ddot{\theta}$ (i)

and (a) becomes

$N - mg\cos\theta = -mr\dot{\theta}^2$ $\rightarrow$ at separation point *N*=0: $-g\cos\theta = -r\dot{\theta}^2$ (ii)

From relation between angular velocity and angular acceleration for radius R and (i)

$$\alpha\, d\theta = \omega\, d\omega \quad \Rightarrow \quad \ddot{\theta}\, d\theta = \dot{\theta}\, d\dot{\theta} \Rightarrow \int_{\theta_0}^{\theta} \ddot{\theta}\, d\theta = \int_{\dot{\theta}_0}^{\dot{\theta}} \dot{\theta}\, d\dot{\theta} \Rightarrow \int_{10}^{\theta} \frac{g}{r}\sin\theta\, d\theta = \int_{0}^{\dot{\theta}} \dot{\theta}\, d\dot{\theta}$$

$\Rightarrow \quad -\frac{g}{r}\left[\cos\theta\right]_{10}^{\theta} = \frac{g}{r}\left[\cos 10 - \cos\theta\right] = \frac{\dot{\theta}^2}{2}$ (iii) is found.

(iii) → (ii): $-g\cos\theta = -r*2*\frac{g}{r}\left[\cos 10 - \cos\theta\right] = -2g\left[\cos 10 - \cos\theta\right]$

$\Rightarrow \quad \cos\theta = \frac{2}{3}\cos 10$ $\boxed{\Rightarrow \quad \theta_{seperation} = 48.96°}$