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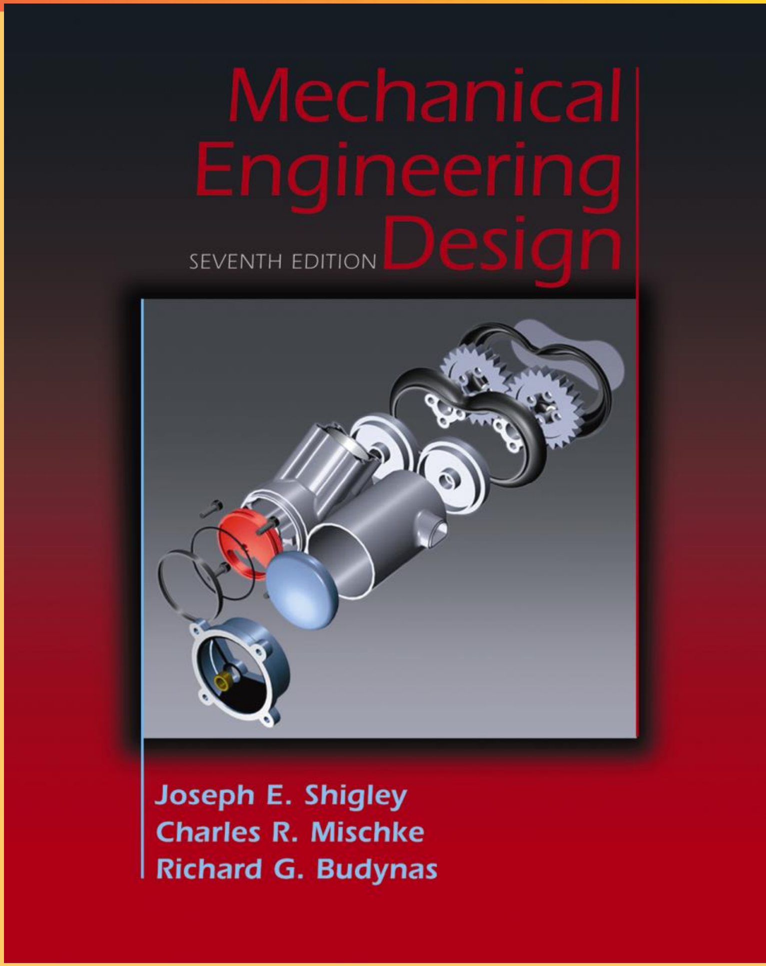
Chapter 7

Failures Resulting from Variable Loading



Mechanical Engineering Design

Seventh Edition

Shigley • Mischke • Budynas

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Strain-Life Relationships

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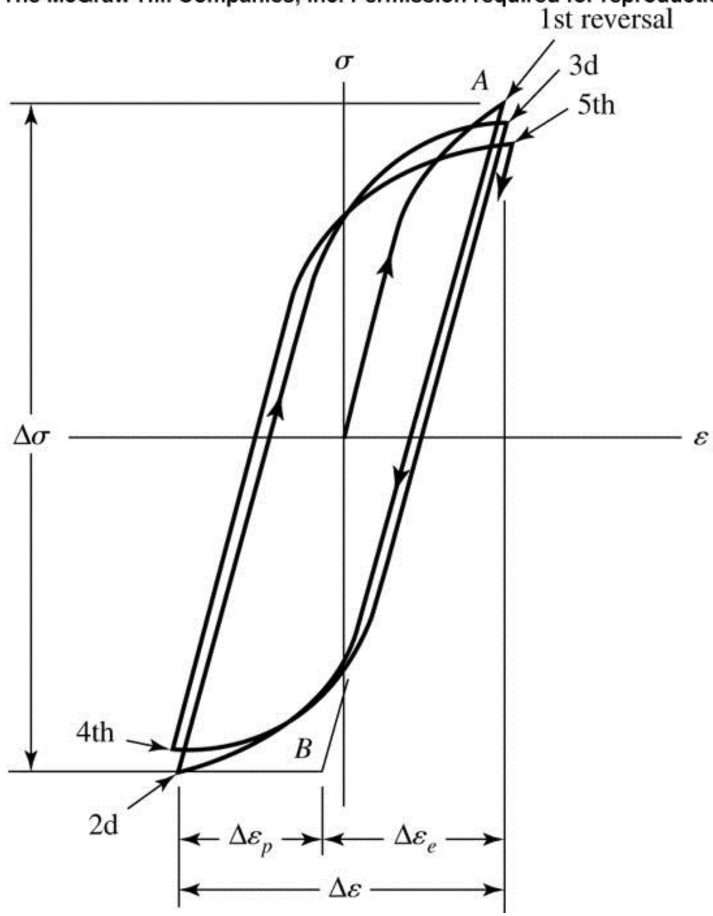

Fig. 7.12 True stress-true strain hysteresis loops showing the first five stress reversals of a cyclic-softening materials.

$$\Delta\varepsilon = \Delta\varepsilon_e + \Delta\varepsilon_p$$

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_F'}{E}(2N)^b + \varepsilon_F'(2N)^c$$

σ_F' : fatigue strength coefficient

ε_F' : fatigue ductility coefficient

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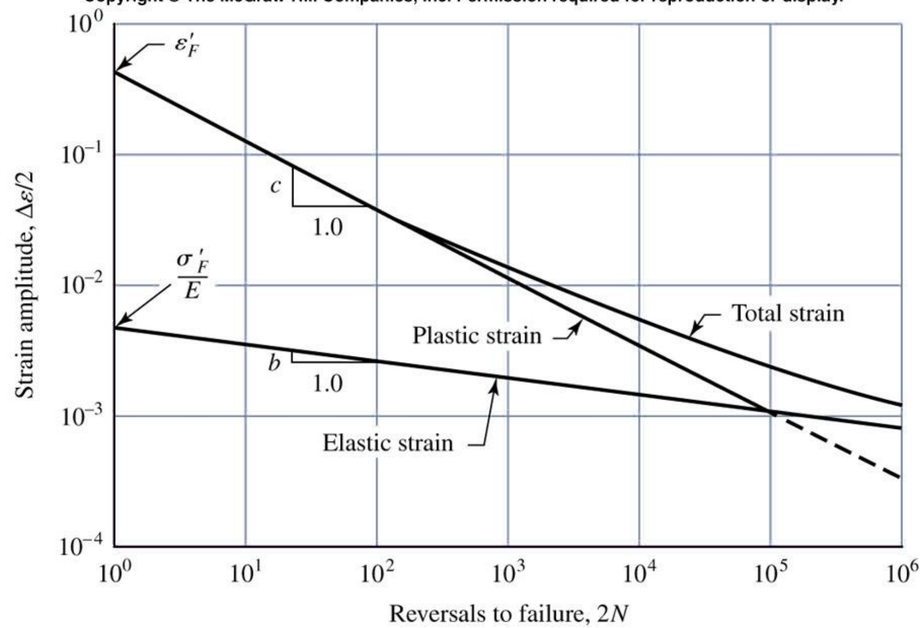

Fig. 7.14 A log-log plot showing how the fatigue life is related to the true strain amplitude for hot-rolled SAE 1020 steel.

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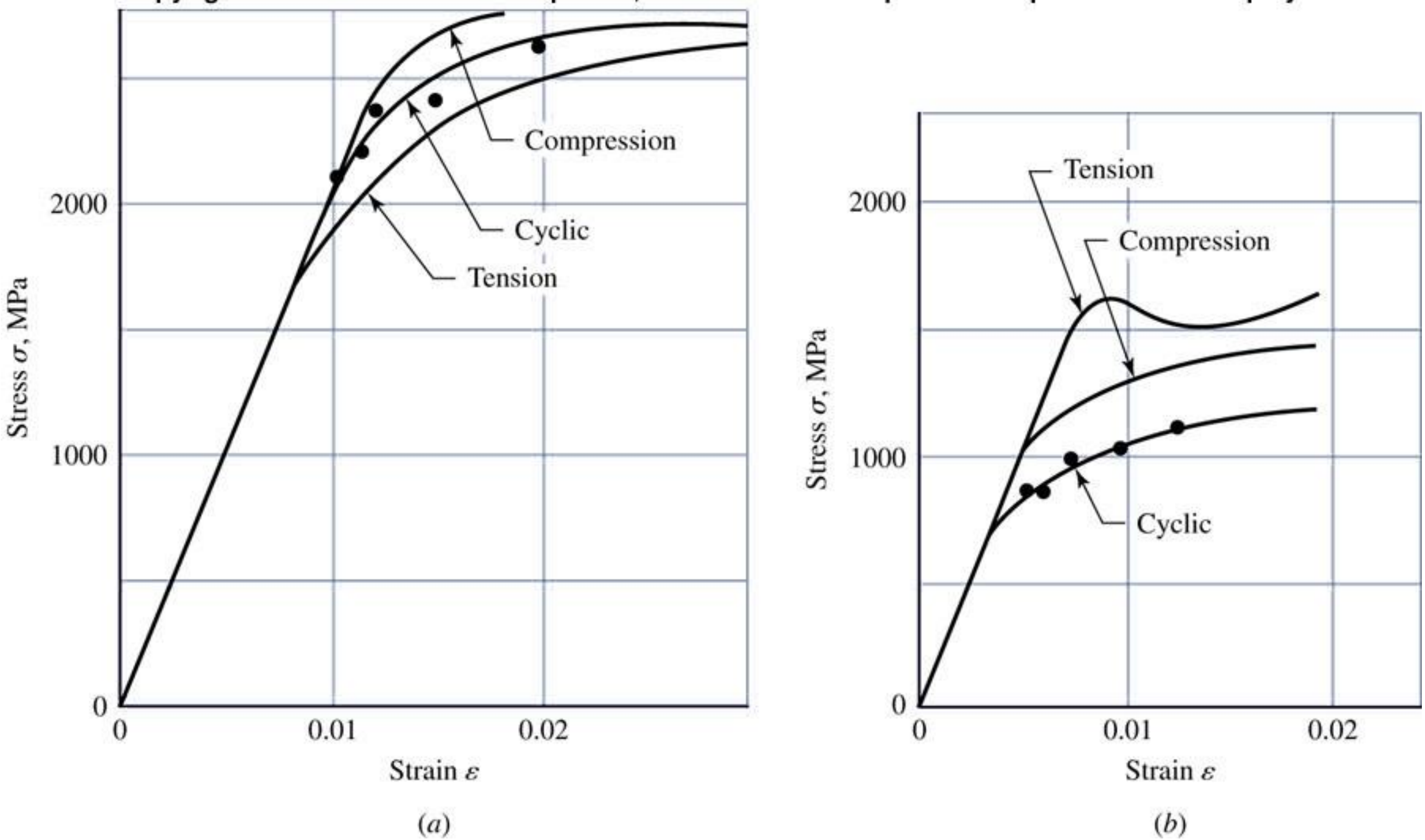

Fig. 7.13 Monotonic and cyclic stress-strain results. (a) Ausformed H-11 steel, 660 Brinell; (b) SAE 4142 steel, 400 Brinell.

Stress-Life Relationships

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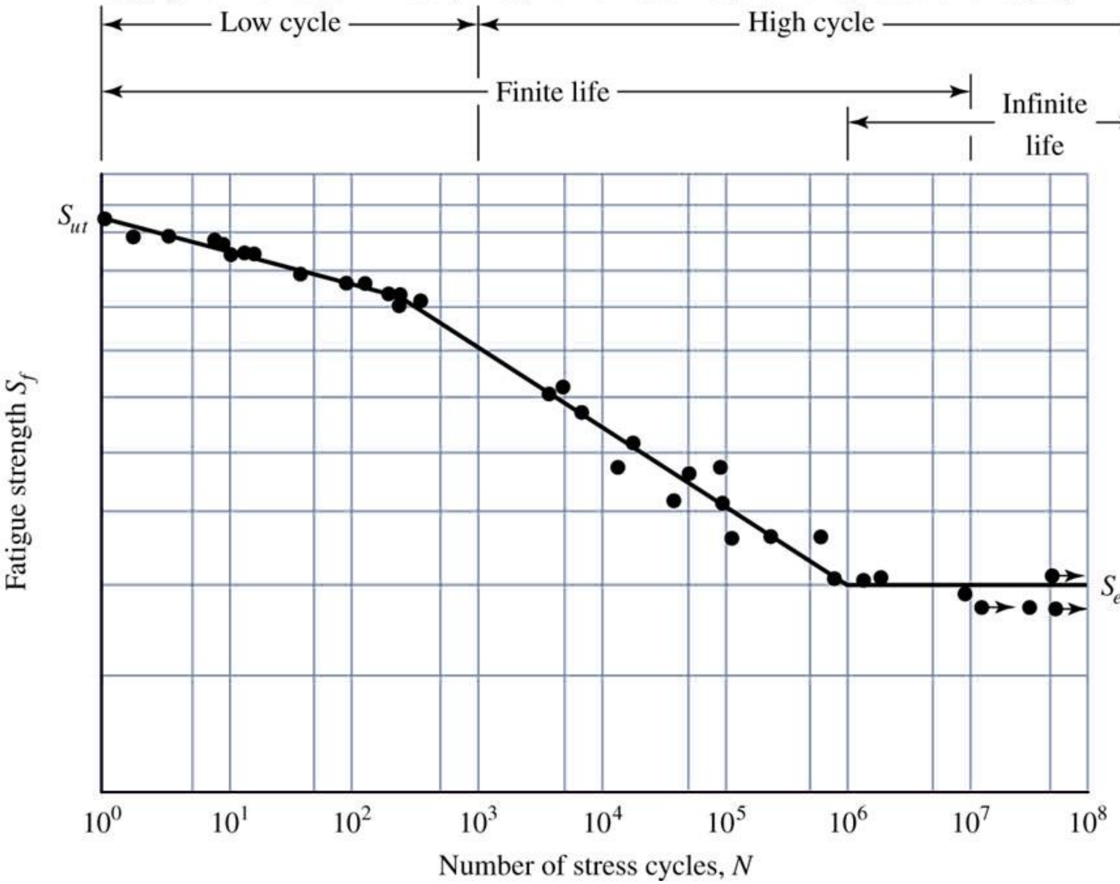

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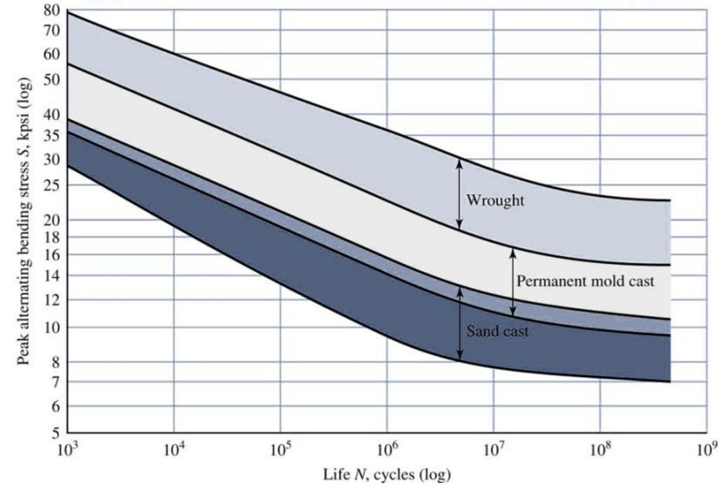

Fig. 7.10 An S-N diagram plotted from the results of completely reversed axial fatigue tests.

Endurance Limit

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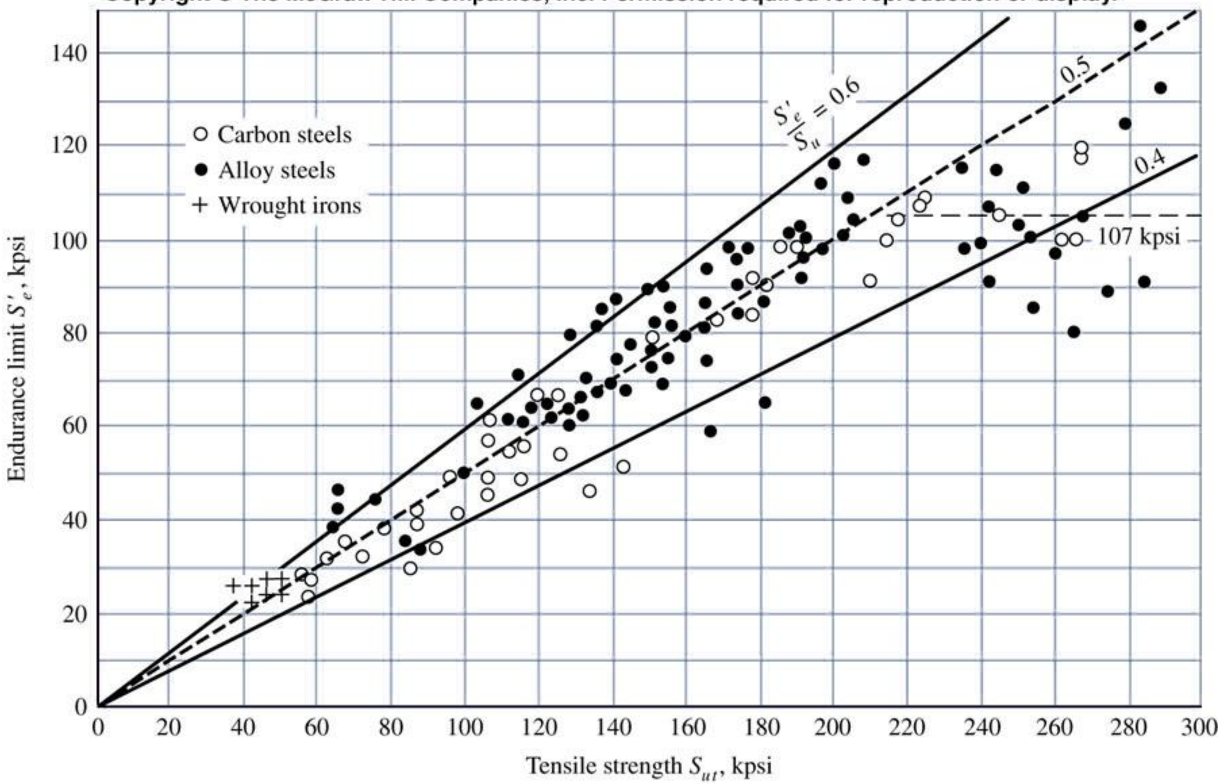


Fig. 7.18 Graph of endurance limits versus tensile strength from actual test rsults for a large number of wrought irons and steels.

Endurance-Limit Modifying Factors

$$S_e = k_a k_b k_c k_d k_e k_f S'_e \tag{6–18}$$

k_a = surface condition modification factor

k_b = size modification factor

k_c = load modification factor

k_d = temperature modification factor

k_e = reliability factor[13]

k_f = miscellaneous-effects modification factor

S'_e = rotary-beam test specimen endurance limit

S_e = endurance limit at the critical location of a machine part in the geometry and condition of use

Endurance-Limit Modifying Factors

- Stresses tend to be high at the surface
- Surface finish has an impact on initiation of cracks at localized stress concentrations
- Surface factor is a function of ultimate strength. Higher strengths are more sensitive to rough surfaces.

$$k_a = aS_{ut}^b \qquad (6\text{–}19)$$

Table 6–2

Parameters for Marin Surface Modification Factor, Eq. (6–19)

	Factor a		Exponent
Surface Finish	**S_{ut}, kpsi**	**S_{ut}, MPa**	**b**
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995

From C.J. Noll and C. Lipson, "Allowable Working Stresses," *Society for Experimental Stress Analysis*, vol. 3, no. 2, 1946 p. 29. Reproduced by O.J. Horger (ed.) *Metals Engineering Design ASME Handbook*, McGraw-Hill, New York. Copyright © 1953 by The McGraw-Hill Companies, Inc. Reprinted by permission.

Example 6-4

A steel has a minimum ultimate strength of 520 MPa and a machined surface. Estimate k_a.

Solution From Table 6–2, $a = 4.51$ and $b = -0.265$. Then, from Eq. (6–19)

Answer
$$k_a = 4.51(520)^{-0.265} = 0.860$$

Size Factor k_b

- Larger parts have greater surface area at high stress levels
- Likelihood of crack initiation is higher
- Size factor is obtained from experimental data with wide scatter
- For bending and torsion loads, the trend of the size factor data is given by

$$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.11 \le d \le 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \le 10 \text{ in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 2.79 \le d \le 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < d \le 254 \text{ mm} \end{cases} \quad (6\text{–}20)$$

- Applies only for round, rotating diameter
- For axial load, there is no size effect, so $k_b = 1$

Size Factor k_b

- For parts that are not round and rotating, an equivalent round rotating diameter is obtained.
- Equate the volume of material stressed at and above 95% of the maximum stress to the same volume in the rotating-beam specimen.
- Lengths cancel, so equate the areas.
- For a rotating round section, the 95% stress area is the area of a ring,

$$A_{0.95\sigma} = \frac{\pi}{4}[d^2 - (0.95d)^2] = 0.0766d^2 \quad (6\text{–}22)$$

- Equate 95% stress area for other conditions to Eq. (6–22) and solve for *d* as the equivalent round rotating diameter

Size Factor k_b

- For non-rotating round,

$$A_{0.95\sigma} = 0.01046d^2 \qquad (6\text{–}23)$$

- Equating to Eq. (6-22) and solving for equivalent diameter,

$$d_e = 0.370d \qquad (6\text{–}24)$$

- Similarly, for rectangular section *h x b*, $A_{95\sigma} = 0.05\,hb$. Equating to Eq. (6–22),

$$d_e = 0.808(hb)^{1/2} \qquad (6\text{–}25)$$

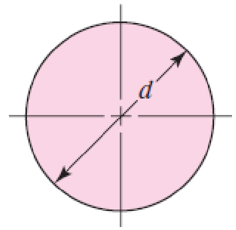
- Other common cross sections are given in Table 6–3

Endurance-Limit Modifying Factors

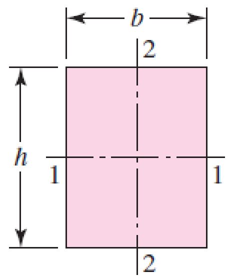
Size Factor k_b

Table 6–3

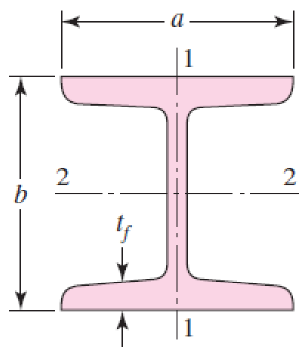
$A_{95\sigma}$ for common non-rotating structural shapes



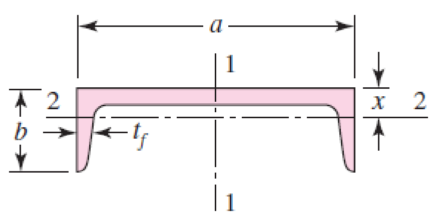
$$A_{0.95\sigma} = 0.01046d^2$$
$$d_e = 0.370d$$



$$A_{0.95\sigma} = 0.05hb$$
$$d_e = 0.808\sqrt{hb}$$



$$A_{0.95\sigma} = \begin{cases} 0.10at_f & \text{axis 1-1} \\ 0.05ba \quad t_f > 0.025a & \text{axis 2-2} \end{cases}$$



$$A_{0.95\sigma} = \begin{cases} 0.05ab & \text{axis 1-1} \\ 0.052xa + 0.1t_f(b - x) & \text{axis 2-2} \end{cases}$$

Example 6-4

A steel shaft loaded in bending is 32 mm in diameter, abutting a filleted shoulder 38 mm in diameter. The shaft material has a mean ultimate tensile strength of 690 MPa. Estimate the Marin size factor k_b if the shaft is used in

(*a*) A rotating mode.

(*b*) A nonrotating mode.

Solution (*a*) From Eq. (6–20)

Answer

$$k_b = \left(\frac{d}{7.62}\right)^{-0.107} = \left(\frac{32}{7.62}\right)^{-0.107} = 0.858$$

(*b*) From Table 6–3,

$$d_e = 0.37d = 0.37(32) = 11.84 \text{ mm}$$

From Eq. (6–20),

Answer

$$k_b = \left(\frac{11.84}{7.62}\right)^{-0.107} = 0.954$$

Loading Factor k_c

- Accounts for changes in endurance limit for different types of fatigue loading.
- Only to be used for single load types. Use Combination Loading method (Sec. 6–14) when more than one load type is present.

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion}^{17} \end{cases} \qquad (6\text{–}26)$$

Temperature Factor k_d

- Endurance limit appears to maintain same relation to ultimate strength for elevated temperatures as at room temperature
- This relation is summarized in Table 6–4

Table 6–4

Effect of Operating Temperature on the Tensile Strength of Steel.* (S_T = tensile strength at operating temperature; S_{RT} = tensile strength at room temperature; $0.099 \leq \hat{\sigma} \leq 0.110$)

Temperature, °C	S_T/S_{RT}	Temperature, °F	S_T/S_{RT}
20	1.000	70	1.000
50	1.010	100	1.008
100	1.020	200	1.020
150	1.025	300	1.024
200	1.020	400	1.018
250	1.000	500	0.995
300	0.975	600	0.963
350	0.943	700	0.927
400	0.900	800	0.872
450	0.843	900	0.797
500	0.768	1000	0.698
550	0.672	1100	0.567
600	0.549		

*Data source: Fig. 2–9.

Temperature Factor k_d

- If ultimate strength is known for operating temperature, then just use that strength. Let $k_d = 1$ and proceed as usual.
- If ultimate strength is known only at room temperature, then use Table 6–4 to estimate ultimate strength at operating temperature. With that strength, let $k_d = 1$ and proceed as usual.
- Alternatively, use ultimate strength at room temperature and apply temperature factor from Table 6–4 to the endurance limit.

$$k_d = \frac{S_T}{S_{RT}} \qquad (6\text{–}28)$$

- A fourth-order polynomial curve fit of the underlying data of Table 6–4 can be used in place of the table, if desired.

$$\begin{aligned} k_d = {} & 0.975 + 0.432(10^{-3})T_F - 0.115(10^{-5})T_F^2 \\ & + 0.104(10^{-8})T_F^3 - 0.595(10^{-12})T_F^4 \end{aligned} \qquad (6\text{–}27)$$

Reliability Factor k_e

- Simply obtain k_e for desired reliability from Table 6–5.

Reliability, %	Transformation Variate z_a	Reliability Factor k_e
50	0	1.000
90	1.288	0.897
95	1.645	0.868
99	2.326	0.814
99.9	3.091	0.753
99.99	3.719	0.702
99.999	4.265	0.659
99.9999	4.753	0.620

Table 6–5

Miscellaneous-Effects Factor k_f

- Reminder to consider other possible factors.
  - Residual stresses
  - Directional characteristics from cold working
  - Case hardening
  - Corrosion
  - Surface conditioning, e.g. electrolytic plating and metal spraying
  - Cyclic Frequency
  - Frettage Corrosion
- Limited data is available.
- May require research or testing.

Application of Fatigue Stress Concentration Factor

- Use K_f as a multiplier to increase the nominal stress.
- Some designers (and previous editions of textbook) sometimes applied 1/ K_f as a Marin factor to reduce S_e .
- For infinite life, either method is equivalent, since
- For finite life, increasing stress is more conservative. Decreasing S_e applies more to high cycle than low cycle.

Calculating Flactuating Stresses

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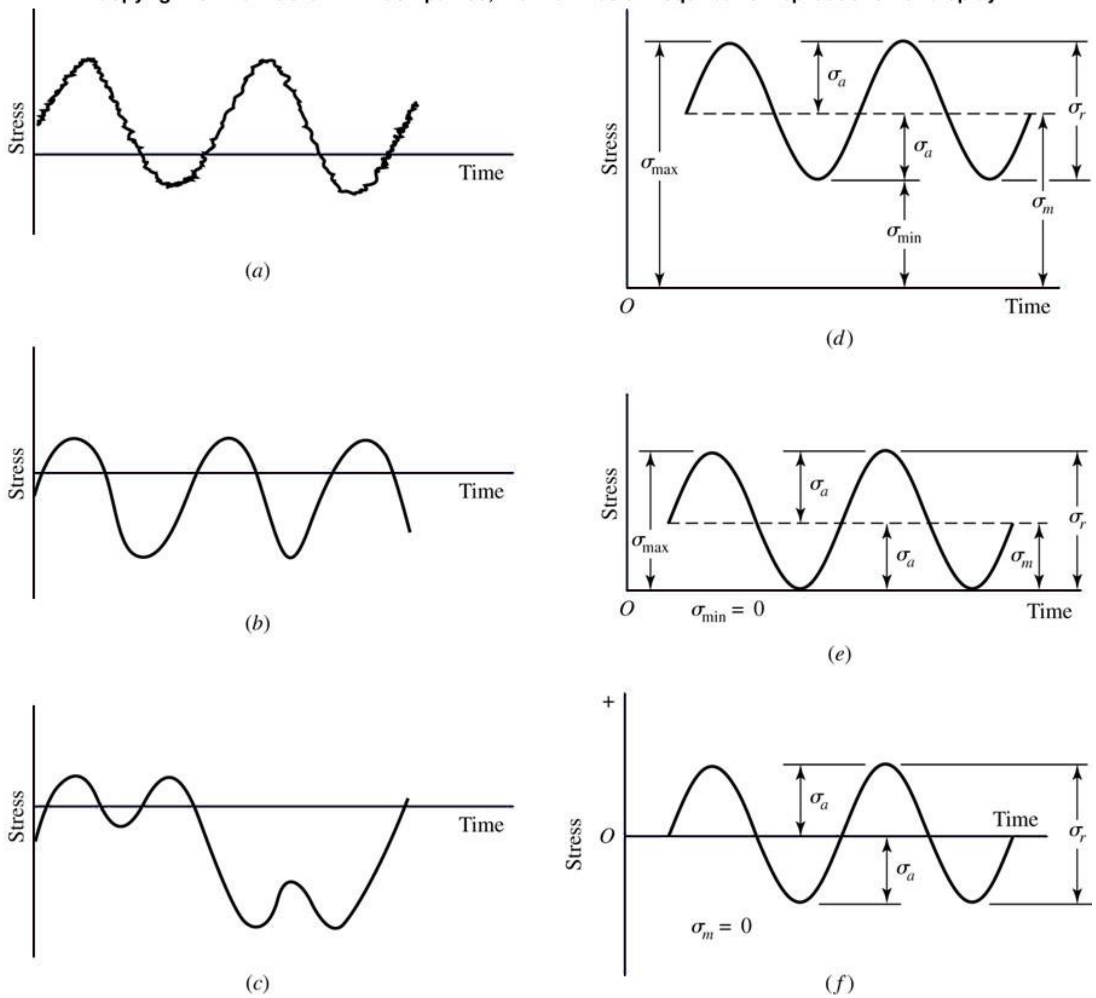

Fig. 7.23 Some stress-time relations : (a) fluctuating stress with high frequency ripple; (b and c) nonsinusoidal fluctuating stress; (d) sinusoidal fluctuating stress; (e) repeated stress; (f) completely reversed sinusoidal stress.

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

σ_m → midrange stress

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

σ_a → stress amplitude

$$R = \frac{\sigma_{min}}{\sigma_{max}} \qquad A = \frac{\sigma_a}{\sigma_m}$$

R → stress ratio ← A

Modified Goodman Relations

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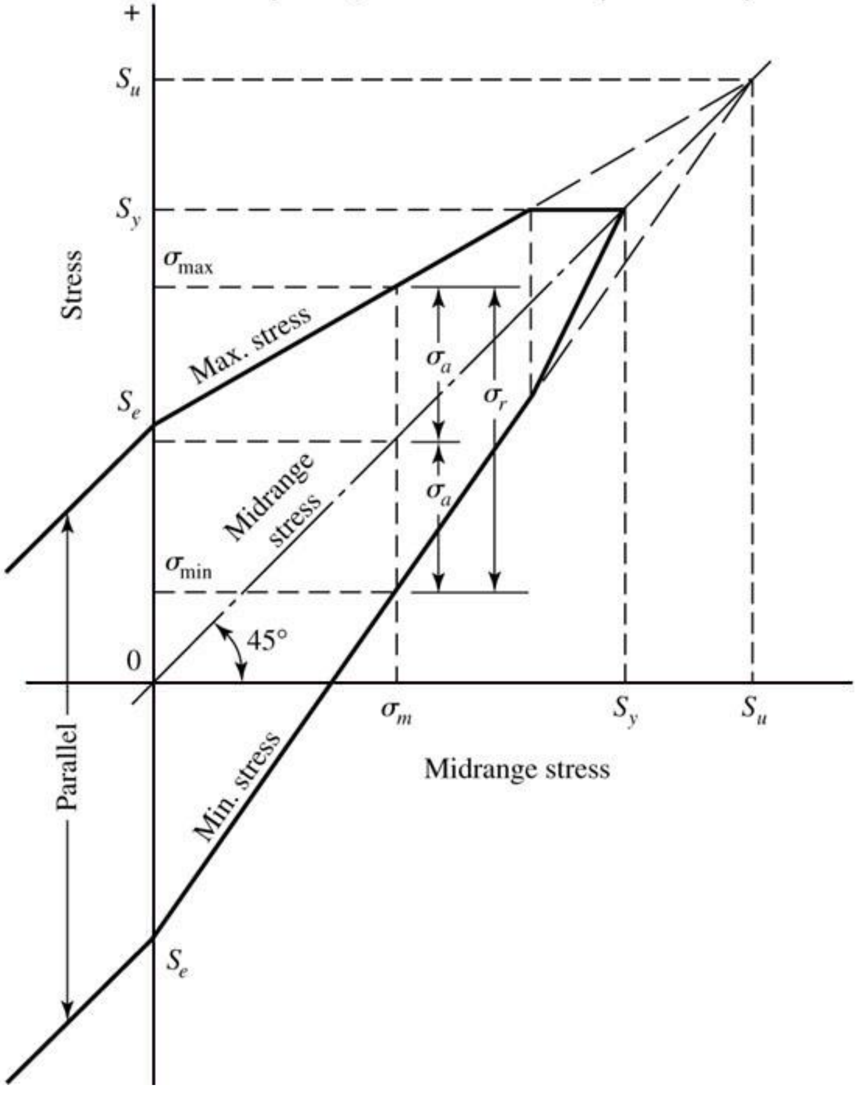

Fig. 7.24 Modified Goodman diagram showing all the strengths and the limiting values of all the stress components for a particular midrange stress.

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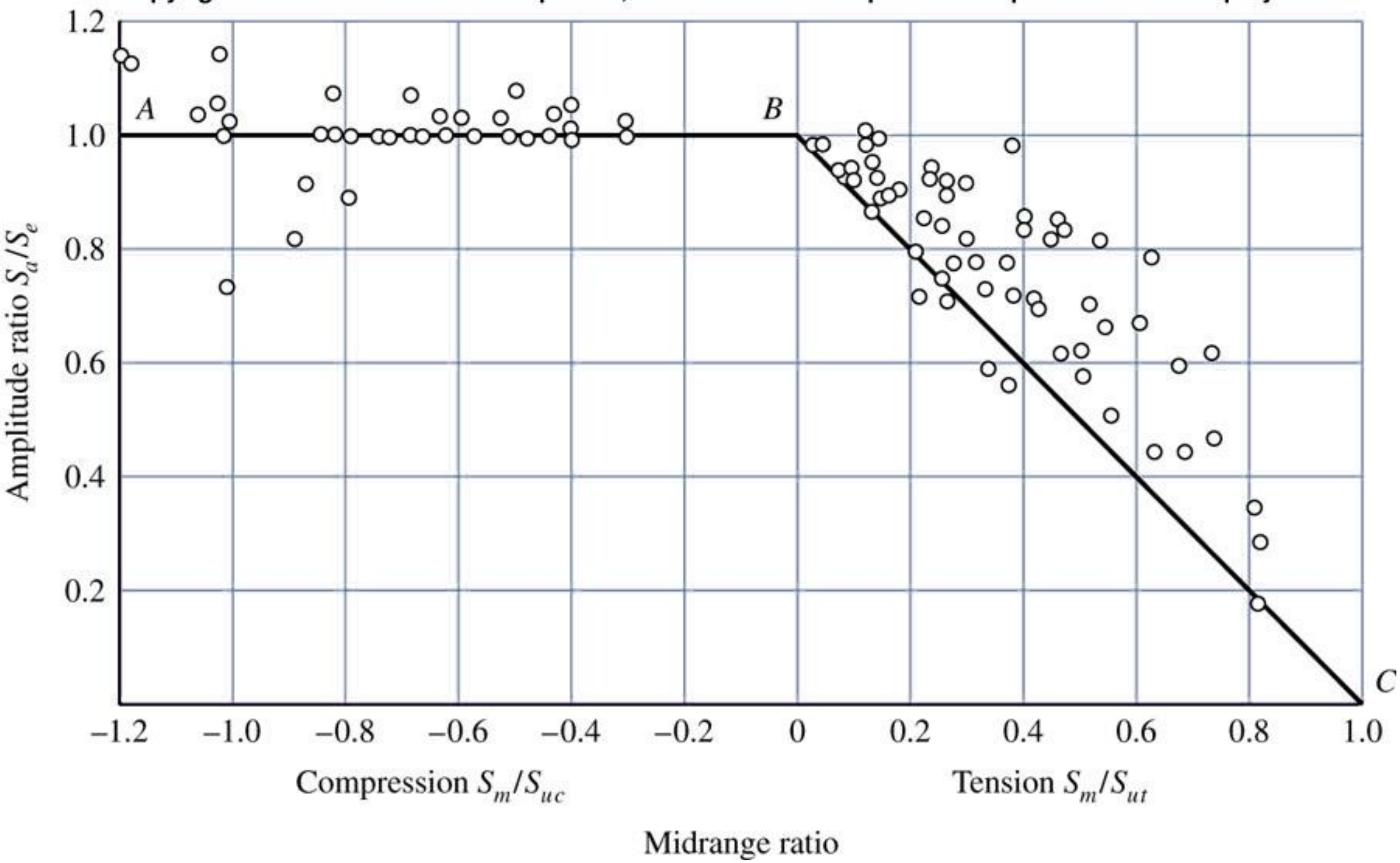

Fig. 7.25 Plot of fatigue failures for midrange stresses in both tensile and compressive regions.

Equations for Commonly Used Failure Criteria

- Intersecting a constant slope load line with each failure criteria produces design equations
- *n* is the design factor or factor of safety for infinite fatigue life

Soderberg $$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{n} \qquad (6\text{–}45)$$

mod-Goodman $$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n} \qquad (6\text{–}46)$$

Gerber $$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1 \qquad (6\text{–}47)$$

ASME-elliptic $$\left(\frac{n\sigma_a}{S_e}\right)^2 + \left(\frac{n\sigma_m}{S_y}\right)^2 = 1 \qquad (6\text{–}48)$$

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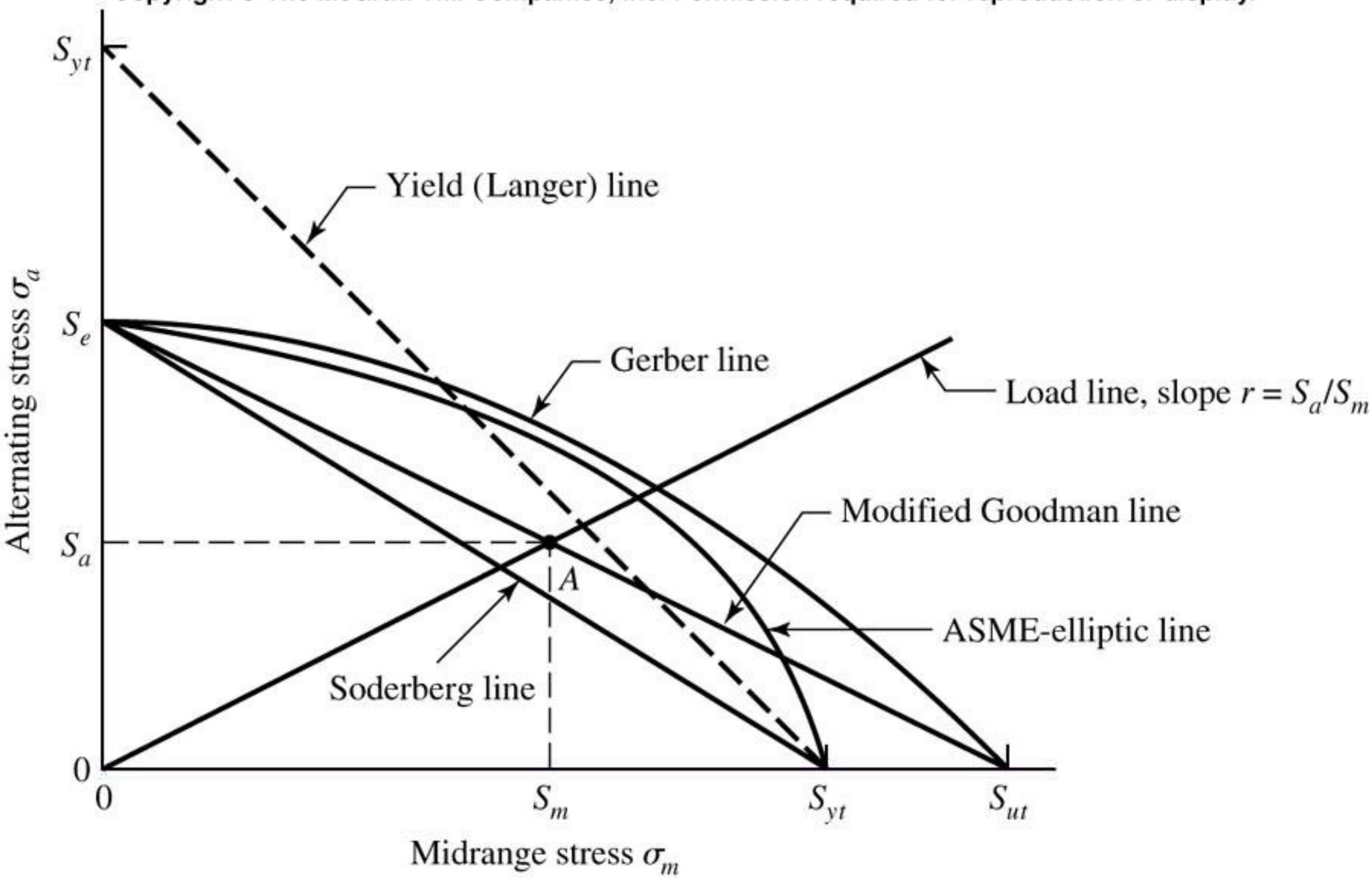

Fig. 7.27 Fatigue diagram showing various criteria of failure.

Summarizing Tables for Failure Criteria

- Tables 6–6 to 6–8 summarize the pertinent equations for Modified Goodman, Gerber, ASME-elliptic, and Langer failure criteria
- The first row gives fatigue criterion
- The second row gives yield criterion
- The third row gives the intersection of static and fatigue criteria
- The fourth row gives the equation for fatigue factor of safety
- The first column gives the intersecting equations
- The second column gives the coordinates of the intersection

Summarizing Table for Modified Goodman

Table 6–6

Amplitude and Steady Coordinates of Strength and Important Intersections in First Quadrant for Modified Goodman and Langer Failure Criteria

Intersecting Equations	Intersection Coordinates
$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$	$S_a = \frac{r S_e S_{ut}}{r S_{ut} + S_e}$
Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_a}{r}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = \frac{r S_y}{1 + r}$
Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_y}{1 + r}$
$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$	$S_m = \frac{(S_y - S_e) S_{ut}}{S_{ut} - S_e}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = S_y - S_m,\ r_{crit} = S_a/S_m$

Fatigue factor of safety

$$n_f = \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}}$$

Summarizing Table for Gerber

Table 6–7

Amplitude and Steady Coordinates of Strength and Important Intersections in First Quadrant for Gerber and Langer Failure Criteria

Intersecting Equations	Intersection Coordinates
$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$	$S_a = \frac{r^2 S_{ut}^2}{2S_e}\left[-1 + \sqrt{1 + \left(\frac{2S_e}{r S_{ut}}\right)^2}\,\right]$
Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_a}{r}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = \frac{r S_y}{1 + r}$
Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_y}{1 + r}$
$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$	$S_m = \frac{S_{ut}^2}{2S_e}\left[1 - \sqrt{1 + \left(\frac{2S_e}{S_{ut}}\right)^2\left(1 - \frac{S_y}{S_e}\right)}\,\right]$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = S_y - S_m,\ r_{\text{crit}} = S_a / S_m$

Fatigue factor of safety

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\,\right] \qquad \sigma_m > 0$$

Summarizing Table for ASME-Elliptic

Table 6–8

Amplitude and Steady Coordinates of Strength and Important Intersections in First Quadrant for ASME-Elliptic and Langer Failure Criteria

Intersecting Equations	Intersection Coordinates
$\left(\frac{S_a}{S_e}\right)^2 + \left(\frac{S_m}{S_y}\right)^2 = 1$ Load line $r = S_a/S_m$	$S_a = \sqrt{\frac{r^2 S_e^2 S_y^2}{S_e^2 + r^2 S_y^2}}$ $S_m = \frac{S_a}{r}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$ Load line $r = S_a/S_m$	$S_a = \frac{r S_y}{1 + r}$ $S_m = \frac{S_y}{1 + r}$
$\left(\frac{S_a}{S_e}\right)^2 + \left(\frac{S_m}{S_y}\right)^2 = 1$ $\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = 0, \ \frac{2 S_y S_e^2}{S_e^2 + S_y^2}$ $S_m = S_y - S_a, r_{crit} = S_a/S_m$

Fatigue factor of safety

$$n_f = \sqrt{\frac{1}{(\sigma_a/S_e)^2 + (\sigma_m/S_y)^2}}$$