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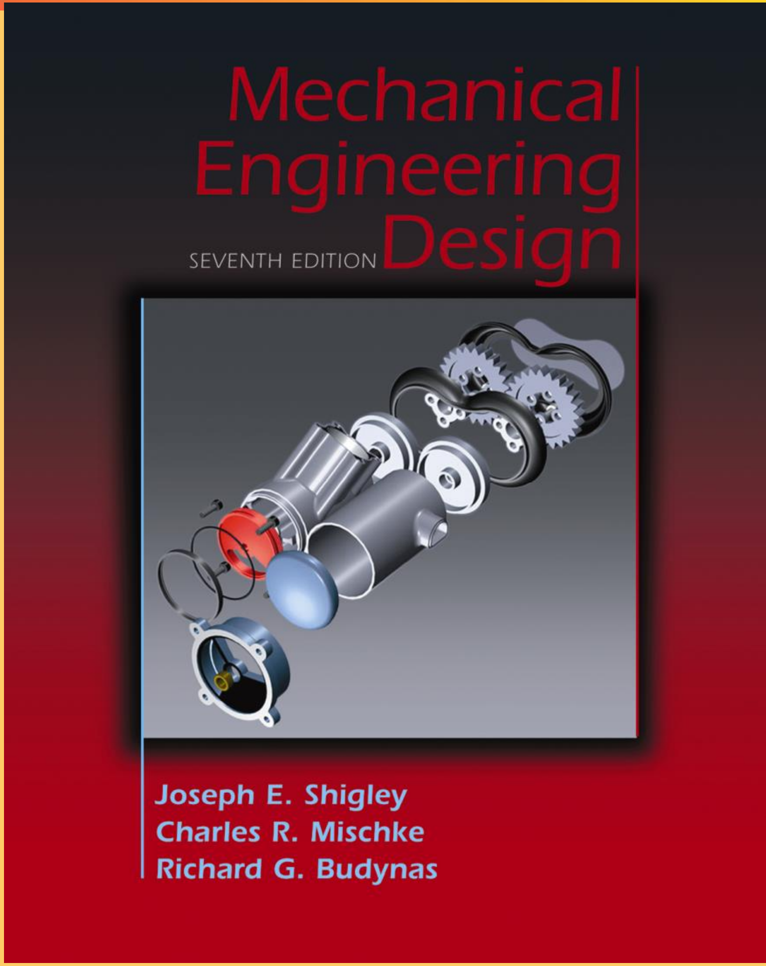
Chapter 5

Deflection and Stiffness

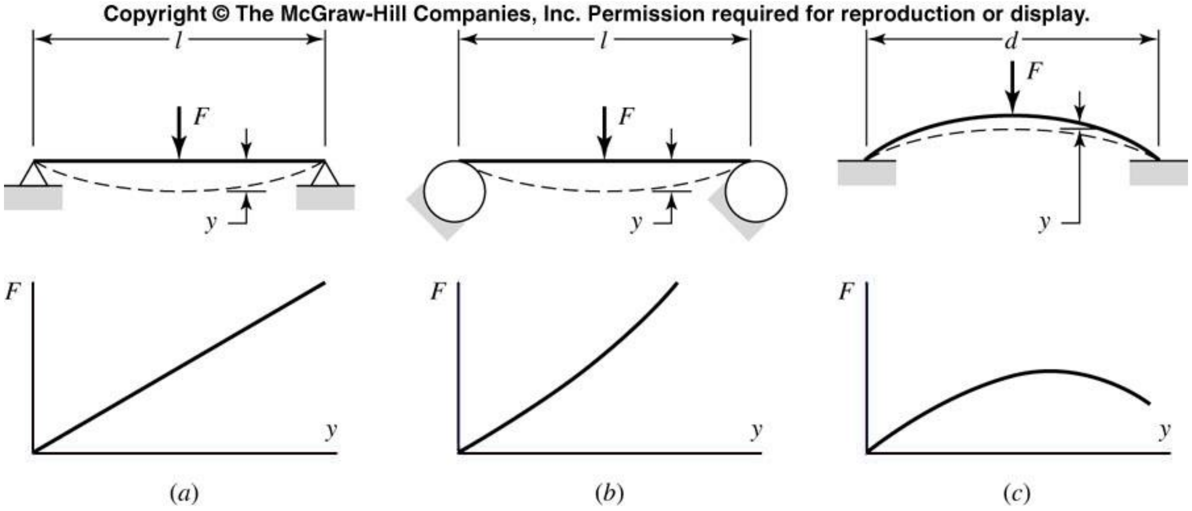


Mechanical Engineering Design

Seventh Edition

Shigley • Mischke • Budynas

Spring Rates, Tension, Compression and Torsion



$$k = \frac{F}{y} \text{ for a linear spring}$$

$$\delta = \frac{Fl}{AE} \qquad k = \frac{AE}{l}$$

$$\theta = \frac{Tl}{GJ} \qquad k = \frac{GJ}{l}$$

Fig. 5.1 (a) A linear spring; (b) a stiffening spring; (c) a softening spring.

Deflection Due to Bending

$$\frac{q}{EI} = \frac{d^4 y}{dx^4}$$

$$\frac{V}{EI} = \frac{d^3 y}{dx^3}$$

$$\frac{M}{EI} = \frac{d^2 y}{dx^2}$$

Deflection can be found by
- Integration,
- Area-moment method,
- Use of singularity functions, and also
- Energy methods.

Strain Energy and Castigliano's Theorem

The external work done on elastic member in deforming it is transformed into strain, or potential, energy.

$$U = \frac{F^2 L}{2AE} \quad \text{tension and compression}$$

$$U = \frac{T^2 L}{2GJ} \quad \text{torsion}$$

$$U = \frac{F^2 L}{2AG} \quad \text{direct shear}$$

$$U = \int \frac{M^2 dx}{2EI} \quad \text{bending}$$

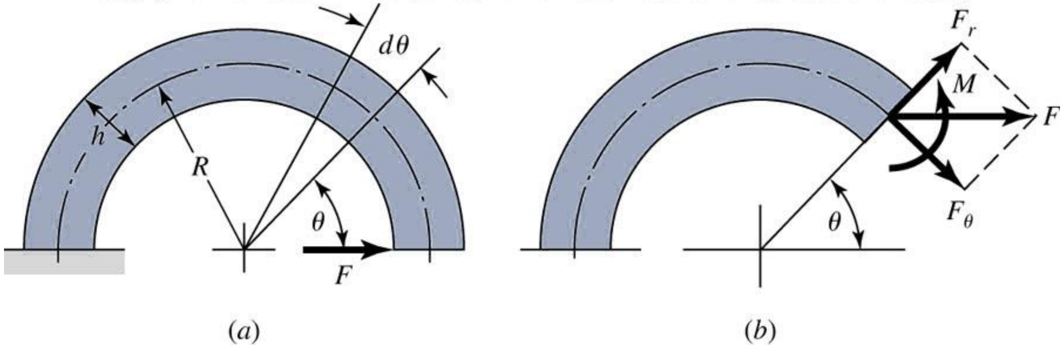
Castigliano's theorem states that when forces act on elastic systems subject to small displacements, the displacement corresponding to any force, collinear with the force, is equal to the partial derivative of the total strain energy with respect to that force.

$$\delta_i = \frac{\partial U}{\partial F_i}$$

Statically Indeterminate Problems

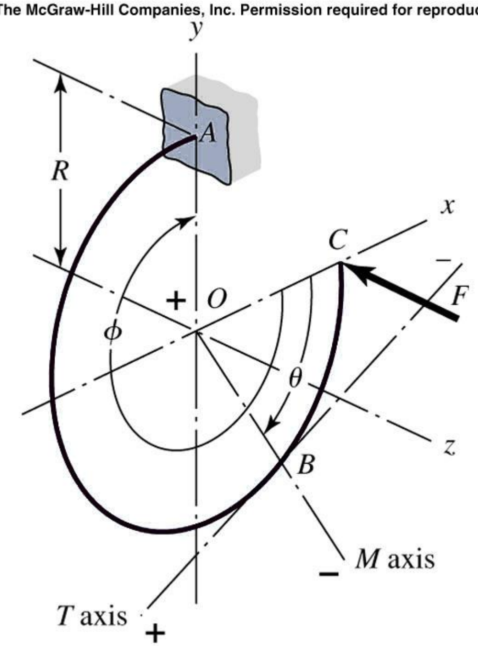
A system in which the laws of statics are not sufficient to determine all the unknown forces or moments is said to be statically indeterminate.
In addition to the static equilibrium, you need to write compatibility conditions.

Deflection of Curved Members



$$U = \int \frac{M^2 d\theta}{2AeE} + \int \frac{F_\theta^2 R d\theta}{2AE} - \int \frac{MF_\theta d\theta}{AE} + \int \frac{F_r^2 R d\theta}{2AG}$$

Fig. 5.11 (a) Curve bar loaded by force F. (b) Diagram showing forces acting on section taken at angle θ.



$$U = \int_0^\phi \frac{M^2 R d\theta}{2EI} + \int_0^\phi \frac{T^2 R d\theta}{2GJ}$$

Fig. 5.12 Ring ABC in the xy plane subject to force F parallel to the z axis.

Compression Members - General

1. Long columns with central loading
2. Intermediate-length columns with central loading
3. Columns with eccentric loading
4. Struts or short columns with eccentric loading

Long Columns with Central Loading

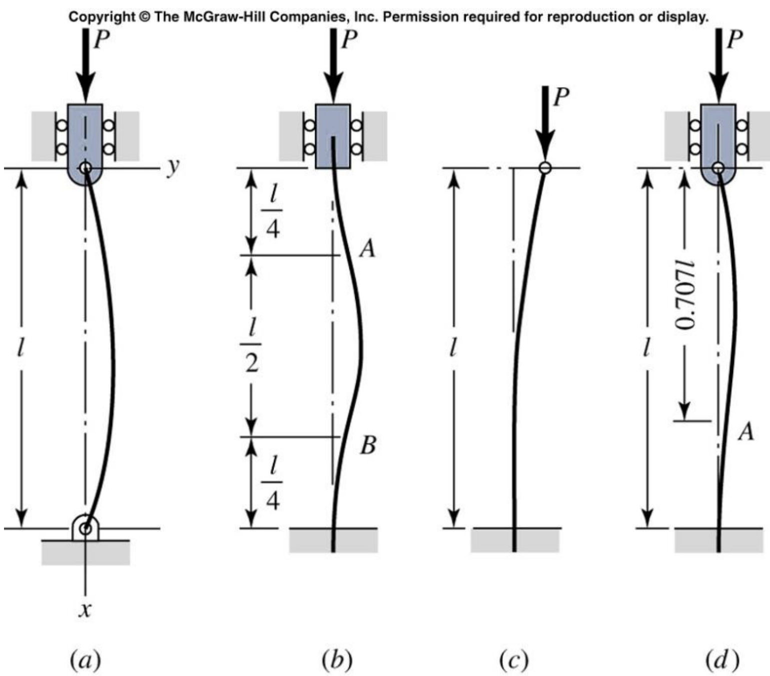


Fig. 5.18 (a) Both ends rounded or pivoted; (b) both ends fixed; (c) one end free, one end fixed; (d) one and rounded and pivoted, one end fixed.

$$P_{cr} = \frac{C\pi^2 EI}{l^2} \qquad \frac{P_{cr}}{A} = \frac{C\pi^2 E}{(l/k)^2}$$

(l/k) is the slenderness ratio $\qquad I = Ak^2$

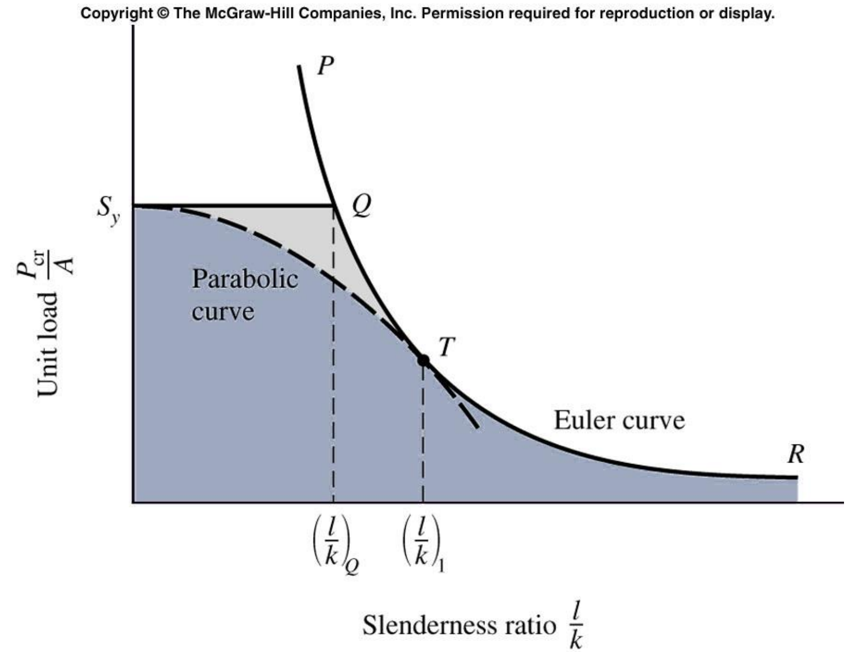


Fig. 5.19 Euler curve plotted for C=1.

$$\left(\frac{l}{k}\right)_1 = \left(\frac{2\pi^2 CE}{S_y}\right)$$

Intermediate-Length Columns with Central Loading (Parabolic or J.B. Johnson formula)

$$\frac{P_{cr}}{A} = S_y - \left(\frac{S_y}{2\pi} \frac{l}{k} \right)^2 \frac{1}{CE} \qquad \left(\frac{l}{k} \right) \le \left(\frac{l}{k} \right)_1$$

Columns with Eccentric Loading (Secant formula)

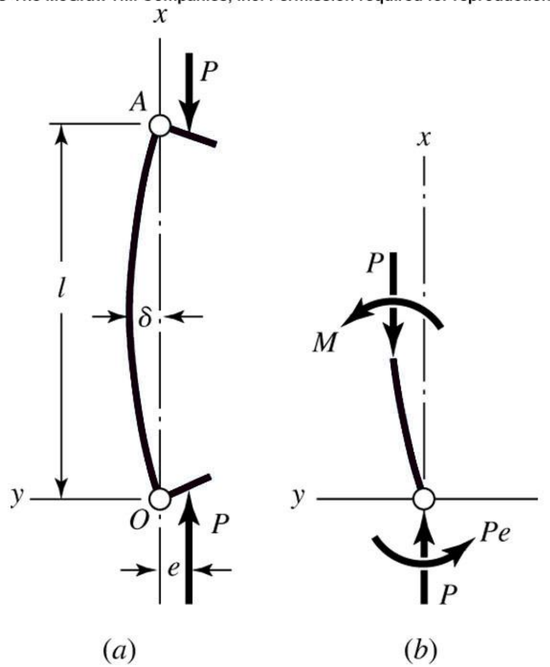


Fig. 5.20 Notation for an eccentrically loaded column.

$$\frac{P}{A} = \frac{S_{yc}}{1 + (ec/k^2)\sec\left[(l/2k)\sqrt{P/AE} \right]}$$

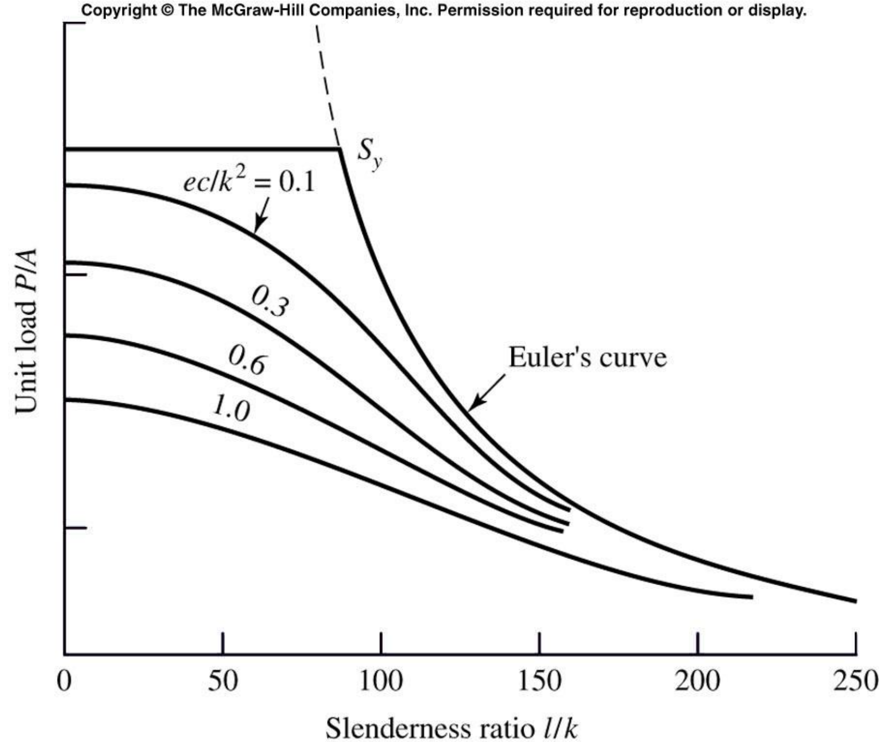


Fig. 5.21 Comparison of secant and Euler equations.

Struts, or Short Compression Members

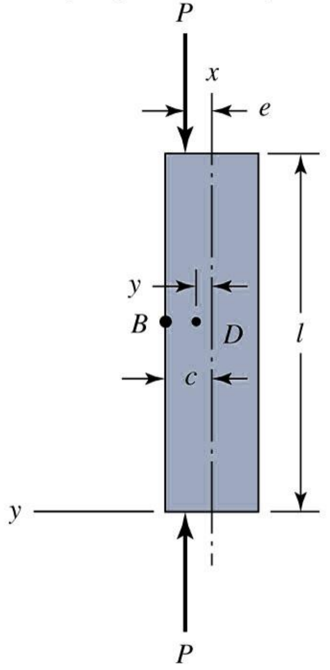


Fig. 5.22 Eccentrically loaded strut.

$$\sigma_c = \frac{P}{A}\left(1 + \frac{ec}{k^2}\right)$$

$$\left(\frac{l}{k}\right)_2 = 0.282\left(\frac{AE}{P_{cr}}\right)^{1/2}$$