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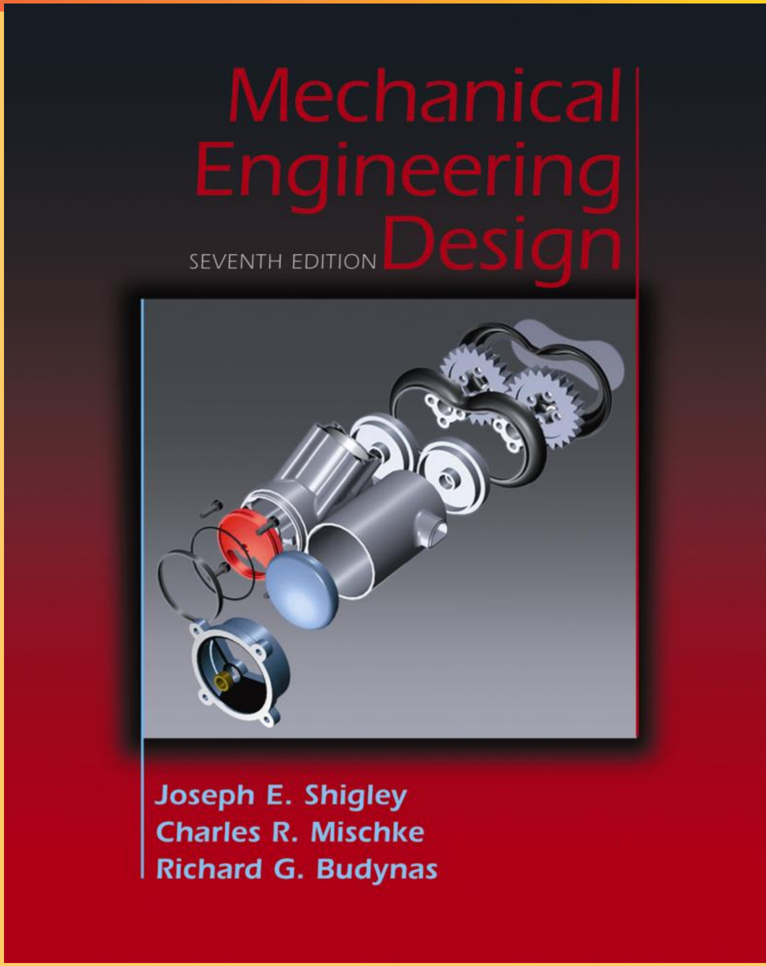
Chapter 4

Stress



Mechanical Engineering Design

Seventh Edition

Shigley • Mischke • Budynas

Stress Components

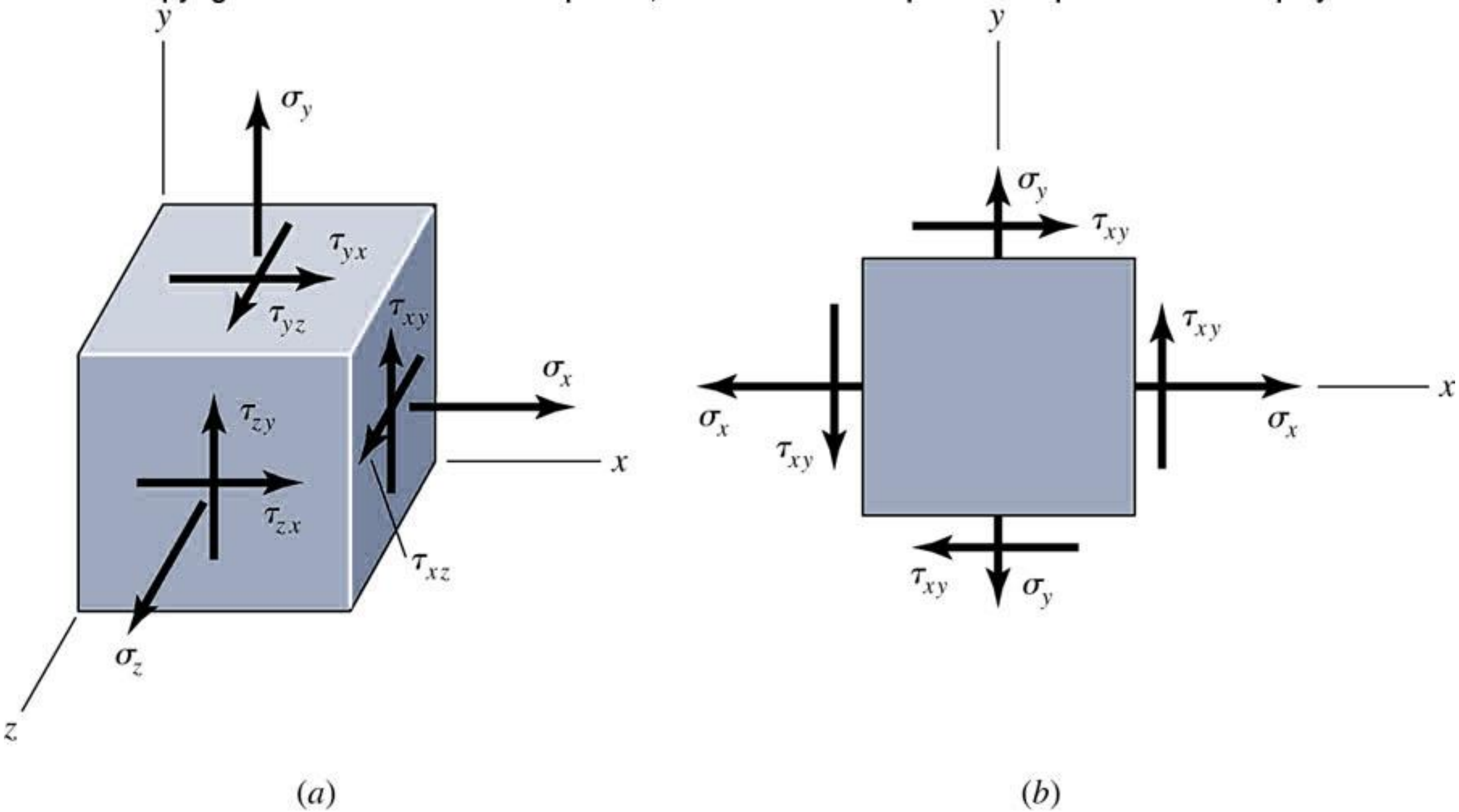


Fig. 4.10 The general three dimensional stress elements.

The element is in equilibrium, hence

$$\tau_{xy} = \tau_{yx}$$

$$\tau_{yz} = \tau_{zy}$$

$$\tau_{zx} = \tau_{xz}$$

Stress Transformation

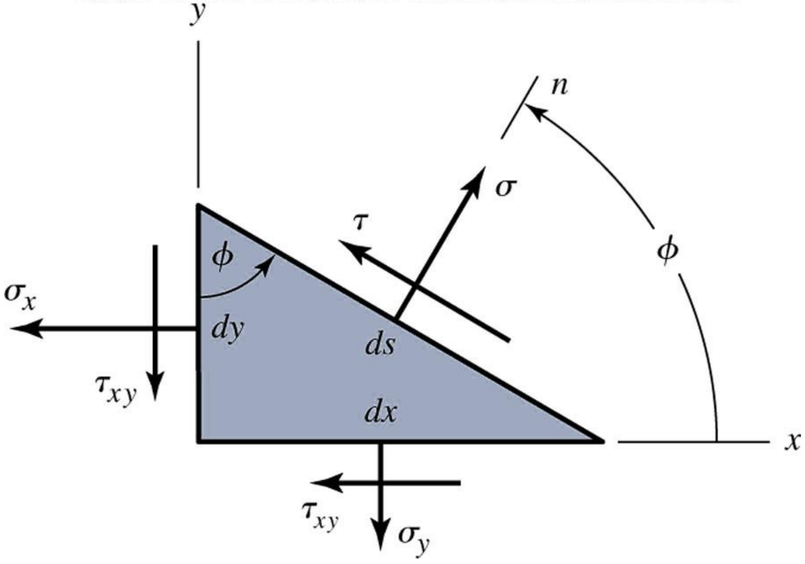


Fig. 4.11 Two dimensional stress state.

$$\sigma = \sigma_x \cos^2\phi + \sigma_y \sin^2\phi + 2\tau_{xy}\cos\phi\sin\phi$$

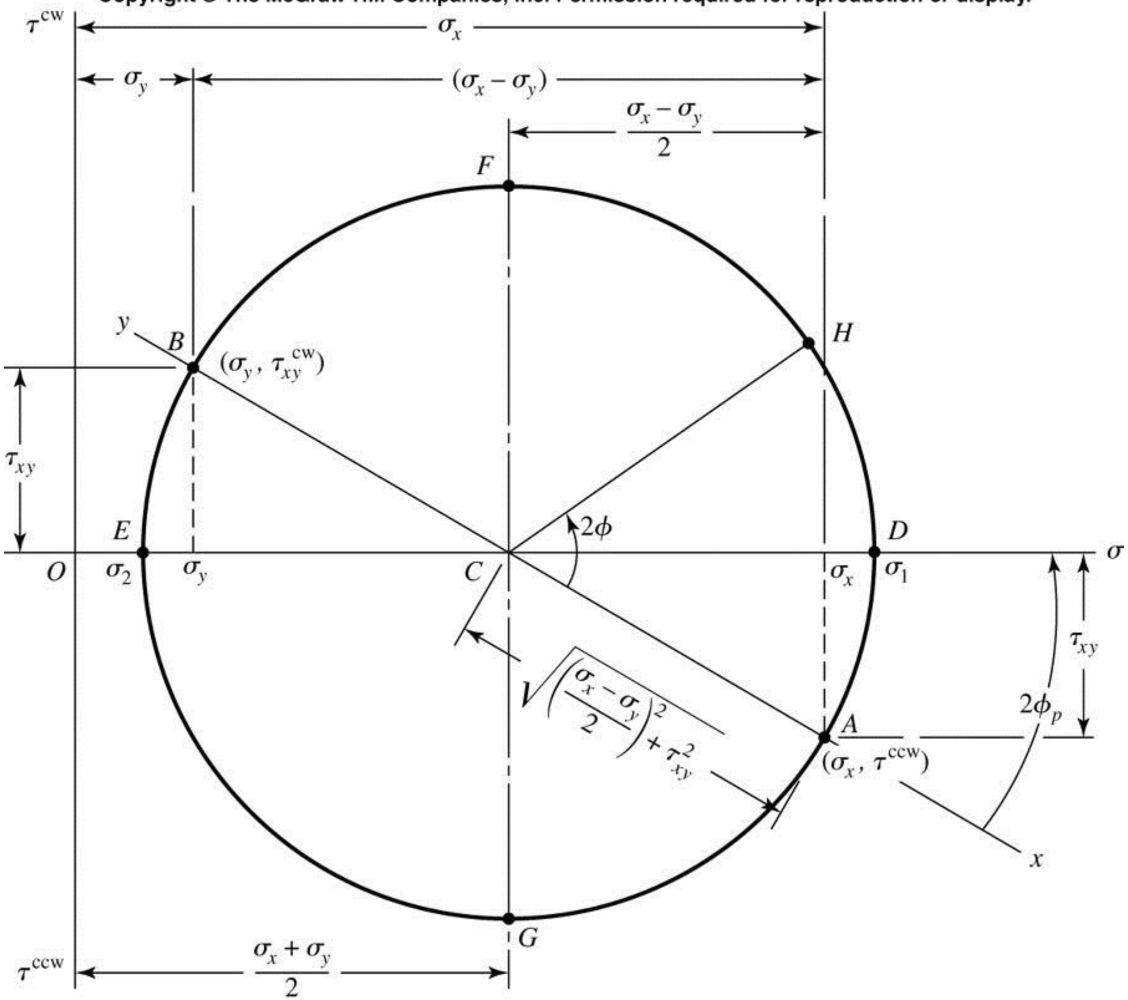
$$\tau = -(\sigma_x - \sigma_y)\sin\phi\cos\phi + \tau_{xy}(\cos^2\phi - \sin^2\phi)$$

$$\sigma = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\phi + \tau_{xy}\sin 2\phi$$

$$\tau = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\varphi + \tau_{xy}\cos 2\varphi$$

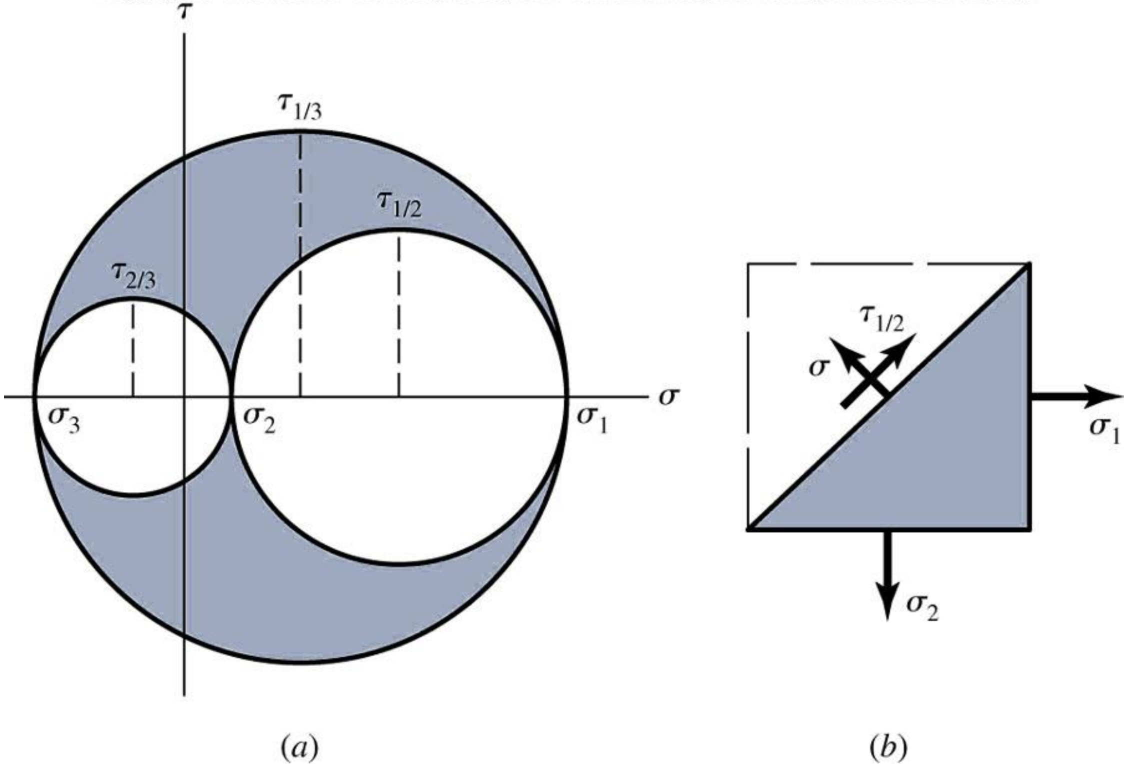
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad \tan 2\varphi_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\tau_{\substack{\max\\ \min}} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{\sigma_1 - \sigma_2}{2} \qquad \tan 2\varphi_s = \frac{\sigma_y - \sigma_x}{2\tau_{xy}}$$



$$\left(\sigma - \frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

Fig. 4.12 Mohr circle diagram.



$$\tau_{1/2} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\tau_{2/3} = \frac{\sigma_2 - \sigma_3}{2}$$

$$\tau_{1/3} = \frac{\sigma_1 - \sigma_3}{2}$$

Fig. 4.14 Mohr circles for triaxial stresses.

Octahedral stresses

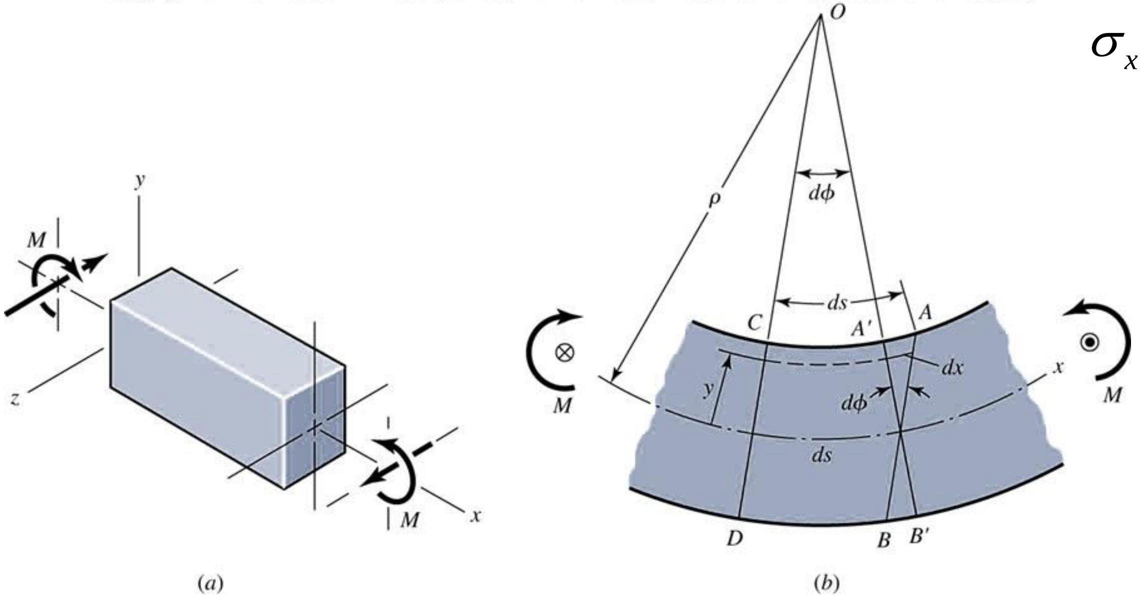
$$\tau_{oct} = \frac{1}{3}\left[\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2\right]^{1/2}$$

$$= \frac{1}{3}\left[\left(\sigma_x - \sigma_y\right)^2 + \left(\sigma_y - \sigma_z\right)^2 + \left(\sigma_z - \sigma_x\right)^2 + 6\left(\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2\right)\right]^{1/2}$$

$$\sigma_{oct} = \frac{1}{3}\left(\sigma_1 + \sigma_2 + \sigma_3\right) = \frac{1}{3}\left(\sigma_x + \sigma_y + \sigma_z\right)$$

Normal Stresses in Flexure

$$\sigma_x = \frac{M_y(zI_z - yI_{yz})}{I_y I_z - I_{yz}^2} + \frac{M_z(zI_{yz} - yI_y)}{I_y I_z - I_{yz}^2}$$



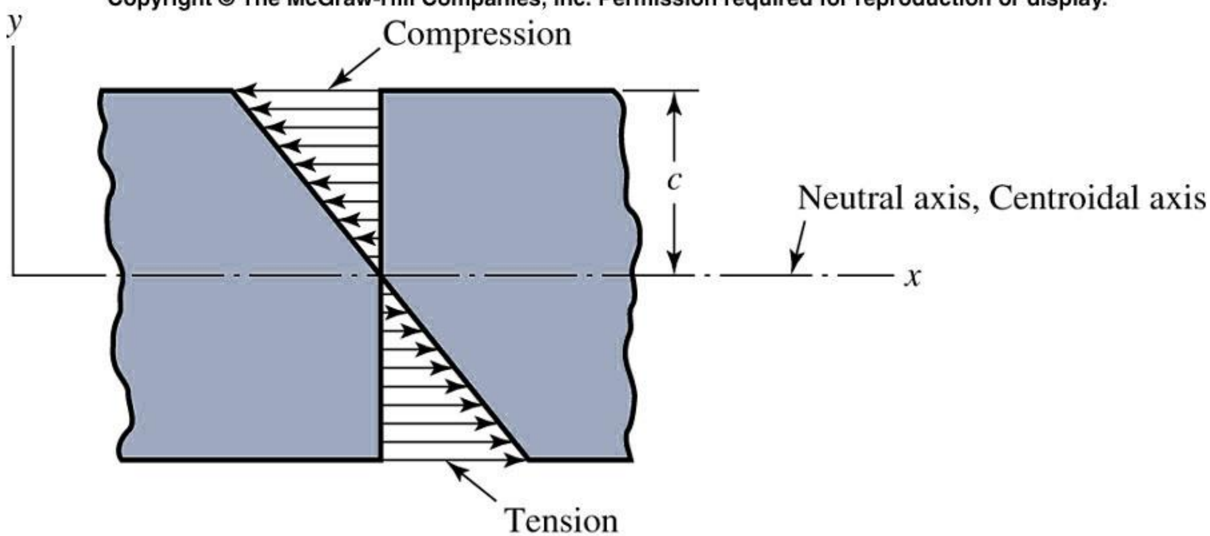
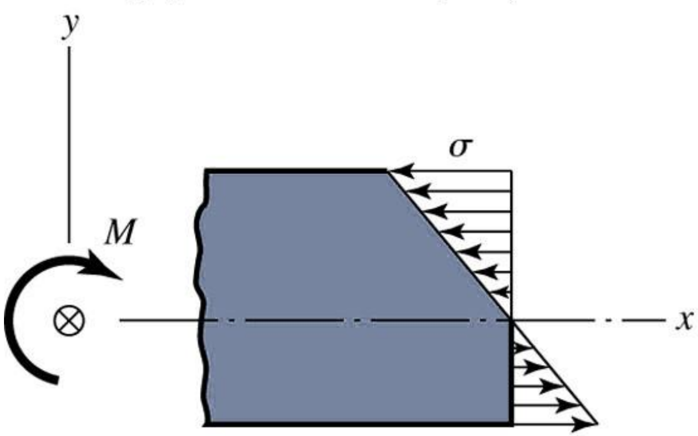
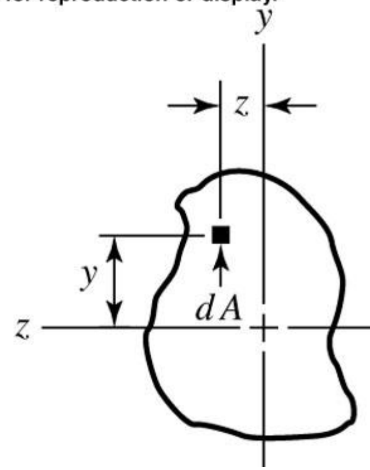
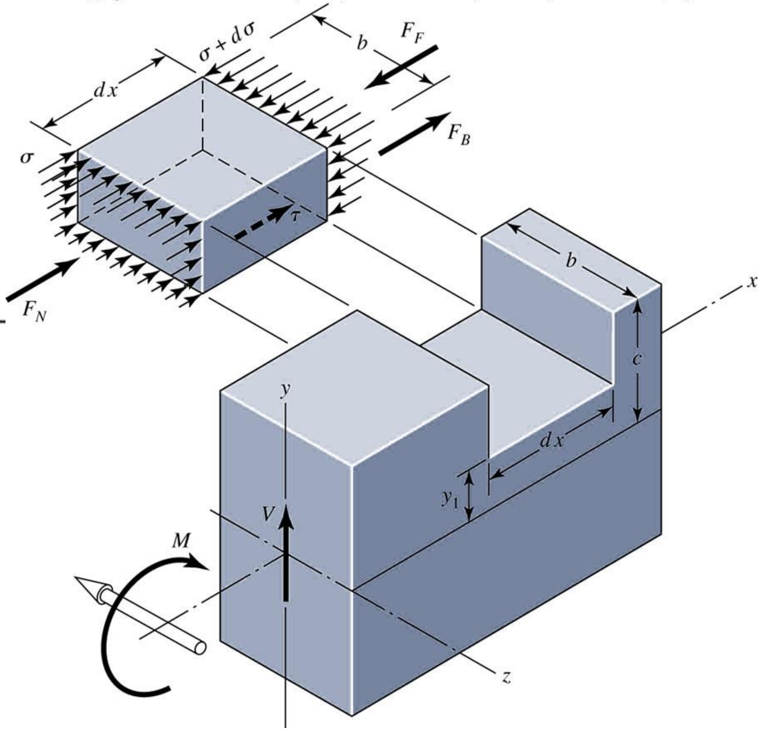


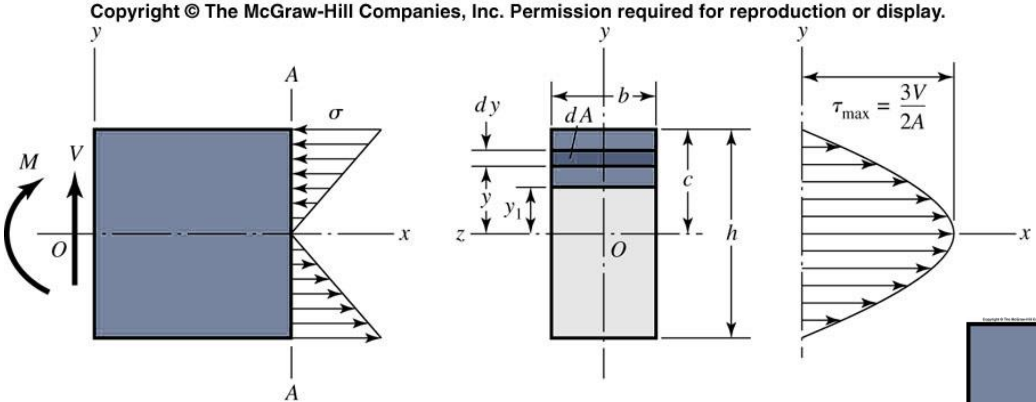
Fig. 4.15 Bending of a beam.

Shear Stresses in Beams

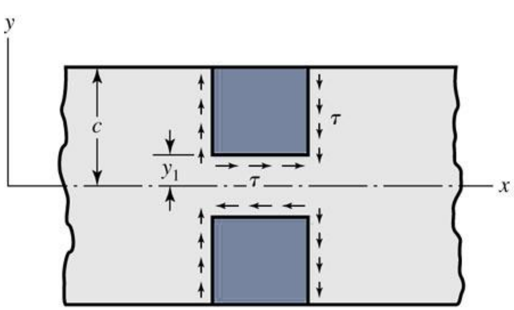








(a) (b) (c)

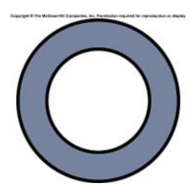


(d)

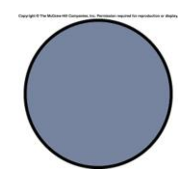
$$\tau = \frac{VQ}{Ib}$$

$$Q = \int_{y_1}^{c} y\,dA$$

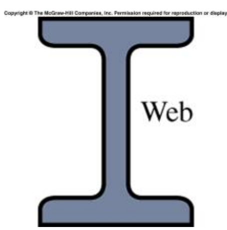
$$\tau_{\max} = \frac{3V}{2A}$$



$$\tau_{\max} = \frac{2V}{A}$$

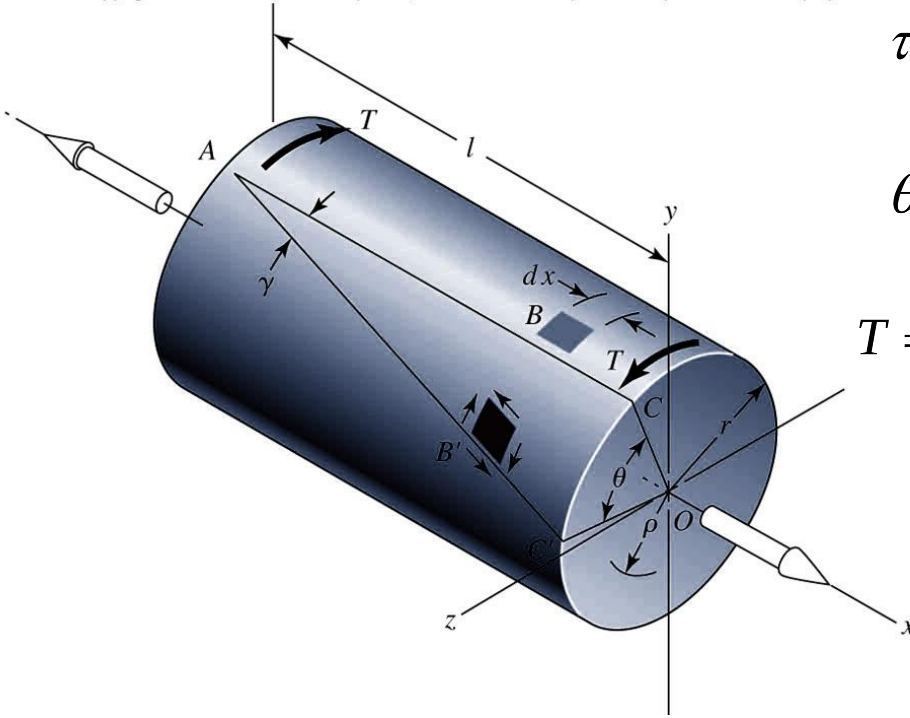


$$\tau_{\max} = \frac{4V}{3A}$$



$$\tau_{\max} = \frac{V}{A_{web}}$$

Torsion

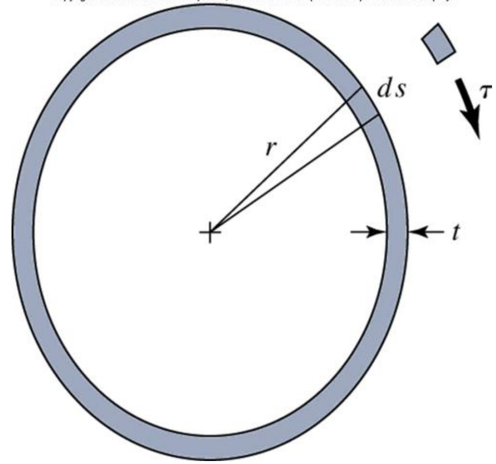


$$\tau = \frac{T\rho}{J}$$

$$\theta = \frac{Tl}{GJ}$$

$$T = 9.55\frac{H}{n}$$

Closed thin-walled tubes



$$\tau = \frac{T}{2At}$$

$$\theta_1 = \frac{Tl}{4GA^2t}$$

Rectangular cross sections

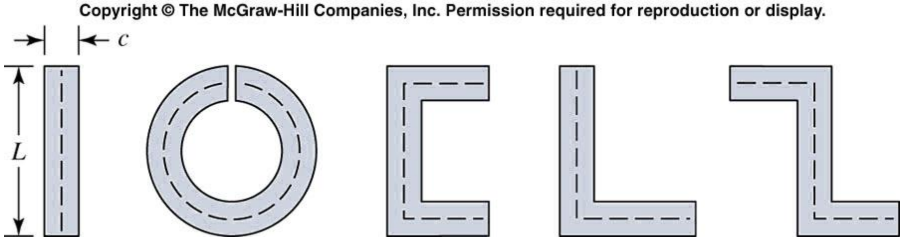
$$\tau_{\max} = \frac{T}{\alpha bc^2} = \frac{T}{bc^2}\left(3 + \frac{1.8}{b/c}\right) \qquad \theta = \frac{Tl}{\beta bc^3G}$$

b is the longer side

c is the shorter side

b/c	1.0	1.5	2.0	2.5	4.0	8.0	∞
α	0.208	0.231	0.246	0.258	0.282	0.307	0.333
β	0.141	0.196	0.228	0.249	0.281	0.307	0.333

Open thin-walled sections



$$\tau = G\theta_1 c = \frac{3T}{Lc^2}$$

Stress Concentration

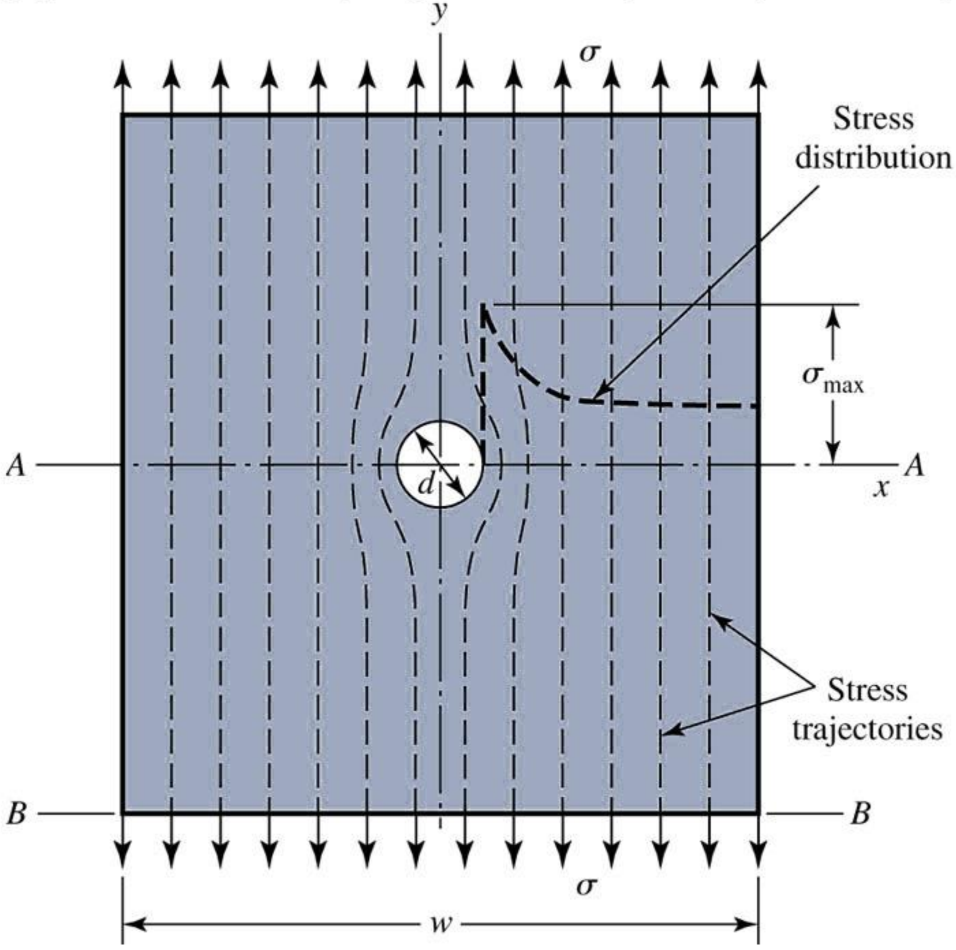


Fig. 4.31 Stress distribution near a hole in a plate loaded in tension.

$$K_t = \frac{\sigma_{\max}}{\sigma_0} \qquad K_{ts} = \frac{\tau_{\max}}{\tau_0}$$

For an infinite plate containing an elliptical hole

$$K_t = 1 + \frac{2b}{a}$$

b is the half-width

a is the half-height

In ductile materials the stress-concentration factor is not usually applied to predict the critical stresses, because plastic strain in the region of the stress has a strengthening effect.

In brittle materials the geometric stress-concentration factor K_t is applied to the nominal stress before comparing with strength.

Stresses in Cylinders

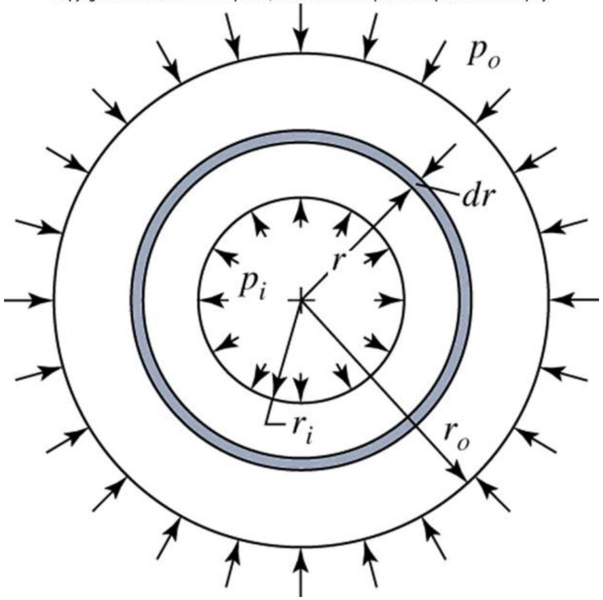
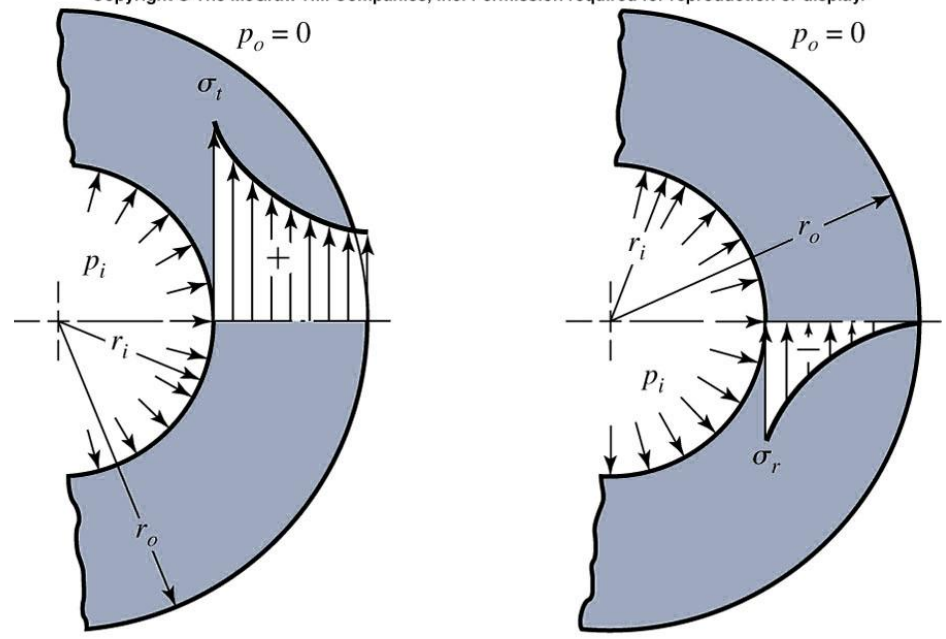


Fig. 4.33 A cylinder subjected to both internal and externel pressure

$$\sigma_t = \frac{p_i r_i^2 - p_0 r_0^2 - r_i^2 r_0^2 (p_0 - p_i)/r^2}{r_0^2 - r_i^2}$$

$$\sigma_r = \frac{p_i r_i^2 - p_0 r_0^2 + r_i^2 r_0^2 (p_0 - p_i)/r^2}{r_0^2 - r_i^2}$$

$$\sigma_l = \frac{p_i r_i^2}{r_0^2 - r_i^2}$$



(*a*) Tangential stress distribution

(*b*) Radial stress distribution

Fig. 4.34 Distribution of stresses in a thick-walled cylinder subjected to internal pressure.

Thin-walled vessels

$$\sigma_t = \frac{p_i r_i}{t} \qquad \sigma_l = \frac{p_i r_i}{2t}$$

Rotating Rings

$$\sigma_t = \rho\omega^2\left(\frac{3+\nu}{8}\right)\left(r_i^2 + r_o^2 + \frac{r_i^2 r_o^2}{r^2} - \frac{1+3\nu}{3+\nu} r^2\right)$$

$$\sigma_r = \rho\omega^2\left(\frac{3+\nu}{8}\right)\left(r_i^2 + r_o^2 - \frac{r_i^2 r_o^2}{r^2} - r^2\right)$$

Press and Shrink Fits

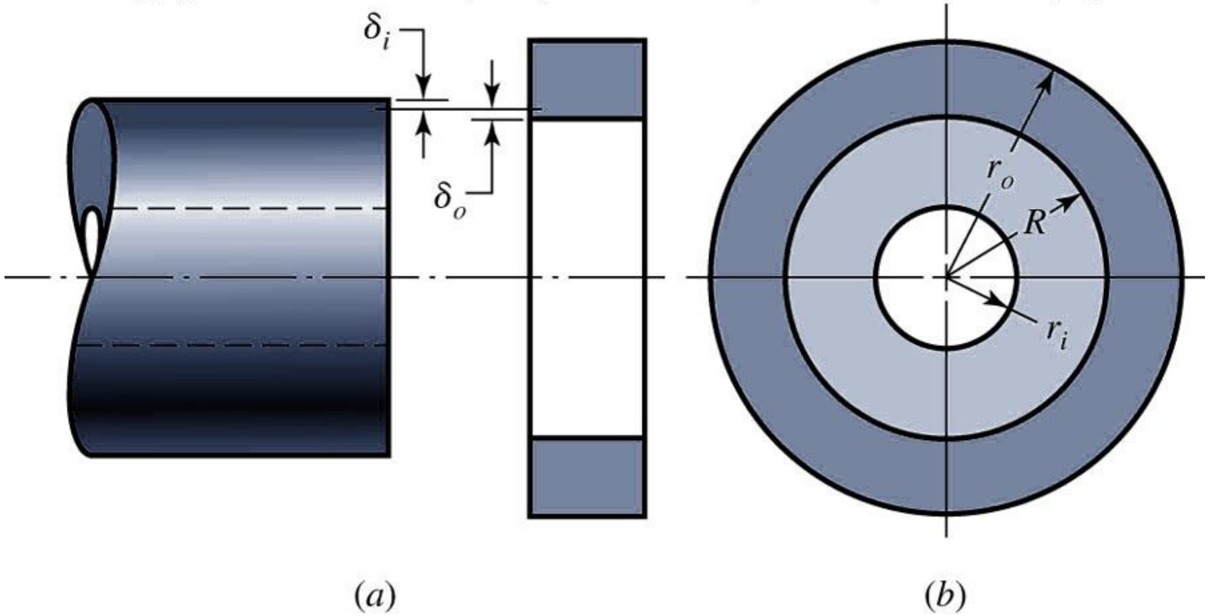


Fig. 4.35 Notation for press and shrink fits. (a) Unassembled parts. (b) after assembly.

$$\delta = \frac{pR}{E_0}\left(\frac{r_0^2 + R^2}{r_0^2 - R^2} + \nu_0\right) + \frac{pR}{E_i}\left(\frac{R^2 + r_i^2}{R^2 - r_i^2} - \nu_i\right)$$

$$\sigma_{it}(at\ R) = -p\left(\frac{R^2 + r_i^2}{R^2 - r_i^2}\right)$$

$$\sigma_{ot}(at\ R) = p\left(\frac{r_0^2 + R^2}{r_0^2 - R^2}\right)$$

Temperature Effects

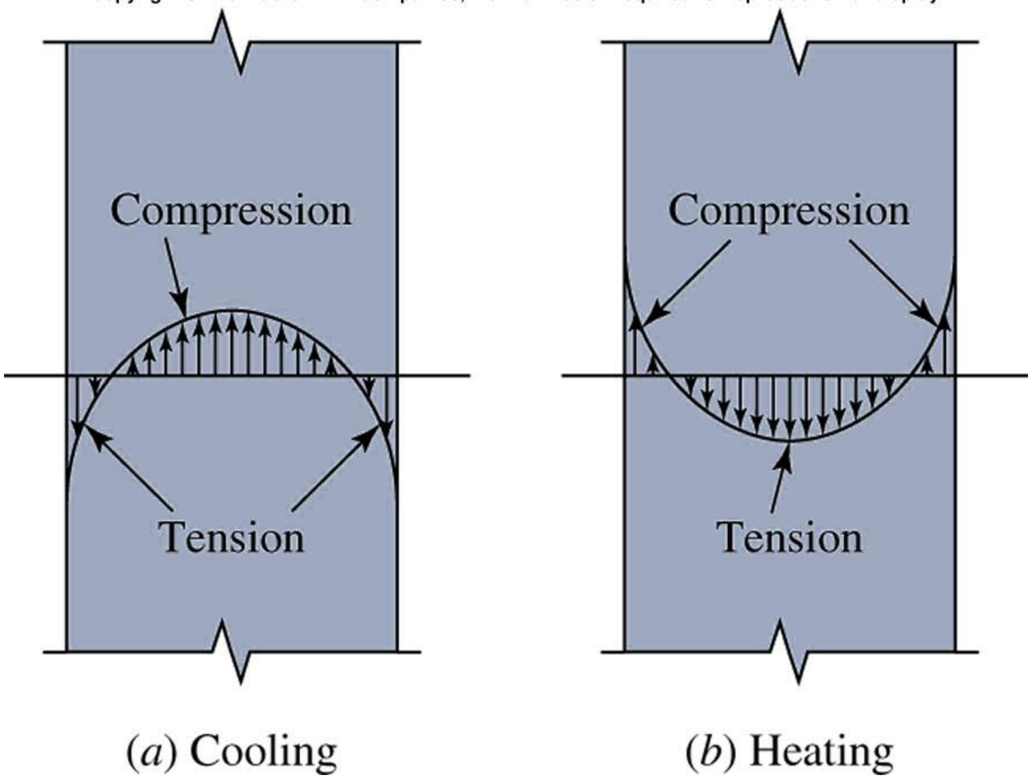


Fig. 4.36 Thermal stresses in an infinite slab during heating and cooling.

A thermal stress is one which arises because of the existence of a temperature gradient in a member.

$$\sigma = \alpha(\Delta T)E$$

Curved Members in Flexure

$$r_n = \frac{A}{\int \frac{dA}{r}} \qquad \sigma = \frac{My}{Ae(r_n - y)}$$

$$\sigma_i = \frac{Mc_i}{Aer_i} \qquad \sigma_o = -\frac{Mc_o}{Aer_o}$$

$$\sigma = \frac{M(r_n - r)}{Ar(r_c - r_n)}$$

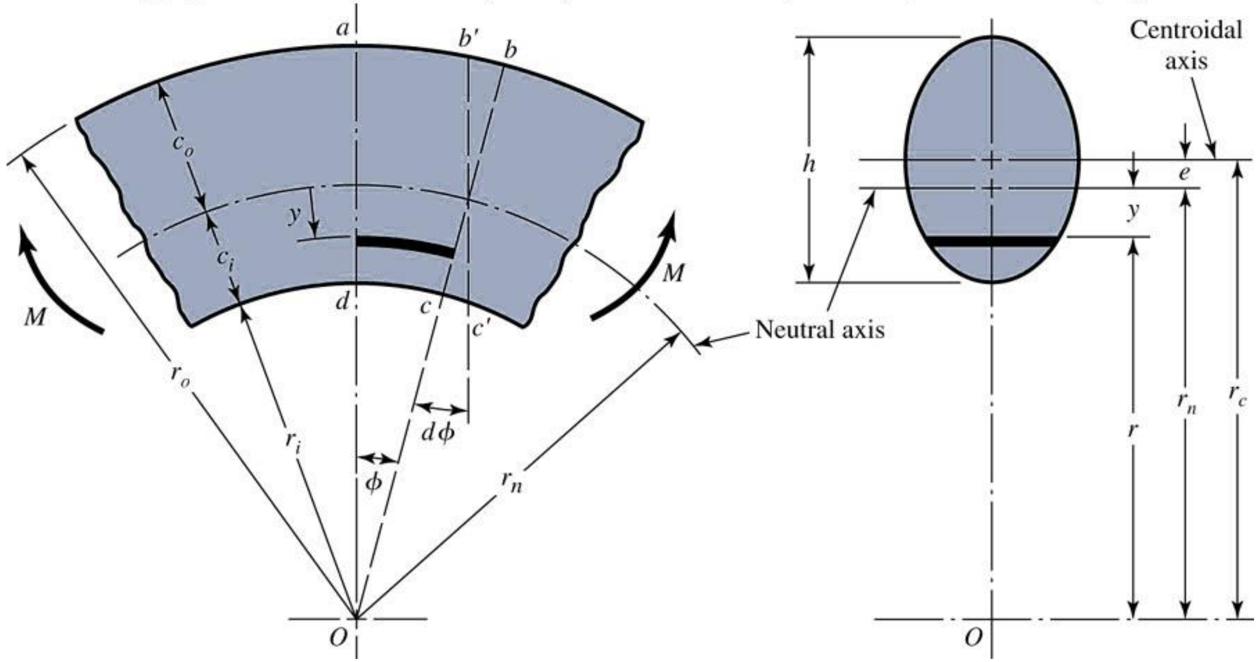
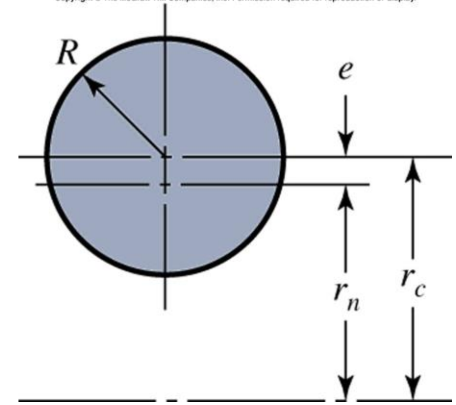
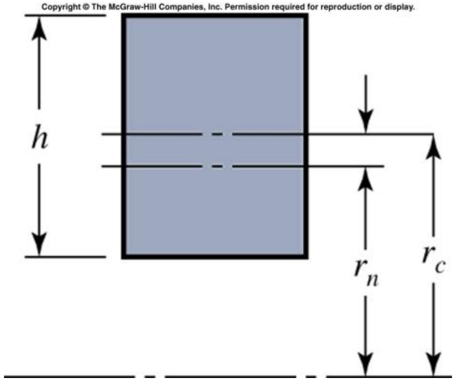


Fig. 4.37 Note that y is positive in the direction toward point O.

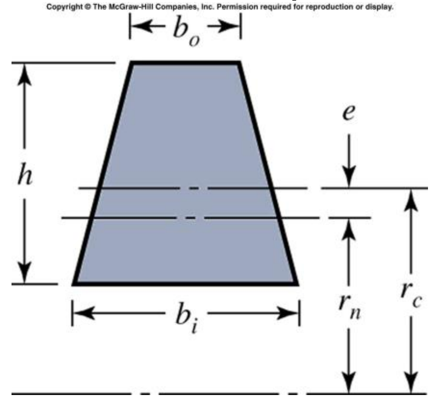


$$r_c = r_i + \frac{d}{2}$$

$$r_n = \frac{d^2}{4(2r_c - \sqrt{4r_c^2 - d^2})}$$



$$r_c = r_i + \frac{h}{2} \qquad r_n = \frac{h}{\ln(r_o / r_i)}$$



$$r_c = r_i + \frac{h}{3}\frac{b_i + 2b_o}{b_i + b_o} \qquad r_n = \frac{A}{b_o - b_i + [(b_i r_o - b_o r_i)/h]\ln(r_o / r_i)}$$

Contact Stresses

Contact stress problems arise in the contact of a wheel and a rail, in automotive valve cams and tappets, in mating gear teeth, and in the action of rolling bearings.
Typical failures are seen as cracks, pits, or flaking in the surface material.
The results presented here are due to Hertz and so are frequently known as Hertzian stresses.

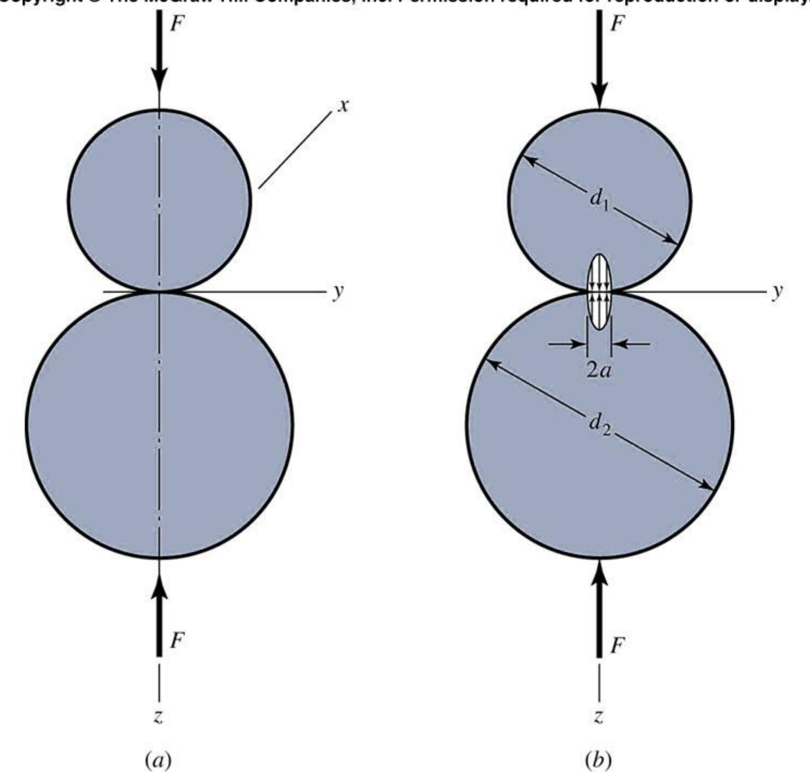


Fig. 4.42 (a) Two spheres held in contact by force F; (b) contact stress has an elliptical distribution across contact zone diameter 2a.

$$a = \sqrt[3]{\frac{3F}{8}\frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{1/d_1 + 1/d_2}} \qquad p_{\max} = \frac{3F}{2\pi a^2}$$

The maximum stress occur on the z axis, and these are principal stresses.

$$\sigma_x = \sigma_y = -p_{\max}\left[\left(1 - \frac{z}{a}\tan^{-1}\frac{1}{z/a}\right)(1+\nu) - \frac{1}{2\left(1+\frac{z^2}{a^2}\right)}\right]$$

$$\sigma_z = \frac{-p_{\max}}{1+\frac{z^2}{a^2}}$$

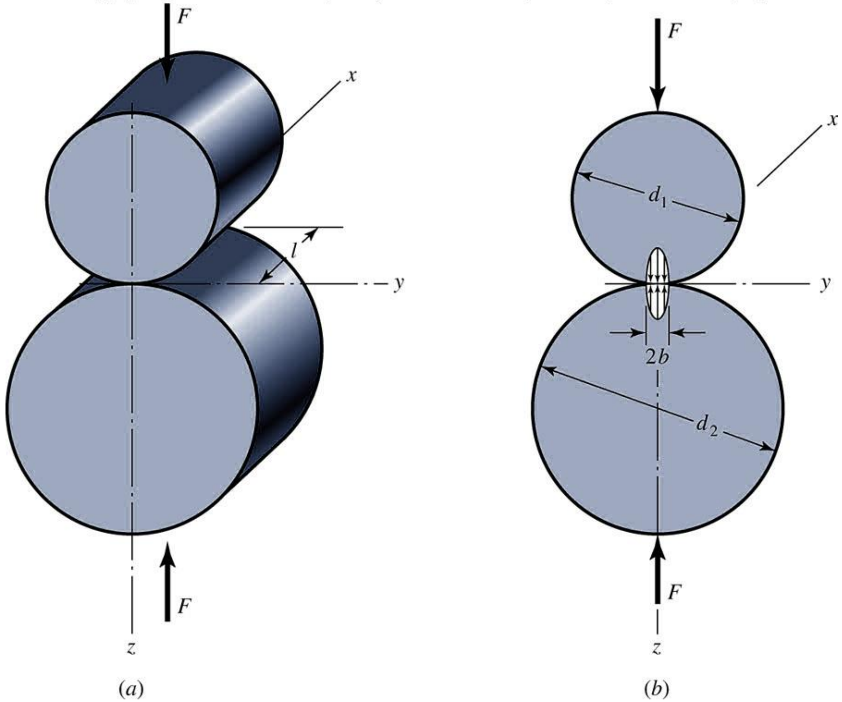


Fig. 4.44 (a) Two right circular cylinders held in contact by forces F uniformly distributed along cylinder length l. (b) Contact stress has an elliptical distribution across the contact zone width 2b.

$$b = \sqrt{\frac{2F}{\pi l} \frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{1/d_1 + 1/d_2}}$$

$$p_{\max} = \frac{3F}{\pi b l}$$

$$\sigma_x = -2\nu\, p_{\max}\left(\sqrt{1+\frac{z^2}{b^2}} - \frac{z}{b}\right)$$

$$\sigma_y = -p_{\max}\left[\left(2 - \frac{1}{\left(1+\frac{z^2}{b^2}\right)}\right)\sqrt{1+\frac{z^2}{b^2}} - 2\frac{z}{b}\right]$$

$$\sigma_z = \frac{-p_{\max}}{\sqrt{1+\frac{z^2}{b^2}}}$$

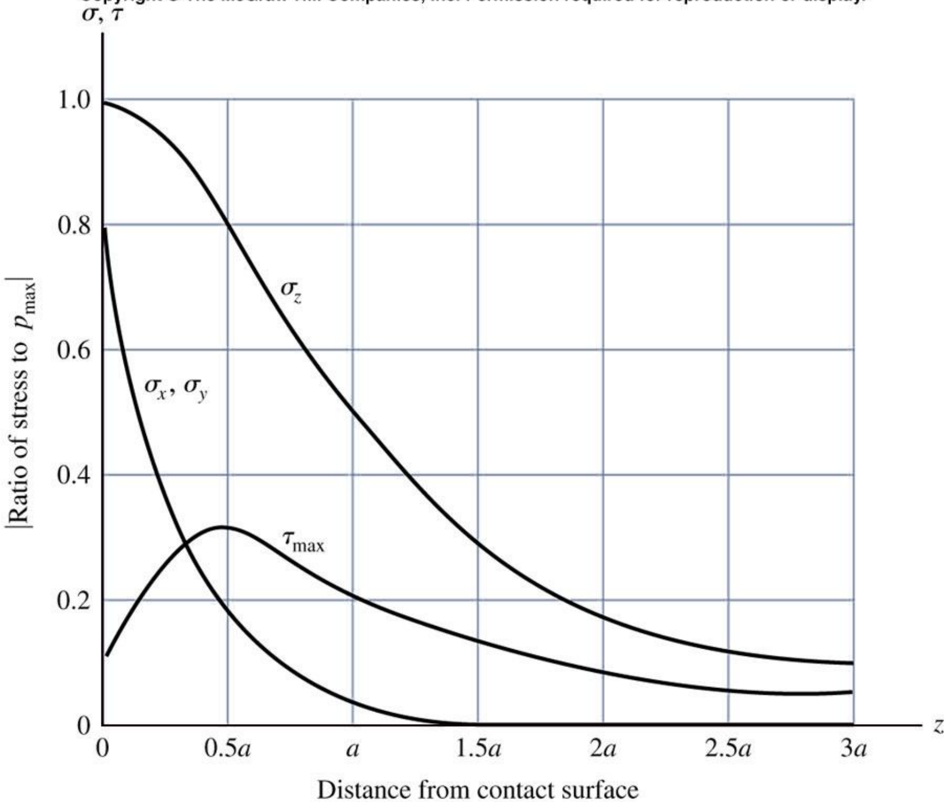


Fig. 4.43 Magnitude of the stress components below the surface as a function of the maximum pressure of contacting spheres.

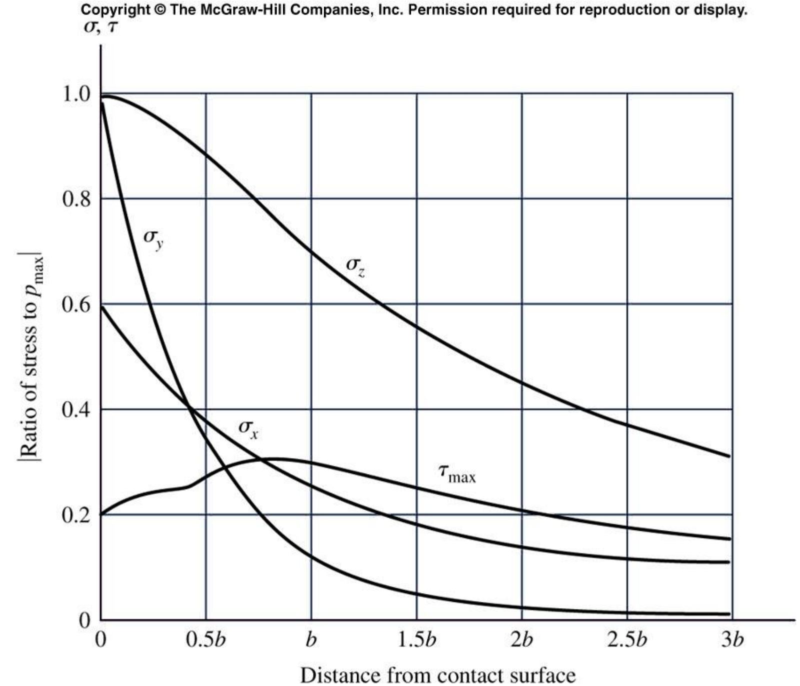


Fig. 4.45 Magnitude of the stress components below the surface as a function of the maximum pressure of contacting cylinders.