Sketch a free-body diagram of each element in the figure. Compute the magnitude and direction of each force using an algebraic or vector method, as specified.


Problem 3-3


Problem 3-4

3-3

$$
\begin{aligned}
& R_{O}=\frac{0.8}{\tan 30^{\circ}}=1.39 \mathrm{kN} \quad \text { Ans. } \\
& R_{A}=\frac{0.8}{\sin 30^{\circ}}=1.6 \mathrm{kN} \quad \text { Ans. }
\end{aligned}
$$



Step 1: Find $R_{A} \& R_{E}$
$h=\frac{4.5}{\tan 30^{\circ}}=7.794 \mathrm{~m}$
$\Sigma M_{A}=0$
$9 R_{E}-7.794\left(400 \cos 30^{\circ}\right)$
$-4.5\left(400 \sin 30^{\circ}\right)=0$
$R_{E}=400 \mathrm{~N}$ Ans.
$\sum F_{x}=0 \quad R_{\text {fx }}+400 \cos 30^{\circ}=0$

$$
R_{A x}=-346.4 \mathrm{~N}
$$


$\sum F_{y}=0 \quad R_{1 y}+400-400 \sin 30^{\circ}=0$
$R_{\text {ty }}=-200 \mathrm{~N}$
$R_{A}=\sqrt{346.4^{2}+200^{2}}=400 \mathrm{~N}$ Ans.

Step 2: Find components of $R_{C}$ on link 4 and $R_{D}$

$$
\begin{aligned}
& \sum M_{C}=0 \\
& \quad 400(4.5)-(7.794-1.9) R_{D}=0 \\
& \quad R_{D}=305.4 \mathrm{~N} \quad \text { Ans. } \\
& \sum F_{x}=0 \Rightarrow \quad\left(R_{C x}\right)_{4}=305.4 \mathrm{~N} \\
& \sum F_{y}=0 \Rightarrow \quad\left(R_{C y}\right)_{4}=-400 \mathrm{~N}
\end{aligned}
$$



Step 3: Find components of $R_{C}$ on link 2

$$
\begin{aligned}
& \sum F_{x}=0 \\
& \quad\left(R_{C x}\right)_{2}+305.4-346.4=0 \\
& \quad\left(R_{C x}\right)_{2}=41 \mathrm{~N} \\
& \sum F_{y}=0 \\
& \quad\left(R_{C y}\right)_{2}=200 \mathrm{~N}
\end{aligned}
$$



For each section illustrated, find the second moment of area, the location of the neutral axis, and the distances from the neutral axis to the top and bottom surfaces. Consider that the section is transmitting a positive bending moment about the $z$ axis, $M z=1.13 \mathrm{kN} . \mathrm{m}$. Determine the resulting stresses at the top and bottom surfaces and at every abrupt change in the cross section.

(a)

(c) Dimensions in mm
(a) Let $a=$ total area of entire envelope Let $b=$ area of side notch
$A=a-2 b=40(3)(25)-25(34)=2150 \mathrm{~mm}^{2}$
$I=I_{a}-2 I_{b}=\frac{1}{12}(40)(75)^{3}-\frac{1}{12}(34)(25)^{3}$
$I=1.36\left(10^{6}\right) \mathrm{mm}^{4} \quad$ Ans.


Dimensions in mm.
(c)

Use two negative areas.
$A_{a}=625 \mathrm{~mm}^{2}, A_{b}=5625 \mathrm{~mm}^{2}, A_{c}=10000 \mathrm{~mm}^{2}$
$A=10000-5625-625=3750 \mathrm{~mm}^{2}$;


$$
\begin{aligned}
& \bar{y}_{a}=6.25 \mathrm{~mm}, \bar{y}_{b}=50 \mathrm{~mm}, \bar{y}_{c}=50 \mathrm{~mm} \\
& \bar{y}=\frac{10000(50)-5625(50)-625(6.25)}{3750}=57.29 \mathrm{~mm} \quad \text { Ans. } \\
& c_{1}=100-57.29=42.71 \mathrm{~mm} \text { Ans. } \\
& I_{a}=\frac{50(12.5)^{3}}{12}=8138 \mathrm{~mm}^{4} \\
& I_{b}=\frac{75(75)^{3}}{12}=2.637\left(10^{6}\right) \mathrm{mm}^{4} \\
& I_{c}=\frac{100(100)^{3}}{12}=8.333\left(10^{6}\right) \mathrm{in}^{4} \\
& I_{1}=\left[8.333\left(10^{6}\right)+10000(7.29)^{2}\right]-\left[2.637\left(10^{6}\right)+5625(7.29)^{2}\right]-\left[8138+625(57.29-6.25)^{2}\right] \\
& I_{1}=4.29\left(10^{6}\right) \mathrm{in}^{4} \quad \text { Ans. }
\end{aligned}
$$

The cast-iron bell-crank lever depicted in the figure is acted upon by forces $F_{1}$ of 2.4 kN and $F_{2}$ of 3.2 kN . The section $A-A$ at the central pivot has a curved inner surface with a radius of $r_{i}=25 \mathrm{~mm}$. Estimate the stresses at the inner and outer surfaces of the curved portion of the lever.


Section A-A
Dimensions in mm

## $r_{i}=25 \mathrm{~mm}, r_{o}=r_{i}+h=25+87=112 \mathrm{~mm}, r_{c}=25+87 / 2=68.5 \mathrm{~mm}$

 The radius of the neutral axis is found from Eq. (3-63), given below.$$
r_{n}=\frac{A}{\int(d A / r)}
$$

For a rectangular area with constant width $b$, the denominator is

$$
\int_{r_{i}}^{r_{o}}\left(\frac{b d r}{r}\right)=b \ln \frac{r_{o}}{r_{i}}
$$

Applying this equation over each of the four rectangular areas,

$$
\begin{aligned}
& \int \frac{d A}{r}=9\left(\ln \frac{45}{25}\right)+31\left(\ln \frac{54.5}{45}\right)+31\left(\ln \frac{92}{82.5}\right)+9\left(\ln \frac{112}{92}\right)=16.3769 \\
& A=2[20(9)+31(9.5)]=949 \mathrm{~mm}^{2} \\
& r_{n}=\frac{A}{\int(d A / r)}=\frac{949}{16.3769}=57.9475 \mathrm{~mm} \\
& e=r_{c}-r_{n}=68.5-57.9475=10.5525 \mathrm{~mm} \\
& c_{i}=r_{n}-r_{i}=57.9475-25=32.9475 \mathrm{~mm} \\
& c_{o}=r_{o}-r_{n}=112-57.9475=54.0525 \mathrm{~mm} \\
& M=150 F_{2}=150(3.2)=480 \mathrm{kN} \cdot \mathrm{~mm}
\end{aligned}
$$

We need to find the forces transmitted through the section in order to determine the axial stress. It is not immediately obvious which plane should be used for resolving the axial versus shear directions. It is convenient to use the plane containing the reaction force at the bushing, which assumes its contribution resolves entirely into shear force. To find the angle of this plane, find the resultant of $F_{1}$ and $F_{2}$.

$$
\begin{aligned}
& F_{x}=F_{1 x}+F_{2 x}=2.4 \cos 60^{\circ}+3.2 \cos 0^{\circ}=4.40 \mathrm{kN} \\
& F_{y}=F_{1 y}+F_{2 y}=2.4 \sin 60^{\circ}+3.2 \sin 0^{\circ}=2.08 \mathrm{kN} \\
& F=\left(4.40^{2}+2.08^{2}\right)^{1 / 2}=4.87 \mathrm{kN}
\end{aligned}
$$

This is the pin force on the lever which acts in a direction

$$
\theta=\tan ^{-1} \frac{F_{y}}{F_{x}}=\tan ^{-1} \frac{2.08}{4.40}=25.3^{\circ}
$$



On the surface $25.3^{\circ}$ from the horizontal, find the internal forces in the tangential and normal directions. Resolving $F_{1}$ into components,

$$
\begin{aligned}
& F_{t}=2.4 \cos \left(60^{\circ}-25.3^{\circ}\right)=1.97 \mathrm{kN} \\
& F_{n}=2.4 \sin \left(60^{\circ}-25.3^{\circ}\right)=1.37 \mathrm{kN}
\end{aligned}
$$

The transverse shear stress is zero at the inner and outer surfaces. Using Eq. (3-65) for the bending stress, and combining with the axial stress due to $F_{n}$,

$$
\begin{aligned}
& \sigma_{i}=\frac{F_{n}}{A}+\frac{M c_{i}}{A e r_{i}}=\frac{1370}{949}+\frac{[(3200)(150)](32.9475)}{949(10.5525)(25)}=64.6 \mathrm{MPa} \quad \text { Ans. } \\
& \sigma_{o}=\frac{F_{n}}{A}-\frac{M c_{o}}{A e r_{o}}=\frac{1370}{949}-\frac{[(3200)(150)](54.0525)}{949(10.5525)(112)}=-21.7 \mathrm{MPa} \quad \text { Ans. }
\end{aligned}
$$

