A ductile hot-rolled steel bar has a minimum yield strength in tension and compression of 350 MPa. Using the distortion-energy and maximum-shear-stress theories determine the factors of safety for the following plane stress states:

(*a*) $\sigma_x = 100$ MPa, $\sigma_y = 100$ MPa

(*b*) $\sigma_x = 100$ MPa, $\sigma_y = 50$ MPa

(*c*) $\sigma_x = 100$ MPa, $\tau_{xy} = -75$ MPa

(*d*) $\sigma_x = -50$ MPa, $\sigma_y = -75$ MPa, $\tau_{xy} = -50$ MPa

(*e*) $\sigma_x = 100$ MPa, $\sigma_y = 20$ MPa, $\tau_{xy} = -20$ MPa

$S_y = 350$ MPa.

MSS: $\sigma_1 - \sigma_3 = S_y / n \quad \Rightarrow \quad n = \dfrac{S_y}{(\sigma_1 - \sigma_3)}$

DE: $\sigma' = \left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2} = \left(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2\right)^{1/2}$

$$n = \frac{S_y}{\sigma'}$$

(a) MSS: $\sigma_1 = 100$ MPa, $\sigma_2 = 100$ MPa, $\sigma_3 = 0$

$$n = \frac{350}{100 - 0} = 3.5 \quad \textit{Ans.}$$

DE: $\sigma' = (100^2 - 100(100) + 100^2)^{1/2} = 100$ MPa, $\quad n = \dfrac{350}{100} = 3.5 \quad \textit{Ans.}$

(b) MSS: $\sigma_1 = 100 \text{ MPa}, \sigma_2 = 50 \text{ MPa}, \sigma_3 = 0$

$$n = \frac{350}{100 - 0} = 3.5 \quad \textit{Ans.}$$

DE: $$\sigma' = (100^2 - 100(50) + 50^2)^{1/2} = 86.6 \text{ MPa}, \quad n = \frac{350}{86.6} = 4.04 \quad \textit{Ans.}$$

(c) $$\sigma_A,\ \sigma_B = \frac{100}{2} \pm \sqrt{\left(\frac{100}{2}\right)^2 + (-75)^2} = 140,\ -40 \text{ MPa}$$

$$\sigma_1 = 140,\ \sigma_2 = 0,\ \sigma_3 = -40 \text{ MPa}$$

MSS: $$n = \frac{350}{140 - (-40)} = 1.94 \quad \textit{Ans.}$$

DE: $$\sigma' = \left[100^2 + 3\left(-75^2\right)\right]^{1/2} = 164 \text{ MPa}, \quad n = \frac{350}{164} = 2.13 \quad \textit{Ans.}$$

$$\textbf{(d)}\ \sigma_A, \sigma_B = \frac{-50-75}{2} \pm \sqrt{\left(\frac{-50+75}{2}\right)^2 + (-50)^2} = -11.0, -114.0 \text{ MPa}$$

$$\sigma_1 = 0,\ \sigma_2 = -11.0,\ \sigma_3 = -114.0 \text{ MPa}$$

MSS: $$n = \frac{350}{0-(-114.0)} = 3.07 \quad \textit{Ans.}$$

DE: $$\sigma' = [(-50)^2 - (-50)(-75) + (-75)^2 + 3(-50)^2]^{1/2} = 109.0 \text{ MPa}$$

$$n = \frac{350}{109.0} = 3.21 \quad \textit{Ans.}$$

$$\textbf{(e)}\ \sigma_A, \sigma_B = \frac{100+20}{2} \pm \sqrt{\left(\frac{100-20}{2}\right)^2 + (-20)^2} = 104.7,\ 15.3 \text{ MPa}$$

$$\sigma_1 = 104.7,\ \sigma_2 = 15.3,\ \sigma_3 = 0 \text{ MPa}$$

MSS: $$n = \frac{350}{104.7-0} = 3.34 \quad \textit{Ans.}$$

DE: $$\sigma' = \left[100^2 - 100(20) + 20^2 + 3(-20)^2\right]^{1/2} = 98.0 \text{ MPa}$$

$$n = \frac{350}{98.0} = 3.57 \quad \textit{Ans.}$$

A countershaft carrying two V-belt pulleys is shown in the figure. Pulley A receives power from a motor through a belt with the belt tensions shown. The power is transmitted through the shaft and delivered to the belt on pulley B. Assume the belt tension on the loose side at B is 15 percent of the tension on the tight side.

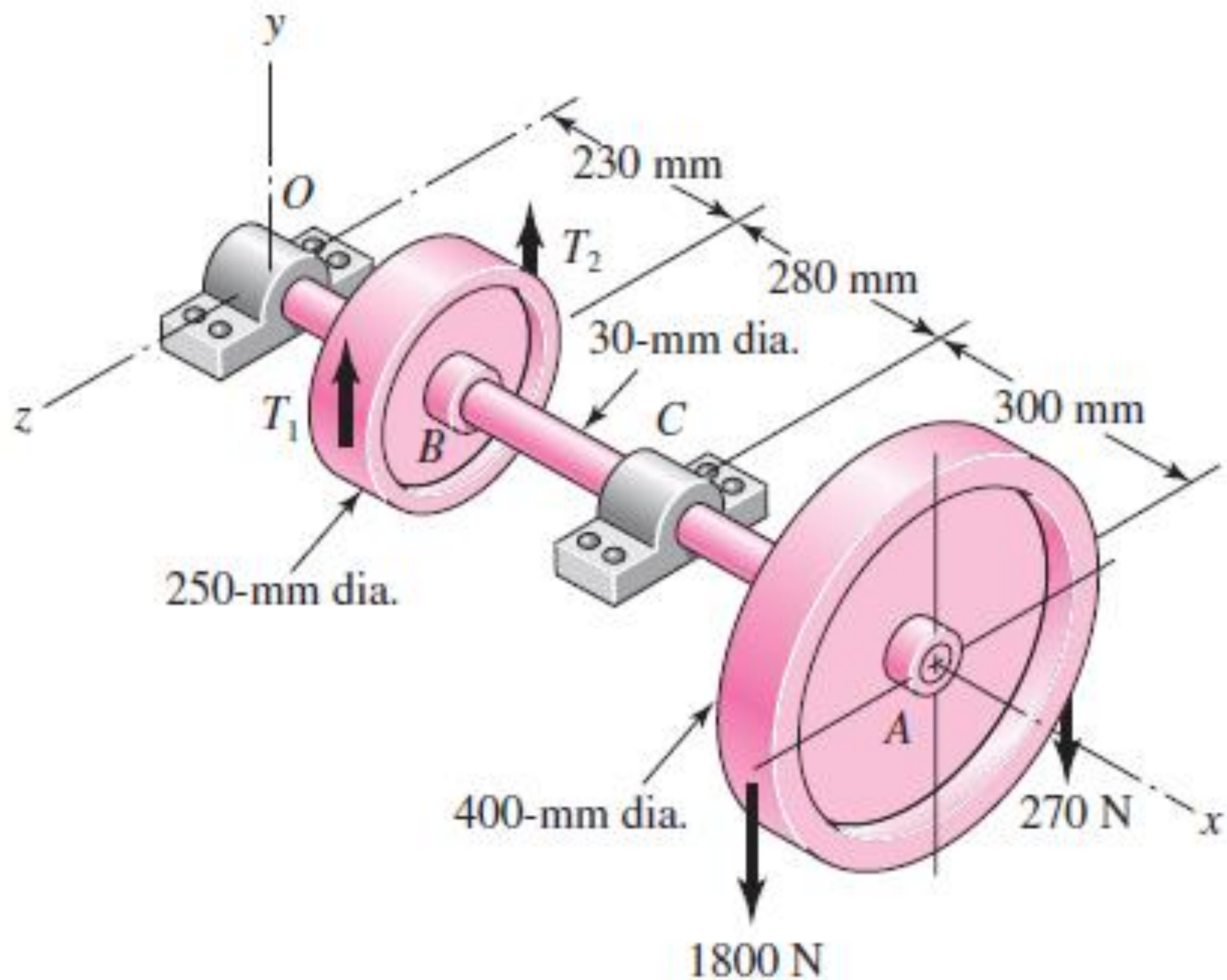
(*a*) Determine the tensions in the belt on pulley B, assuming the shaft is running at a constant speed.

(*b*) Find the magnitudes of the bearing reaction forces, assuming the bearings act as simple supports.

(*c*) Draw shear-force and bending-moment diagrams for the shaft. If needed, make one set for the horizontal plane and another set for the vertical plane.

(*d*) At the point of maximum bending moment, determine the bending stress and the torsional shear stress.

(*e*) At the point of maximum bending moment, determine the principal stresses and the maximum shear stress.



y
O
z
230 mm
T_2
280 mm
30-mm dia.
T_1
B
C
300 mm
250-mm dia.
A
400-mm dia.
270 N
x
1800 N

(a)

$$T_2 = 0.15T_1$$

$$\sum T = 0 = (1800 - 270)(200) + (T_2 - T_1)(125) = 306\left(10^3\right) + 125\left(0.15T_1 - T_1\right)$$

$$306\left(10^3\right) - 106.25T_1 = 0 \qquad \Rightarrow \qquad T_1 = 2880 \text{ N} \quad \textit{Ans.}$$

$$T_2 = 0.15\left(2880\right) = 432 \text{ N} \quad \textit{Ans.}$$

(b)

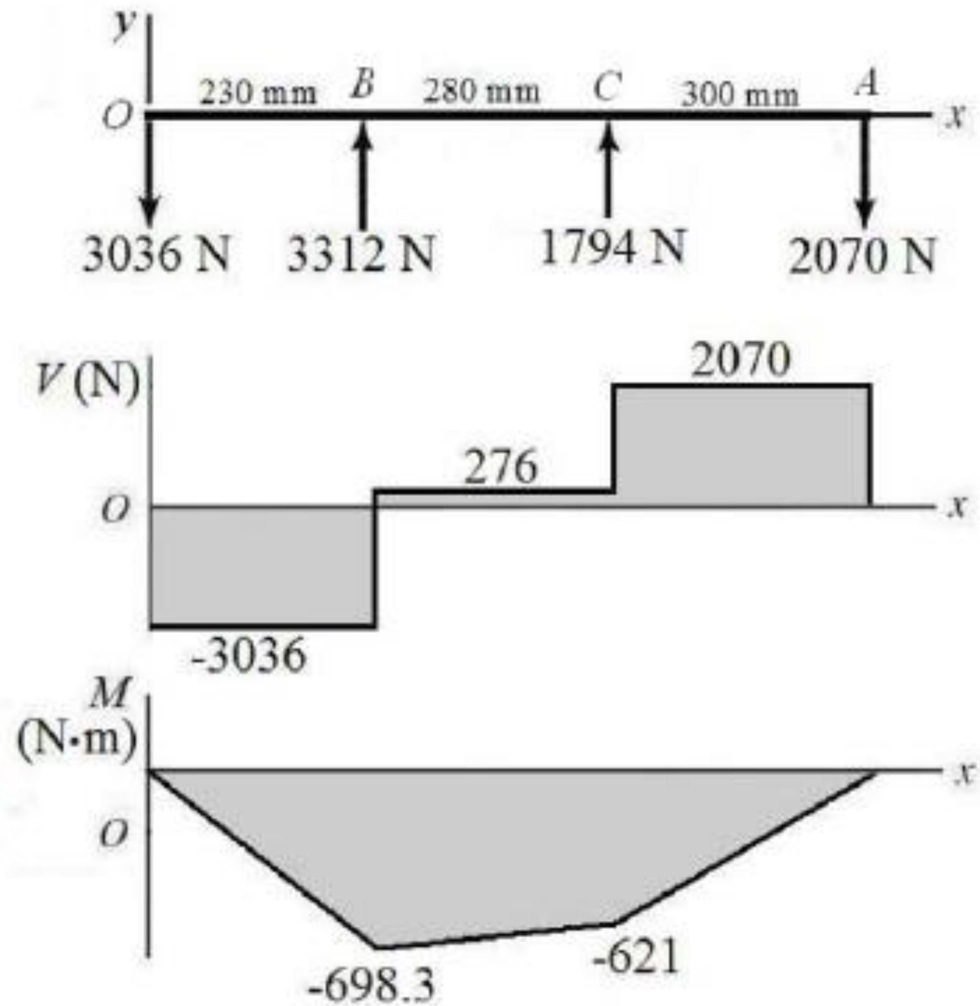
$$\sum M_O = 0 = 3312(230) + R_C(510) - 2070(810)$$

$$R_C = 1794 \text{ N} \quad \textit{Ans.}$$

$$\sum F_y = 0 = R_O + 3312 + 1794 - 2070$$

$$R_O = -3036 \text{ N} \quad \textit{Ans.}$$

(c)



y
O
230 mm
B
280 mm
C
300 mm
A
x
3036 N
3312 N
1794 N
2070 N
V (N)
2070
276
O
x
-3036
M
(N·m)
x
O
-698.3
-621

(d) The maximum bending moment is at $x = 230$ mm, and is $M = -698.3$ N·m. Since the shaft rotates, each stress element will experience both positive and negative bending stress as it moves from tension to compression. The torque transmitted through the shaft from A to B is $T = (1800 - 270)(0.200) = 306$ N·m. For a stress element on the outer surface where the bending stress and the torsional stress are both maximum,

$$\sigma = \frac{Mc}{I} = \frac{32M}{\pi d^3} = \frac{32(698.3)}{\pi(0.030)^3} = 263\left(10^3\right)\text{ Pa} = 263\text{ MPa} \quad \textit{Ans.}$$

$$\tau = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(306)}{\pi(0.030)^3} = 57.7\left(10^6\right)\text{ Pa} = 57.7\text{MPa} \qquad \textit{Ans.}$$

(e)

$$\sigma_1,\ \sigma_2 = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xy}\right)^2} = \frac{263}{2} \pm \sqrt{\left(\frac{263}{2}\right)^2 + (57.7)^2}$$

$$\sigma_1 = 275\text{ MPa} \qquad \textit{Ans.}$$

$$\sigma_2 = -12.1\text{ MPa} \qquad \textit{Ans.}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \left(\tau_{xy}\right)^2} = \sqrt{\left(\frac{263}{2}\right)^2 + (57.7)^2} = 144\text{ MPa} \qquad \textit{Ans.}$$

f) Use both the maximum-shear-stress theory and the distortion-energy theory, and compare the results. The material is 1018 CD steel.

$S_y = 370$ MPa.

$\sigma_1 = 275$ MPa, $\sigma_2 = -12.1$ MPa, and $\tau_{max} = 144$ MPa

The state of stress is *plane stress*. Thus, the three-dimensional principal stresses are

$\sigma_1 = 275$ MPa, $\sigma_2 = 0$, and $\sigma_3 = -12.1$ MPa

MSS: From Eq. (5-3), $n = \dfrac{S_y}{\sigma_1 - \sigma_3} = \dfrac{370}{275-(-12.1)} = 1.29$ *Ans.*

DE: From Eqs. (5-13) and (5-19)

$$n = \frac{S_y}{\sigma'} = \frac{S_y}{\left(\sigma_A^2 - \sigma_A\sigma_B + \sigma_B^2\right)^{1/2}} = \frac{370}{\left[275^2 - 275(-12.1) + (-12.1)^2\right]^{1/2}}$$

$= 1.32$ *Ans.*