EXAMPLE 5-5

A solid steel shaft AB shown in Fig. 5–14 is to be used to transmit 3750 W from the motor M to which it is attached. If the shaft rotates at $\omega = 175$ rpm and the steel has an allowable shear stress of $\tau_{allow} = 100$ MPa, determine the required diameter of the shaft to the nearest mm.

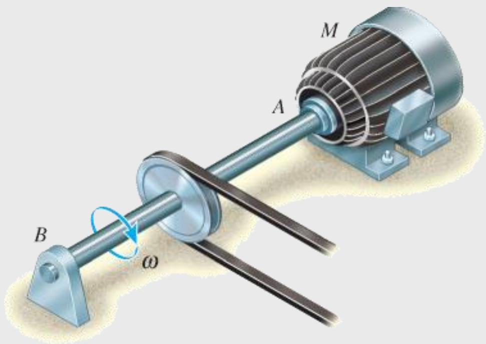


Fig. 5–14

SOLUTION

The torque on the shaft is determined from Eq. 5–10, that is, $P = T\omega$. Expressing P in Newton-metres per second and ω in radians/second, we have

$$P = 3750 \text{ N·m/s}$$

$$\omega = \frac{175 \text{ rev}}{\text{min}}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 18.33 \text{ rad/s}$$

Thus,

$P = T\omega;$ $$3750 \text{ N·m/s} = T(18.33) \text{ rad/s}$$

$$T = 204.6 \text{ N·m}$$

Applying Eq. 5–12 yields

$$\frac{J}{c} = \frac{\pi}{2}\frac{c^4}{c} = \frac{T}{\tau_{\text{allow}}}$$

$$c = \left(\frac{2T}{\pi\tau_{\text{allow}}}\right)^{1/3} = \left(\frac{2(204.6 \text{ N}\cdot\text{m})(1000 \text{ mm/m})}{\pi(100 \text{ N/mm}^2)}\right)^{1/3}$$

$$c = 10.92 \text{ mm}$$

Since $2c = 21.84$ mm, select a shaft having a diameter of

$$d = 22 \text{ mm}$$ ***Ans.***

EXAMPLE 5–6

A tubular shaft, having an inner diameter of 30 mm and an outer diameter of 42 mm, is to be used to transmit 90 kW of power. Determine the frequency of rotation of the shaft so that the shear stress will not exceed 50 MPa.

SOLUTION

The maximum torque that can be applied to the shaft is determined from the torsion formula.

$$\tau_{max} = \frac{Tc}{J}$$

$$50(10^6)\ \text{N/m}^2 = \frac{T(0.021\ \text{m})}{(\pi/2)[(0.021\ \text{m})^4 - (0.015\ \text{m})^4]}$$

$$T = 538\ \text{N} \cdot \text{m}$$

Applying Eq. 5–11, the frequency of rotation is

$$P = 2\pi f T$$

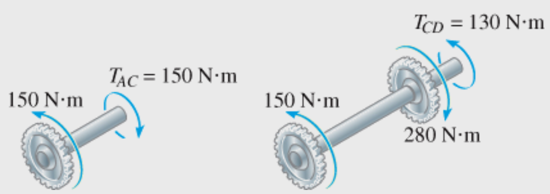
$$90(10^3)\ \text{N} \cdot \text{m/s} = 2\pi f(538\ \text{N} \cdot \text{m})$$

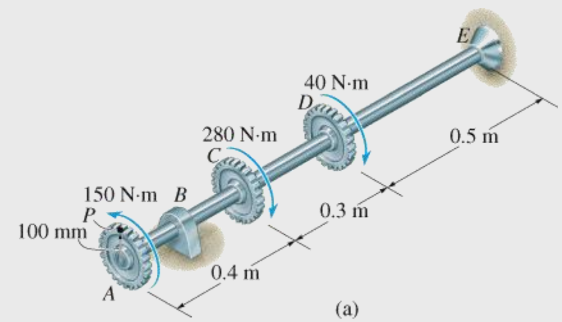
$$f = 26.6\ \text{Hz}$$

Ans.

EXAMPLE 5–7

The gears attached to the fixed-end steel shaft are subjected to the torques shown in Fig. 5–20*a*. If the shear modulus of elasticity is $G = 80$ GPa and the shaft has a diameter of 14 mm, determine the displacement of the tooth P on gear A. The shaft turns freely within the bearing at B.





(a)

SOLUTION

Internal Torque. By inspection, the torques in segments AC, CD, and DE are different yet *constant* throughout each segment. Free-body diagrams of appropriate segments of the shaft along with the calculated internal torques are shown in Fig. 5–20*b*. Using the right-hand rule and the established sign convention that positive torque is directed away from the sectioned end of the shaft, we have

$$T_{AC} = +150\ \text{N}\cdot\text{m} \qquad T_{CD} = -130\ \text{N}\cdot\text{m} \qquad T_{DE} = -170\ \text{N}\cdot\text{m}$$

These results are also shown on the torque diagram, Fig. 5–20*c*.

Angle of Twist. The polar moment of inertia for the shaft is

$$J = \frac{\pi}{2}(0.007\ \text{m})^4 = 3.77(10^{-9})\ \text{m}^4$$

Applying Eq. 5–16 to each segment and adding the results algebraically, we have

$$\phi_A = \sum \frac{TL}{JG} = \frac{(+\,150\ \text{N}\cdot\text{m})(0.4\ \text{m})}{3.77(10^{-9})\ \text{m}^4[80(10^9)\ \text{N/m}^2]}$$
$$+ \frac{(-130\ \text{N}\cdot\text{m})(0.3\ \text{m})}{3.77(10^{-9})\ \text{m}^4[80(10^9)\ \text{N/m}^2)]}$$
$$+ \frac{(-170\ \text{N}\cdot\text{m})(0.5\ \text{m})}{3.77(10^{-9})\ \text{m}^4[80(10^9)\ \text{N/m}^2)]} = -0.212\ \text{rad}$$

Since the answer is negative, by the right-hand rule the thumb is directed *toward* the end E of the shaft, and therefore gear A will rotate as shown in Fig. 5–20*d*.

The displacement of tooth P on gear A is

$$s_P = \phi_A r = (0.212\ \text{rad})(100\ \text{mm}) = 21.2\ \text{mm} \qquad \textit{Ans.}$$

Remember that this analysis is valid only if the shear stress does not exceed the proportional limit of the material.

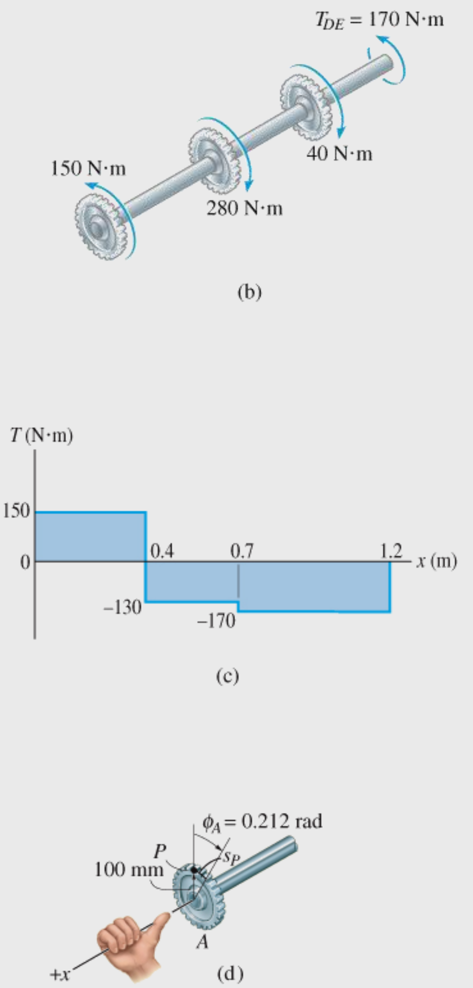
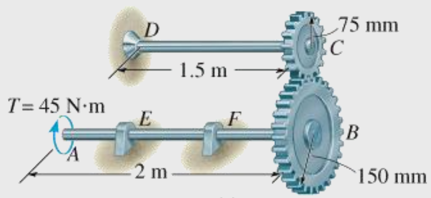


Fig. 5–20

EXAMPLE 5-8

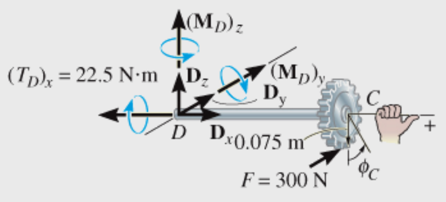
The two solid steel shafts shown in Fig. 5–21*a* are coupled together using the meshed gears. Determine the angle of twist of end A of shaft AB when the torque $T = 45\ \text{N}\cdot\text{m}$ is applied. Take $G = 80$ GPa. Shaft AB is free to rotate within bearings E and F, whereas shaft DC is fixed at D. Each shaft has a diameter of 20 mm.



(a)

SOLUTION

Internal Torque. Free-body diagrams for each shaft are shown in Fig. 5–21*b* and 5–21*c*. Summing moments along the x axis of shaft AB yields the tangential reaction between the gears of $F = 45\ \text{N}\cdot\text{m}/0.15\ \text{m} = 300\ \text{N}$. Summing moments about the x axis of shaft DC, this force then creates a torque of $(T_D)_x = 300\ \text{N}(0.075\ \text{m}) = 22.5\ \text{N}\cdot\text{m}$ on shaft DC.



(b)

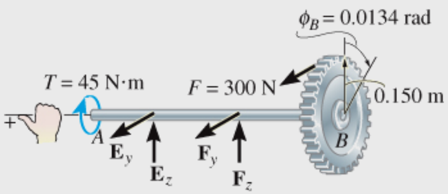
Angle of Twist. To solve the problem, we will first calculate the rotation of gear C due to the torque of $22.5\ \text{N}\cdot\text{m}$ in shaft DC, Fig. 5–21*b*. This angle of twist is

$$\phi_C = \frac{TL_{DC}}{JG} = \frac{(+22.5\ \text{N}\cdot\text{m})(1.5\ \text{m})}{(\pi/2)(0.010\ \text{m})^4[80(10^9)\ \text{N/m}^2]} = +0.0269\ \text{rad}$$

Since the gears at the end of the shaft are in mesh, the rotation ϕ_C of gear C causes gear B to rotate ϕ_B, Fig. 5–21*c*, where

$$\phi_B(0.15\ \text{m}) = (0.0269\ \text{rad})(0.075\ \text{m})$$

$$\phi_B = 0.0134\ \text{rad}$$



(c)

Fig. 5–21

We will now determine the angle of twist of end A with respect to end B of shaft AB caused by the $45\ \text{N}\cdot\text{m}$ torque, Fig. 5–21*c*. We have

$$\phi_{A/B} = \frac{T_{AB}L_{AB}}{JG} = \frac{(+45\ \text{N}\cdot\text{m})(2\ \text{m})}{(\pi/2)(0.010\ \text{m})^4[80(10^9)\ \text{N/m}^2]} = +0.0716\ \text{rad}$$

The rotation of end A is therefore determined by adding ϕ_B and $\phi_{A/B}$, since both angles are in the *same direction*, Fig. 5–21*c*. We have

$$\phi_A = \phi_B + \phi_{A/B} = 0.0134\ \text{rad} + 0.0716\ \text{rad} = +0.0850\ \text{rad} \qquad \textit{Ans.}$$