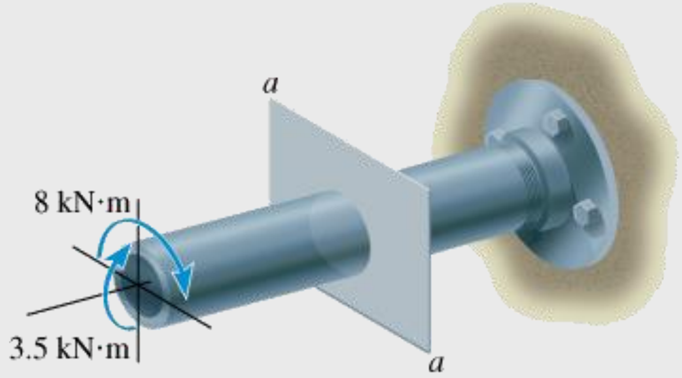
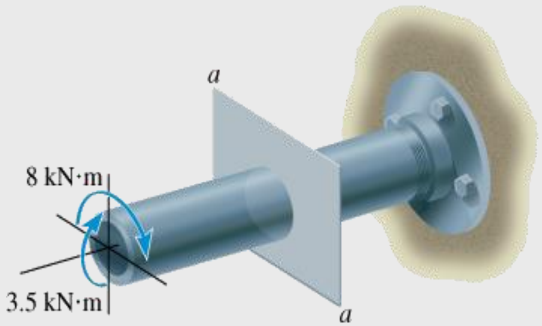
EXAMPLE 10–12

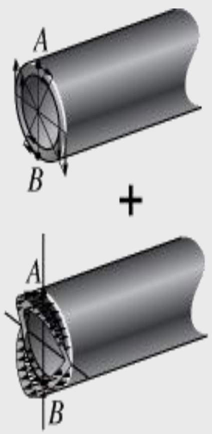
The steel pipe shown in Fig. 10–39*a* has an inner diameter of 60 mm and an outer diameter of 80 mm. If it is subjected to a torsional moment of 8 kN · m and a bending moment of 3.5 kN · m, determine if these loadings cause failure as defined by the maximum-distortion-energy theory. The yield stress for the steel found from a tension test is $\sigma_Y = 250$ MPa.



(a)



(a)



(b)

(c)

SOLUTION

To solve this problem we must investigate a point on the pipe that is subjected to a state of maximum critical stress. Both the torsional and bending moments are uniform throughout the pipe's length. At the arbitrary section *a–a*, Fig. 10–39*a*, these loadings produce the stress distributions shown in Fig. 10–39*b* and 10–39*c*. By inspection, points *A* and *B* are subjected to the same state of critical stress. Here we will investigate the state of stress at *A*. Thus,

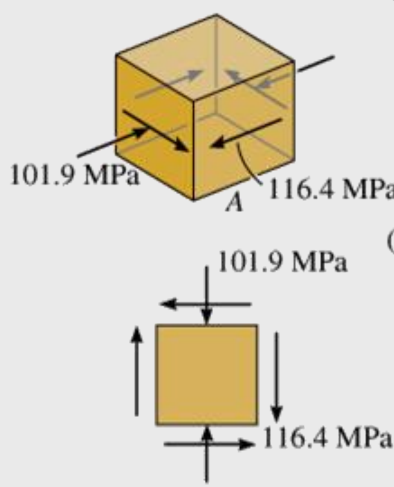
$$\tau_A = \frac{Tc}{J} = \frac{(8000\ \text{N}\cdot\text{m})(0.04\ \text{m})}{(\pi/2)[(0.04\ \text{m})^4 - (0.03\ \text{m})^4]} = 116.4\ \text{MPa}$$

$$\sigma_A = \frac{Mc}{I} = \frac{(3500\ \text{N}\cdot\text{m})(0.04\ \text{m})}{(\pi/4)[(0.04\ \text{m})^4 - (0.03\ \text{m})^4]} = 101.9\ \text{MPa}$$

These results are shown on a three-dimensional view of an element of material at point A, Fig. 10–39d, and also, since the material is subjected to plane stress, it is shown in two dimensions, Fig. 10–39e.

Mohr's circle for this state of plane stress has a center located at

$$\sigma_{avg} = \frac{0 - 101.9}{2} = -50.9 \text{ MPa}$$



(d)

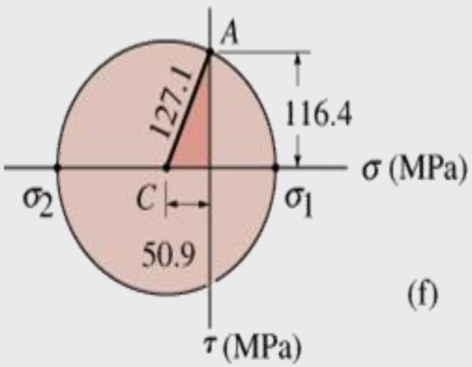


Fig. 10-39

The reference point $A(0, -116.4 \text{ MPa})$ is plotted and the circle is constructed, Fig. 10–39*f*. Here the radius has been calculated from the shaded triangle to be $R = 127.1$ and so the in-plane principal stresses are

$$\sigma_1 = -50.9 + 127.1 = 76.2 \text{ MPa}$$
$$\sigma_2 = -50.9 - 127.1 = -178.0 \text{ MPa}$$

Using Eq. 10–30, we require

$$(\sigma_1{}^2 - \sigma_1\sigma_2 + \sigma_2{}^2) \leq \sigma_Y{}^2$$
$$[(76.2)^2 - (76.2)(-178.0) + (-178.0)^2] \overset{?}{\leq} (250)^2 \quad 51\,100 < 62\,500 \quad \text{OK}$$

Since the criterion has been met, the material within the pipe will *not* yield ("fail") according to the maximum-distortion-energy theory.

EXAMPLE 10-13

The solid cast-iron shaft shown in Fig. 10–40*a* is subjected to a torque of $T = 400\ \text{N} \cdot \text{m}$. Determine its smallest radius so that it does not fail according to the maximum-normal-stress theory. A specimen of cast iron, tested in tension, has an ultimate stress of $(\sigma_{\text{ult}})_t = 150\ \text{MPa}$.

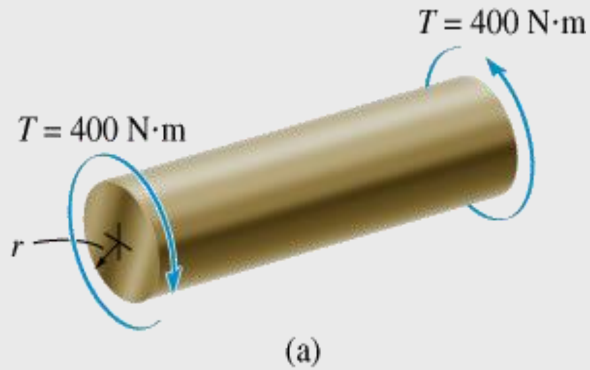


Fig. 10–40

SOLUTION

The maximum or critical stress occurs at a point located on the surface of the shaft. Assuming the shaft to have a radius r, the shear stress is

$$\tau_{max} = \frac{Tc}{J} = \frac{(400\ \text{N}\cdot\text{m})r}{(\pi/2)r^4} = \frac{254.65\ \text{N}\cdot\text{m}}{r^3}$$

Mohr's circle for this state of stress (pure shear) is shown in Fig. 10–40*b*. Since $R = \tau_{max}$, then

$$\sigma_1 = -\sigma_2 = \tau_{max} = \frac{254.65\ \text{N}\cdot\text{m}}{r^3}$$

The maximum-normal-stress theory, Eq. 10–31, requires

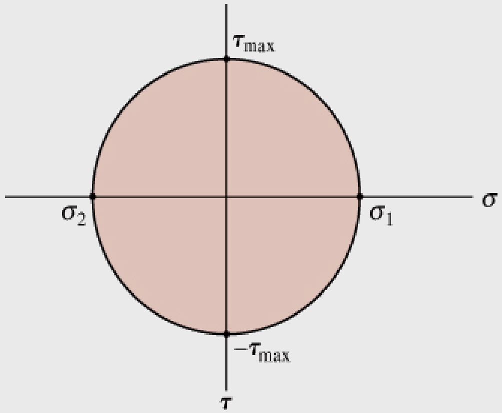
$$|\sigma_1| \le \sigma_{ult}$$

$$\frac{254.65}{r^3} \le 150 \times 10^6\ \text{N/m}^2$$

Thus, the smallest radius of the shaft is determined from

$$\frac{254.65}{r^3} = 150 \times 10^6\ \text{N/m}^2$$

$$r = 0.01193\ \text{m} = 11.93\ \text{mm}$$ ***Ans.***



(b)

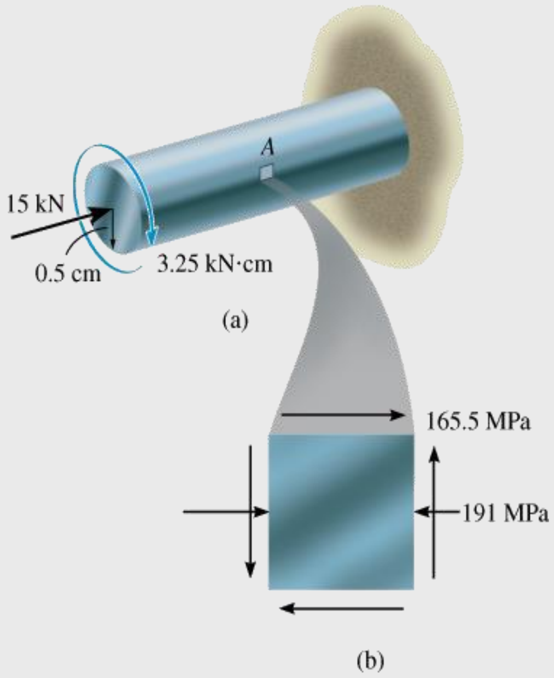


Fig. 10–41

EXAMPLE 10–14

The solid shaft shown in Fig. 10–41*a* has a radius of 0.5 cm and is made of steel having a yield stress of $\sigma_Y = 360$ MPa. Determine if the loadings cause the shaft to fail according to the maximum-shear-stress theory and the maximum-distortion-energy theory.

SOLUTION

The state of stress in the shaft is caused by both the axial force and the torque. Since maximum shear stress caused by the torque occurs in the material at the outer surface, we have

$$\sigma_x = \frac{P}{A} = \frac{15\ \text{kN}}{\pi(0.5\ \text{cm})^2} = 19.10\ \text{kN/cm}^2 = 191\ \text{MPa}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{3.25\ \text{kN}\cdot\text{cm}(0.5\ \text{cm})}{\frac{\pi}{2}(0.5\ \text{cm})^4} = 16.55\ \text{kN/cm}^2 = 165.5\ \text{MPa}$$

The stress components are shown acting on an element of material at point A in Fig. 10–41b. Rather than using Mohr's circle, the principal stresses can also be obtained using the stress-transformation equations, Eq. 9–5.

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2}$$

$$= \frac{-191 + 0}{2} \pm \sqrt{\left(\frac{-191 - 0}{2}\right)^2 + (165.5)^2}$$

$$= -95.5 \pm 191.1$$

$$\sigma_1 = 95.6 \text{ MPa}$$

$$\sigma_2 = -286.6 \text{ MPa}$$

Maximum-Shear-Stress Theory. Since the principal stresses have *opposite signs*, then from Sec. 9.7, the absolute maximum shear stress will occur in the plane, and therefore, applying the second of Eq. 10–27, we have

$$|\sigma_1 - \sigma_2| \leq \sigma_Y$$
$$|9.56 - (-28.66)| \overset{?}{\leq} 36$$
$$38.2 > 36$$

Thus, shear failure of the material will occur according to this theory.

Maximum-Distortion-Energy Theory. Applying Eq. 10–30, we have

$$({\sigma_1}^2 - \sigma_1\sigma_2 + {\sigma_2}^2) \leq \sigma_Y$$
$$[(9.56)^2 - (9.56)(-28.66) + (-28.66)^2] \overset{?}{\leq} (36)^2$$
$$1187 \leq 1296$$

Using this theory, failure will not occur.