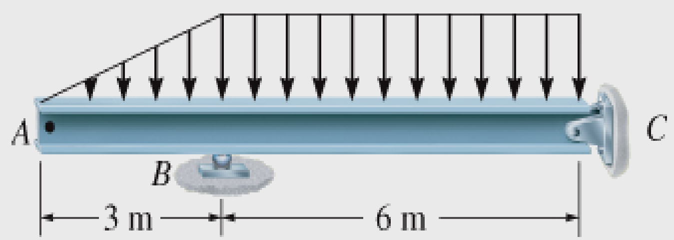
EXAMPLE 14–16

Determine the displacement of point A of the steel beam shown in Fig. 14–38a. $I = 175.8(10^{-6})\ \text{m}^4$, $E_{st} = 200$ GPa.



(a)

SOLUTION

Virtual Moments m. The beam is subjected to the virtual unit load at A and the reactions are computed, Fig. 14–38*b*. By inspection, two coordinates x_1 and x_2 must be chosen to cover all regions of the beam. For purposes of integration it is simplest to use origins at A and C. Using the method of sections, the internal moments m are shown in Fig. 14–38*b*.

Real Moments M. The reactions on the real beam are found first, Fig. 14–38*a*. Then, using the *same* x coordinates as those used for m, the internal moments M are determined.

Virtual-Work Equation. Applying the equation of virtual work to the beam, we have

$$1\text{ kN}\cdot\Delta_A = \int \frac{mM}{EI}\,dx = \int_0^3 \frac{(-1x_1)(-2.5x_1^3)\,dx_1}{EI}$$

$$+ \int_0^6 \frac{(-0.5x_2)(123.75x_2 - 22.5x_2^2)\,dx_2}{EI}$$

or

$$1\text{ kN}\cdot\Delta_A = \frac{0.5(3)^5}{EI} - \frac{20.625(6)^3}{EI} + \frac{2.8125(6)^4}{EI}$$

$$\Delta_A = \frac{-688.5\text{ kN}\cdot\text{m}^3}{EI}$$

Substituting in the data for E and I, we get

$$\Delta_A = \frac{-688.5\text{ kN}\cdot\text{m}^3}{[200(10^6)\text{ kN/m}^2]175.8(10^{-6})\text{m}^4}$$

$$= -0.0196\text{ m} = -19.6\text{ mm} \qquad \textit{Ans.}$$

The negative sign indicates that point A is displaced upward.

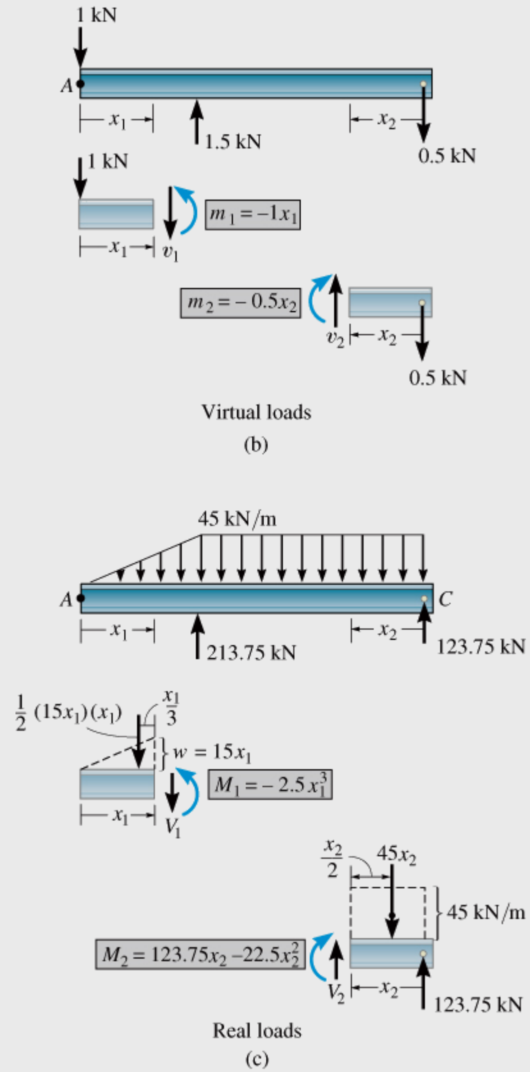
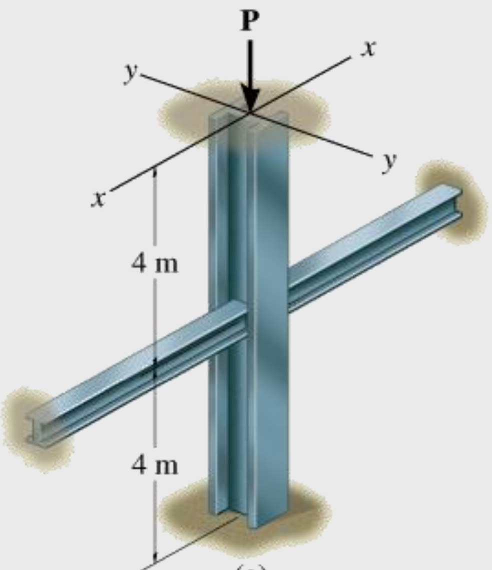


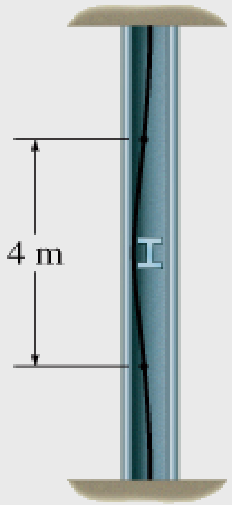
Fig. 14–38

EXAMPLE 13–3

A $W\ 150 \times 24$ steel column is 8 m long and is fixed at its ends as shown in Fig. 13–13*a*. Its load-carrying capacity is increased by bracing it about the $y-y$ (weak) axis using struts that are assumed to be pin-connected to its midheight. Determine the load it can support so that the column does not buckle nor the material exceed the yield stress. Take $E_{st} =$ 200 MPa and $\sigma_Y = 410$ MPa.



(a)



x–x axis buckling

(b)

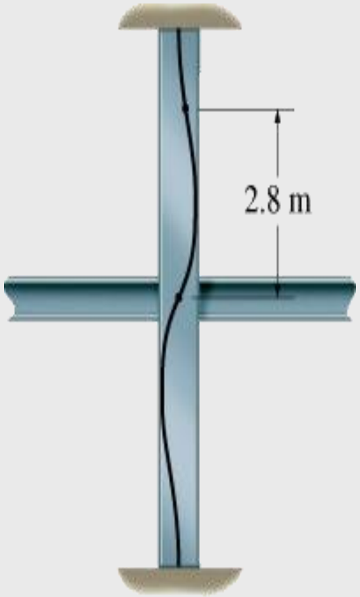
SOLUTION

The buckling behavior of the column will be *different* about the x and y axes due to the bracing. The buckled shape for each of these cases is shown in Figs. 13–13*b* and 13–13*c*. From Fig. 13–13*b*, the effective length for buckling about the $x-x$ axis is $(KL)_x = 0.5(8\text{ m}) = 4\text{ m}$, and from Fig. 13–13*c*, for buckling about the $y-y$ axis, $(KL)_y = 0.7(8\text{ m}/2) = 2.8\text{ m}$. The moments of inertia for a $W\,150 \times 24$ are determined from the table in Appendix B. We have $I_x = 13.4 \times 10^6\text{ mm}^4$, $I_y = 1.83 \times 10^6\text{ mm}^4$.

Applying Eq. 13–11, we have

$$(P_{cr})_x = \frac{\pi^2 E I_x}{(KL)_x^2} = \frac{\pi^2[200(10^6)\text{ kN/m}^2]13.4(10^{-6})\text{ m}^4}{(4\text{ m})^2} = 1653.2\text{ kN} \quad (1)$$

$$(P_{cr})_y = \frac{\pi^2 E I_y}{(KL)_y^2} = \frac{\pi^2[200(10^6)\text{ kN/m}^2]1.83(10^{-6})\text{ m}^4}{(2.8\text{ m})^2} = 460.8\text{ kN} \quad (2)$$



y–y axis buckling

(c)

Fig. 13–13

By comparison, buckling will occur about the $y-y$ axis.

The area of the cross section is 3060 mm², so the average compressive stress in the column will be

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{460.8(10^3)\text{ N}}{3060\text{ mm}^2} = 150.6\text{ N/mm}^2$$

Since this stress is less than the yield stress, buckling will occur before the material yields. Thus,

$$P_{cr} = 461\text{ kN} \qquad \textit{Ans.}$$

Note: From Eq. 13–11 it can be seen that buckling will always occur about the column axis having the *largest* slenderness ratio, since a large slenderness ratio will give a small critical load. Thus, using the data for the radius of gyration from the table in Appendix B, we have

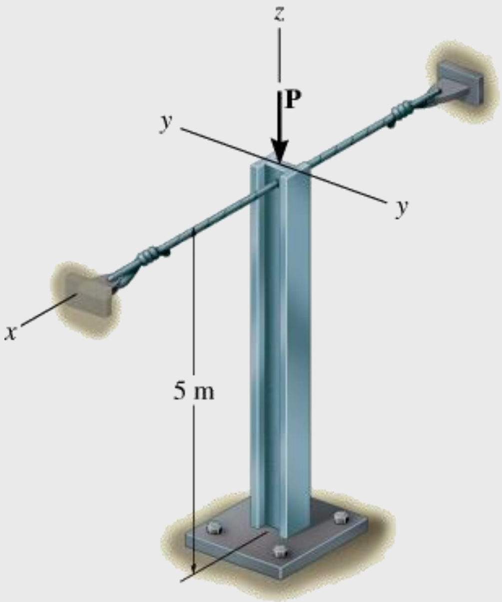
$$\left(\frac{KL}{r}\right)_x = \frac{4\text{ m}(1000\text{ mm/m})}{66.2\text{ mm}} = 60.4$$

$$\left(\frac{KL}{r}\right)_y = \frac{2.8\text{ m}(1000\text{ mm/m})}{24.5\text{ mm}} = 114.3$$

Hence, $y-y$ axis buckling will occur, which is the same conclusion reached by comparing Eqs. 1 and 2.

EXAMPLE 13–4

The aluminum column is fixed at its bottom and is braced at its top by cables so as to prevent movement at the top along the x axis, Fig. 13–14a. If it is assumed to be fixed at its base, determine the largest allowable load P that can be applied. Use a factor of safety for buckling of F.S. = 3.0. Take $E_{al} = 70$ GPa, $\sigma_Y = 215$ MPa, $A = 7.5(10^{-3})\ \text{m}^2$, $I_x = 61.3(10^{-6})\ \text{m}^4$, $I_y = 23.2(10^{-6})\ \text{m}^4$.



(a)

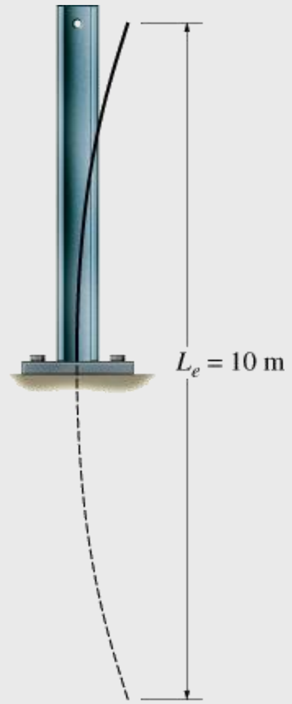
SOLUTION

Buckling about the x and y axes is shown in Fig. 13–14b and 13–14c, respectively. Using Fig. 13–12, for $x-x$ axis buckling, $K = 2$, so $(KL)_x = 2(5\text{ m}) = 10\text{ m}$. Also, for $y-y$ axis buckling, $K = 0.7$, so $(KL)_y = 0.7(5\text{ m}) = 3.5\text{ m}$.

Applying Eq. 13–11, the critical loads for each case are

$$(P_{cr})_x = \frac{\pi^2 EI_x}{(KL)_x^2} = \frac{\pi^2[70(10^9)\text{ N/m}^2](61.3(10^{-6})\text{ m}^4)}{(10\text{ m})^2}$$
$$= 424\text{ kN}$$

$$(P_{cr})_y = \frac{\pi^2 EI_y}{(KL)_y^2} = \frac{\pi^2[70(10^9)\text{ N/m}^2](23.2(10^{-6})\text{ m}^4)}{(3.5\text{ m})^2}$$
$$= 1.31\text{ MN}$$



x—x axis buckling

(b)

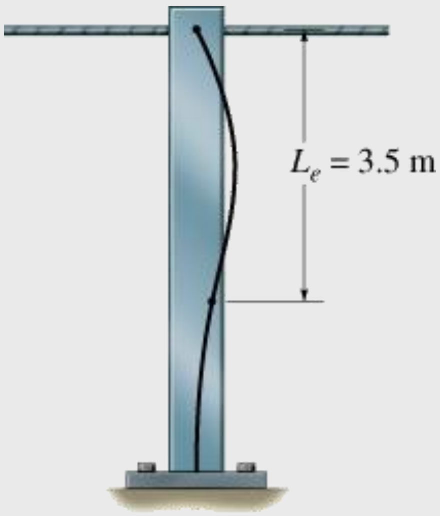
By comparison, as P is increased the column will buckle about the $x-x$ axis. The allowable load is therefore

$$P_{\text{allow}} = \frac{P_{\text{cr}}}{\text{F.S.}} = \frac{424 \text{ kN}}{3.0} = 141 \text{ kN} \qquad \textbf{\textit{Ans.}}$$

Since

$$\sigma_{\text{cr}} = \frac{P_{\text{cr}}}{A} = \frac{424 \text{ kN}}{7.5(10^{-3}) \text{ m}^2} = 56.5 \text{ MPa} < 215 \text{ MPa}$$

Euler's equation can be applied.



y–y axis buckling

(c)