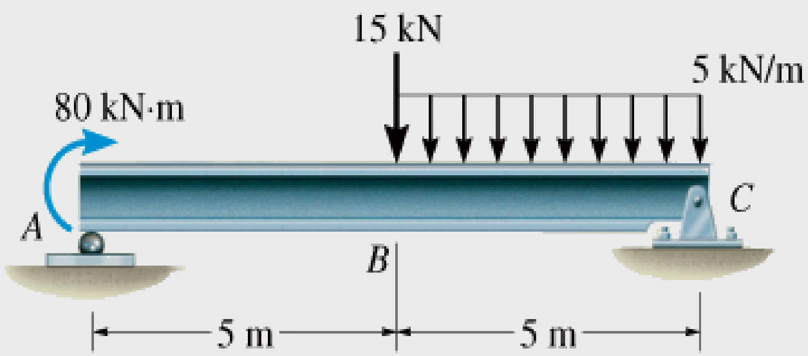
EXAMPLE 6–6

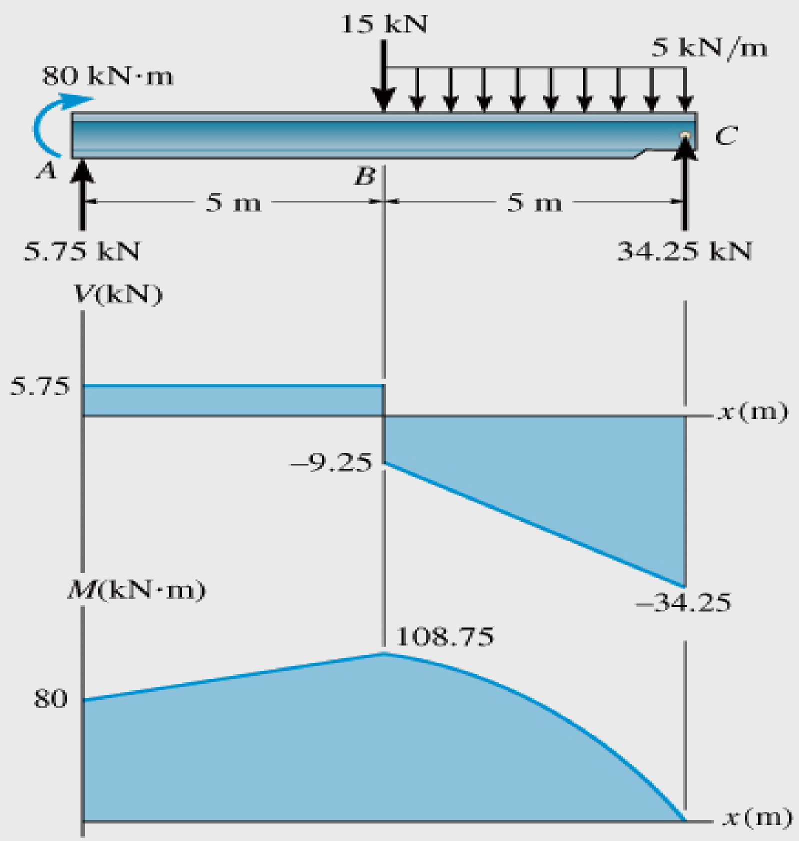
Draw the shear and moment diagrams for the beam shown in Fig. 6–9*a*.



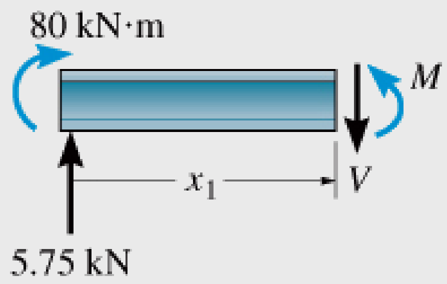
(a)

SOLUTION

Support Reactions. The reactions at the supports have been determined and are shown on the free-body diagram of the beam, Fig. 6–9*d*.



(d)



(b)

Shear and Moment Functions. Since there is a discontinuity of distributed load and also a concentrated load at the beam's center, two regions of x must be considered in order to describe the shear and moment functions for the entire beam.

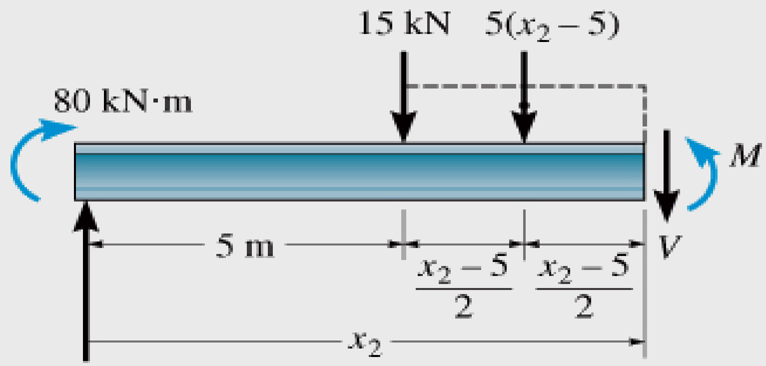
$0 \leq x_1 < 5$ m, Fig. 6–9*b*:

$+\uparrow \Sigma F_y = 0;$ $\quad 5.75 \text{ kN} - V = 0$

$$V = 5.75 \text{ kN} \tag{1}$$

$\circlearrowleft + \Sigma M = 0;$ $\quad -80 \text{ kN} \cdot \text{m} - 5.75 \text{ kN}\, x_1 + M = 0$

$$M = (5.75x_1 + 80) \text{ kN} \cdot \text{m} \tag{2}$$



(c)

$5\text{ m} < x_2 \leq 10\text{ m}$, Fig. 6–9*c*:

$$+\uparrow \Sigma F_y = 0; \quad 5.75\text{ kN} - 15\text{ kN} - 5\text{ kN/m}(x_2 - 5\text{ m}) - V = 0$$

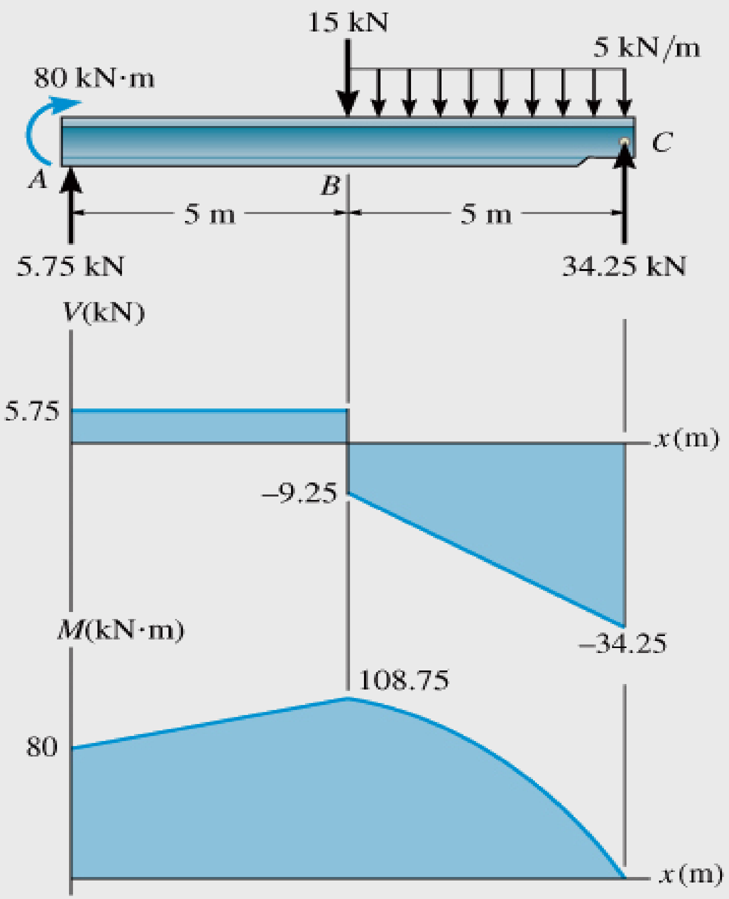
$$V = (15.75 - 5x_2)\text{ kN} \qquad (3)$$

$$\circlearrowleft + \Sigma M = 0; \quad -80\text{ kN}\cdot\text{m} - 5.75\text{ kN}\, x_2 + 15\text{ kN}(x_2 - 5\text{ m}) + 5\text{ kN/m}(x_2 - 5\text{ m})\left(\frac{x_2 - 5\text{ m}}{2}\right) + M = 0$$

$$M = (-2.5x_2{}^2 + 15.75x_2 + 92.5)\text{ kN}\cdot\text{m} \qquad (4)$$

These results can be checked in part by noting that by applying $w = -dV/dx$ and $V = dM/dx$. Also, when $x_1 = 0$, Eqs. 1 and 2 give $V = 5.75$ kN and $M = 80$ kN $\cdot$ m; when $x_2 = 10$ m, Eqs. 3 and 4 give $V = -34.25$ kN and $M = 0$. These values check with the support reactions shown on the free-body diagram, Fig. 6–9*d*.

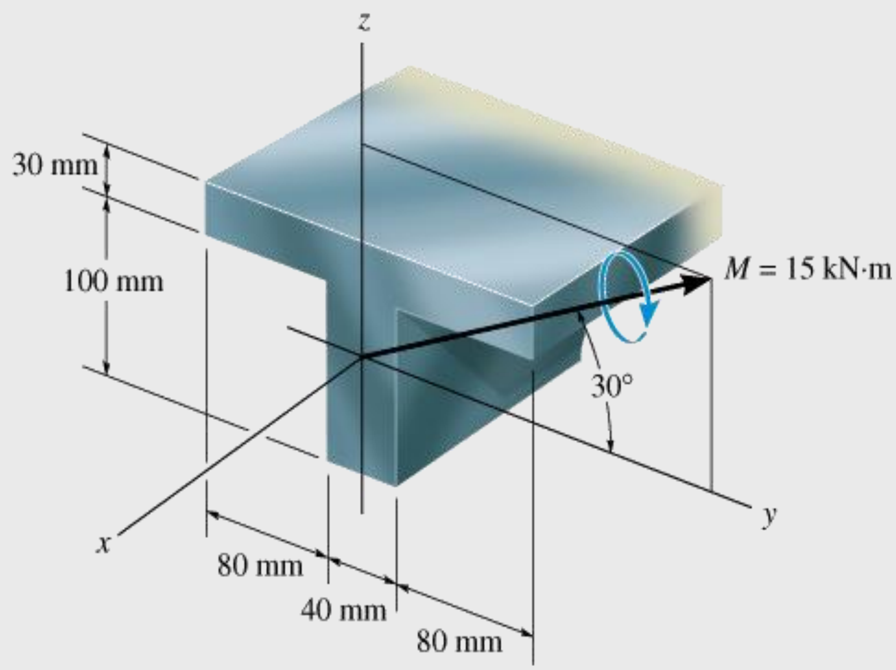
Shear and Moment Diagrams. Equations 1 through 4 are plotted in Fig. 6–9*d*.



(d)

EXAMPLE 6–19

A T-beam is subjected to the bending moment of 15 kN · m as shown in Fig. 6–36*a*. Determine the maximum normal stress in the beam and the orientation of the neutral axis.



(a)

SOLUTION

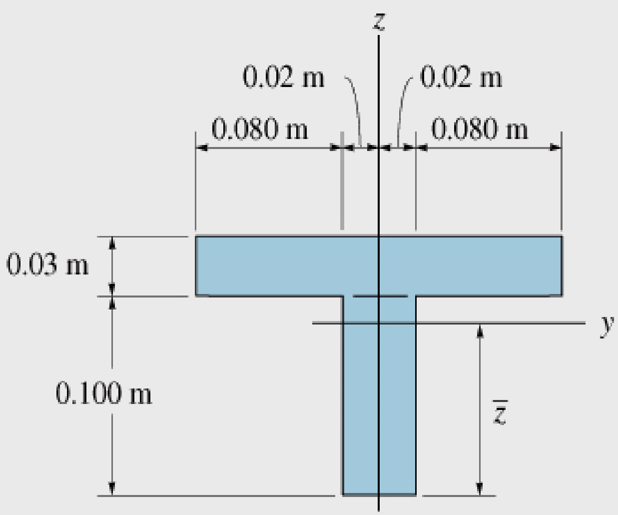
Internal Moment Components. The y and z axes are principal axes of inertia. Why? From Fig. 6–36*a*, both moment components are positive. We have

$$M_y = (15 \text{ kN} \cdot \text{m}) \cos 30^\circ = 12.99 \text{ kN} \cdot \text{m}$$
$$M_z = (15 \text{ kN} \cdot \text{m}) \sin 30^\circ = 7.50 \text{ kN} \cdot \text{m}$$

Section Properties. With reference to Fig. 6–36*b*, working in units of meters, we have

$$\bar{z} = \frac{\Sigma \tilde{z} A}{\Sigma A} = \frac{[0.05 \text{ m}](0.100 \text{ m})(0.04 \text{ m}) + [0.115 \text{ m}](0.03 \text{ m})(0.200 \text{ m})}{(0.100 \text{ m})(0.04 \text{ m}) + (0.03 \text{ m})(0.200 \text{ m})}$$

$$= 0.0890 \text{ m}$$

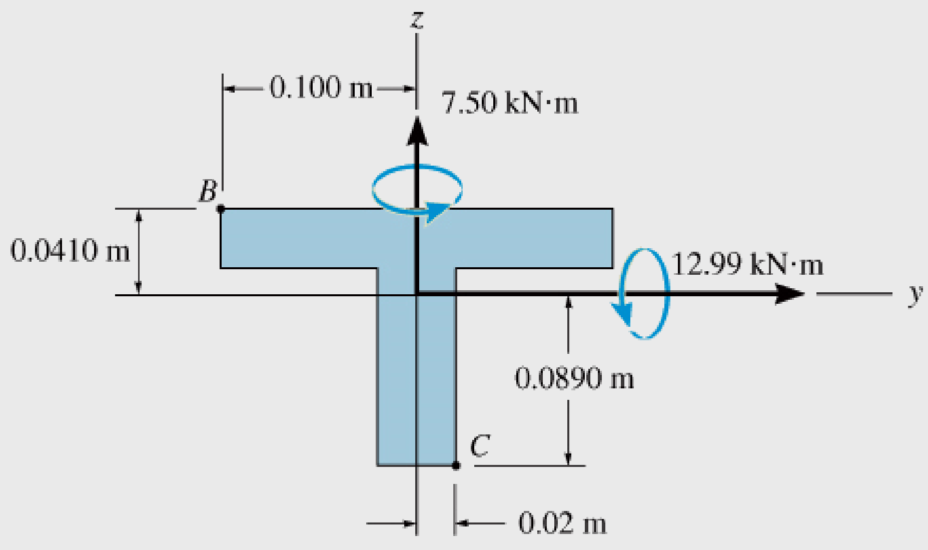


(b)

Using the parallel-axis theorem of Appendix A, $I = \bar{I} + Ad^2$, the principal moments of inertia are thus

$$I_z = \frac{1}{12}(0.100 \text{ m})(0.04 \text{ m})^3 + \frac{1}{12}(0.03 \text{ m})(0.200 \text{ m})^3 = 20.53(10^{-6})\text{m}^4$$

$$I_y = \left[\frac{1}{12}(0.04 \text{ m})(0.100 \text{ m})^3 + (0.100 \text{ m})(0.04 \text{ m})(0.0890 \text{ m} - 0.05 \text{ m})^2\right]$$
$$+ \left[\frac{1}{12}(0.200 \text{ m})(0.03 \text{ m})^3 + (0.200 \text{ m})(0.03 \text{ m})(0.115 \text{ m} - 0.0890 \text{ m})^2\right]$$
$$= 13.92(10^{-6}) \text{ m}^4$$



(c)

Maximum Bending Stress. The moment components are shown in Fig. 6–36*c*. By inspection, the largest *tensile* stress occurs at point *B*, since by superposition both moment components create a tensile stress there. Likewise, the greatest *compressive* stress occurs at point *C*.

Thus,

$$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

$$\sigma_B = -\frac{7.50 \text{ kN} \cdot \text{m } (-0.100 \text{ m})}{20.53(10^{-6}) \text{ m}^4} + \frac{12.99 \text{ kN} \cdot \text{m}(0.0410 \text{ m})}{13.92(10^{-6}) \text{ m}^4}$$

$$= 74.8 \text{ MPa}$$

$$\sigma_C = -\frac{7.50 \text{ kN} \cdot \text{m } (0.020 \text{ m})}{20.53(10^{-6}) \text{ m}^4} + \frac{12.99 \text{ kN} \cdot \text{m } (-0.0890 \text{ m})}{13.92(10^{-6}) \text{ m}^4}$$

$$= -90.4 \text{ MPa}$$ ***Ans.***

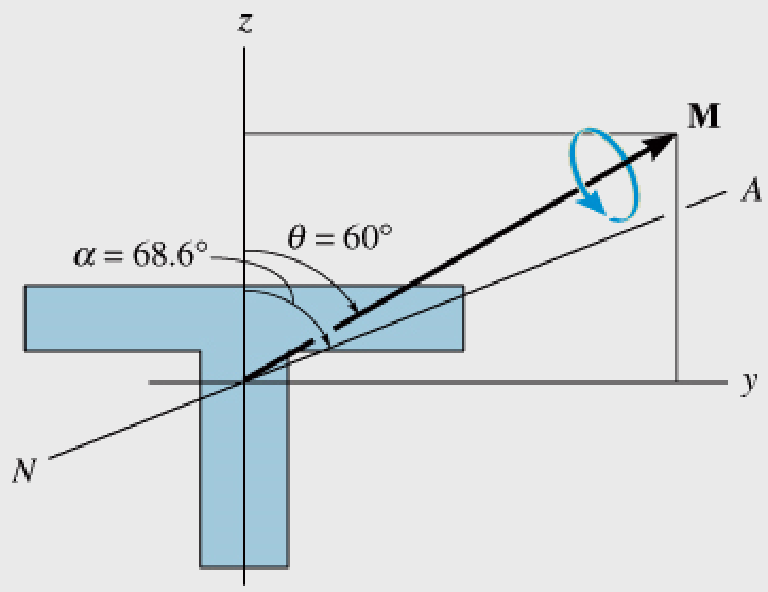
By comparison, the largest normal stress is therefore compressive and occurs at point C.

Orientation of Neutral Axis. When applying Eq. 6–19 it is important to be sure the angles α and θ are defined correctly. As previously stated, y must represent the axis for *minimum* principal moment of inertia, and z must represent the axis for *maximum* principal moment of inertia. These axes are properly positioned here since $I_y < I_z$. Using this setup, θ and α are measured positive from the $+z$ axis toward the $+y$ axis. Hence, from Fig. 6–36*a*, $\theta = +60°$. Thus,

$$\tan \alpha = \left(\frac{20.53(10^{-6})\ \text{m}^4}{13.92(10^{-6})\ \text{m}^4} \right) \tan 60°$$

$$\alpha = 68.6° \qquad \textbf{\textit{Ans.}}$$

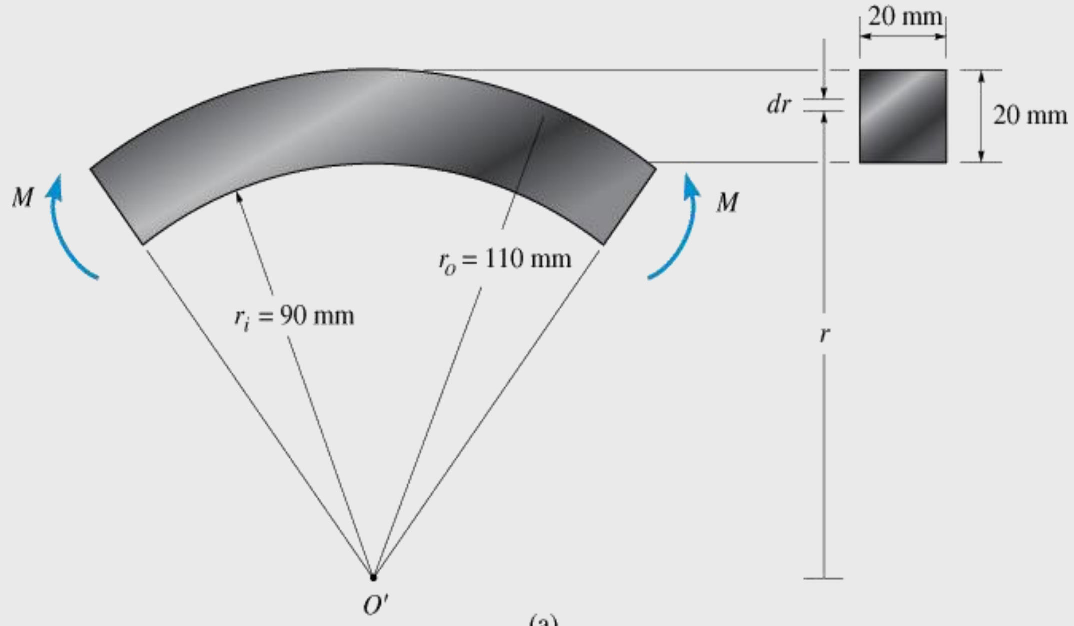
The neutral axis is shown in Fig. 6–36*d*. As expected, it lies between the y axis and the line of action of **M**.



(d)

EXAMPLE 6–24

A steel bar having a rectangular cross section is shaped into a circular arc as shown in Fig. 6–45*a*. If the allowable normal stress is $\sigma_{allow} = 140$ MPa, determine the maximum bending moment M that can be applied to the bar. What would this moment be if the bar was straight?



(a)

SOLUTION

Internal Moment. Since M tends to increase the bar's radius of curvature, it is positive.

Section Properties. The location of the neutral axis is determined using Eq. 6–23. From Fig. 6–45a, we have

$$\int_A \frac{dA}{r} = \int_{90\text{ mm}}^{110\text{ mm}} \frac{(20\text{ mm})\,dr}{r} = (20\text{ mm})\ln r\,\Big|_{90\text{ mm}}^{110\text{ mm}} = 4.0134\text{ mm}$$

This same result can of course be obtained directly from Table 6–2. Thus,

$$R = \frac{A}{\displaystyle\int_A \frac{dA}{r}} = \frac{(20\text{ mm})(20\text{ mm})}{4.0134\text{ mm}} = 99.666\text{ mm}$$

It should be noted that throughout the above calculations R must be determined to several significant figures to ensure that $(\bar{r} - R)$ is accurate to at least three significant figures.

It is unknown if the normal stress reaches its maximum at the top or at the bottom of the bar, and so we must compute the moment M for each case separately. Since the normal stress at the bar's top is compressive, $\sigma = -140$ MPa,

$$\sigma = \frac{M(R - r_o)}{Ar_o(\bar{r} - R)}$$

$$-140\ \text{N/mm}^2 = \frac{M(99.666\ \text{mm} - 110\ \text{mm})}{(20\ \text{mm})(20\ \text{mm})(110\ \text{mm})(100\ \text{mm} - 99.666\ \text{mm})}$$

$$M = 199094\ \text{N} \cdot \text{mm} = 0.199\ \text{kN} \cdot \text{m}$$

Likewise, at the bottom of the bar the normal stress will be tensile, so $\sigma = +140$ MPa. Therefore,

$$\sigma = \frac{M(R - r_i)}{Ar_i(\bar{r} - R)}$$

$$140\ \text{N/mm}^2 = \frac{M(99.666\ \text{mm} - 90\ \text{mm})}{(20\ \text{mm})(20\ \text{mm})(90\ \text{mm})(100\ \text{mm} - 99.666\ \text{mm})}$$

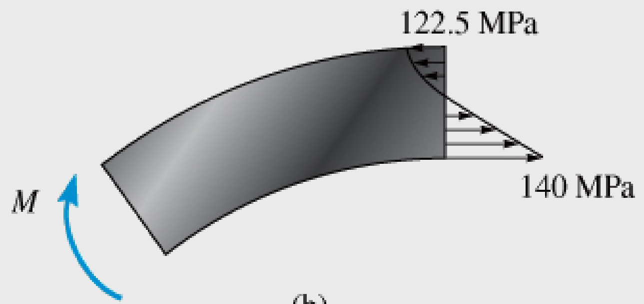
$$M = 174153\ \text{N} \cdot \text{mm} = 0.174\ \text{kN} \cdot \text{m}$$ ***Ans.***

By comparison, the maximum moment that can be applied is $0.174\ \text{kN} \cdot \text{m}$ and so maximum normal stress occurs at the bottom of the bar. The compressive stress at the top of the bar is then

$$\sigma = \frac{174153\ \text{N} \cdot \text{mm}(99.666\ \text{mm} - 110\ \text{mm})}{(20\ \text{mm})(20\ \text{mm})(110\ \text{mm})(100\ \text{mm} - 99.666\ \text{mm})}$$

$$= -122.5\ \text{N/mm}^2$$

The stress distribution is shown in Fig. 6–45*b*.



(b)

If the bar was straight, then

$$\sigma = \frac{Mc}{I}$$

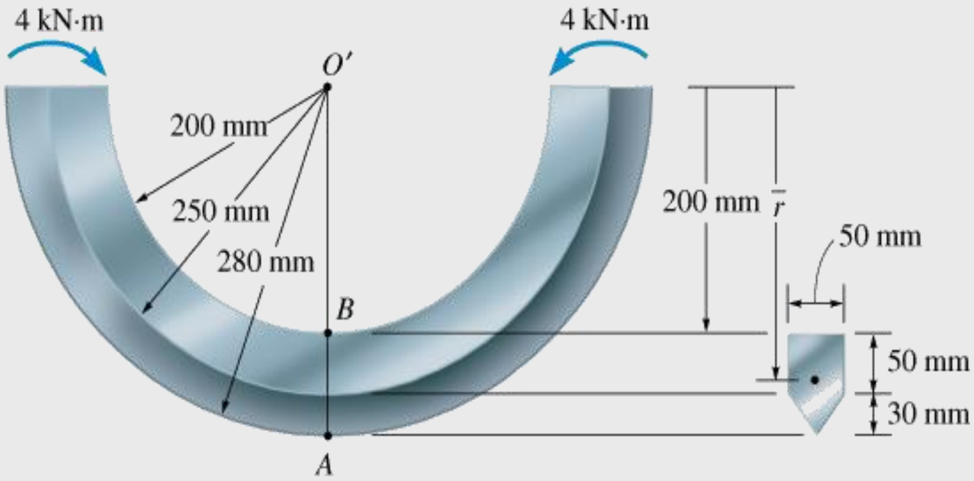
$$140\ \text{N/mm}^2 = \frac{M(10\ \text{mm})}{\frac{1}{12}(20\ \text{mm})(20\ \text{mm})^3}$$

$$M = 186666.7\ \text{N}\cdot\text{mm} = 0.187\ \text{kN}\cdot\text{m} \qquad \textbf{\textit{Ans.}}$$

This represents an error of about 7% from the more exact value determined above.

EXAMPLE 6–25

The curved bar has a cross-sectional area shown in Fig. 6–46*a*. If it is subjected to bending moments of 4 kN · m, determine the maximum normal stress developed in the bar.



(a)

SOLUTION

Internal Moment. Each section of the bar is subjected to the same resultant internal moment of 4 kN · m. Since this moment tends to decrease the bar's radius of curvature, it is negative. Thus $M = -4$ kN ·m.

Section Properties. Here we will consider the cross section to be composed of a rectangle and triangle. The total cross-sectional area is

$$\Sigma A = (0.05 \text{ m})^2 + \frac{1}{2}(0.05 \text{ m})(0.03 \text{ m}) = 3.250(10^{-3}) \text{ m}^2$$

The location of the centroid is determined with reference to the center of curvature, point O', Fig. 6–46*a*.

$$\bar{r} = \frac{\Sigma \tilde{r} A}{\Sigma A}$$

$$= \frac{[0.225 \text{ m}](0.05 \text{ m})(0.05 \text{ m}) + [0.260 \text{ m}]\frac{1}{2}(0.050 \text{ m})(0.030 \text{ m})}{3.250(10^{-3}) \text{ m}^2}$$

$$= 0.23308 \text{ m}$$

We can compute $\int_A dA/r$ for each part using Table 6–2. For the rectangle,

$$\int_A \frac{dA}{r} = 0.05 \text{ m}\left(\ln \frac{0.250 \text{ m}}{0.200 \text{ m}}\right) = 0.011157 \text{ m}$$

And for the triangle,

$$\int_A \frac{dA}{r} = \frac{(0.05 \text{ m})(0.280 \text{ m})}{(0.280 \text{ m} - 0.250 \text{ m})}\left(\ln \frac{0.280 \text{ m}}{0.250 \text{ m}}\right) - 0.05 \text{ m} = 0.0028867 \text{ m}$$

Thus the location of the neutral axis is determined from

$$R = \frac{\Sigma A}{\Sigma \int_A dA/r} = \frac{3.250(10^{-3}) \text{ m}^2}{0.011157 \text{ m} + 0.0028867 \text{ m}} = 0.23142 \text{ m}$$

Note that $R < \bar{r}$ as expected. Also, the calculations were performed with sufficient accuracy so that $(\bar{r} - R) = 0.23308 \text{ m} - 0.23142 \text{ m} = 0.00166 \text{ m}$ is now accurate to three significant figures.

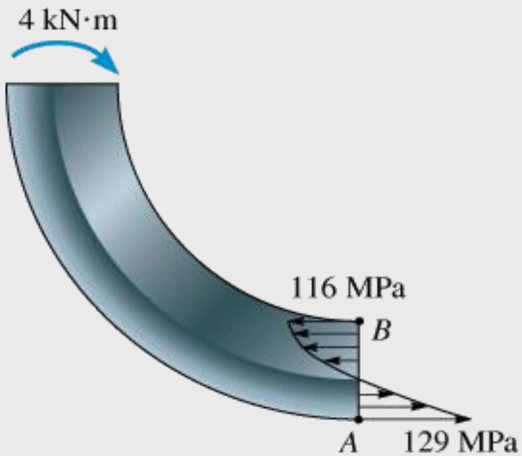
Normal Stress. The maximum normal stress occurs either at A or B. Applying the curved-beam formula to calculate the normal stress at B, $r_B = 0.200$ m, we have

$$\sigma_B = \frac{M(R - r_B)}{Ar_B(\bar{r} - R)} = \frac{(-4\ \text{kN}\cdot\text{m})(0.23142\ \text{m} - 0.200\ \text{m})}{3.2500(10^{-3})\ \text{m}^2(0.200\ \text{m})(0.00166\ \text{m})}$$
$$= -116\ \text{MPa}$$

At point A, $r_A = 0.280$ m and the normal stress is

$$\sigma_A = \frac{M(R - r_A)}{Ar_A(\bar{r} - R)} = \frac{(-4\ \text{kN}\cdot\text{m})(0.23142\ \text{m} - 0.280\ \text{m})}{3.2500(10^{-3})\ \text{m}^2(0.280\ \text{m})(0.00166\ \text{m})}$$
$$= 129\ \text{MPa} \qquad \textbf{\textit{Ans.}}$$

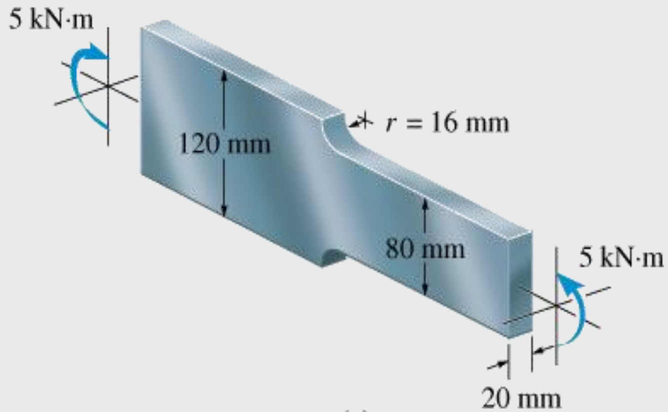
By comparison, the maximum normal stress is at *A*. A two-dimensional representation of the stress distribution is shown in Fig. 6–46*b*.



(b)

EXAMPLE 6–26

The transition in the cross-sectional area of the steel bar is achieved using shoulder fillets as shown in Fig. 6–51*a*. If the bar is subjected to a bending moment of 5 kN · m, determine the maximum normal stress developed in the steel. The yield stress is $\sigma_Y = 500$ MPa.



(a)

SOLUTION

The moment creates the largest stress in the bar at the base of the fillet, where the cross-sectional area is smallest. The stress-concentration factor can be determined by using Fig. 6–48. From the geometry of the bar, we have $r = 16$ mm, $h = 80$ mm, $w = 120$ mm. Thus,

$$\frac{r}{h} = \frac{16\text{ mm}}{80\text{ mm}} = 0.2 \qquad \frac{w}{h} = \frac{120\text{ mm}}{80\text{ mm}} = 1.5$$

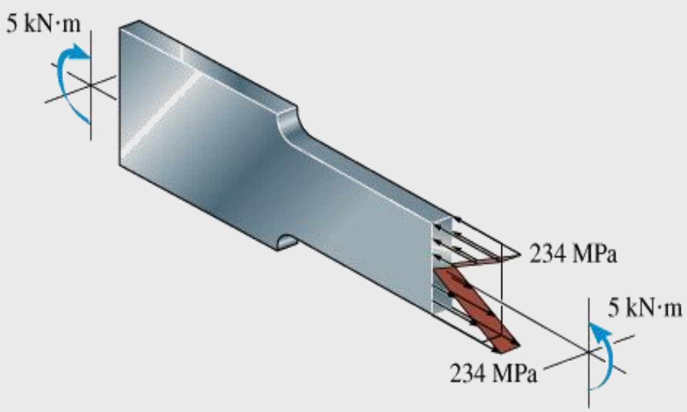
These values give $K = 1.45$.

Applying Eq. 6–26, we have

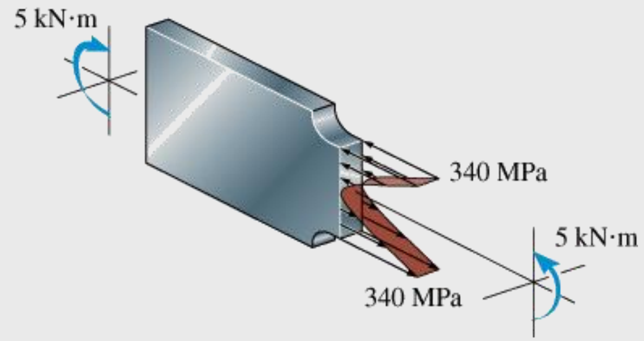
$$\sigma_{\max} = K\frac{Mc}{I} = (1.45)\,\frac{(5\text{ kN}\cdot\text{m})(0.04\text{ m})}{\left[\frac{1}{12}(0.020\text{ m})(0.08\text{ m})^3\right]} = 340\text{ MPa}$$

This result indicates that the steel remains elastic since the stress is below the yield stress (500 MPa).

The normal-stress distribution is nonlinear and is shown in Fig. 6–51*b*. Realize, however, that by Saint-Venant's principle, Sec. 4.1, these localized stresses smooth out and become linear when one moves (approximately) a distance of 80 mm or more to the right of the transition. In this case, the flexure formula gives $\sigma_{max} = 234$ MPa, Fig. 6–51*c*. Also note that the choice of a larger-radius fillet will significantly reduce σ_{max}, since as r increases in Fig. 6–48, K will decrease.



(c)



(b)