UCK361E EQUATIONS GUIDE

Arithmetic mean

$$\bar{x} = \frac{1}{N} \sum x_i$$

Standard deviation

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N-1}}$$

Torsion

$$\theta = \frac{T.L}{G.J}$$

Closed thin-walled tubes

$$\tau = \frac{T}{2 \cdot A \cdot t}$$

A is the area enclosed by the section median line

$$\theta_1 = \frac{T.L}{4 \cdot G \cdot A^2 \cdot t}$$

L is the perimeter of the section median line

θ_1 → twist per unit length

Gaussian (normal) distribution

$$f(x) = \frac{1}{s\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\bar{x}}{s}\right)^2\right]$$

Lognormal distribution

$$f(x) = \frac{1}{x \cdot \bar{s}\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln x - \bar{x}}{\bar{s}}\right)^2\right]$$

$$\bar{s} = \sqrt{\ln(1+C^2)} \qquad C = \frac{s}{\bar{x}}$$

Open thin-walled sections

$$\tau = G \cdot \theta_1 \cdot c = \frac{3 \cdot T}{L \cdot c^2}$$

Weibull distribution

$$R(x) = \exp\left[-\left(\frac{x - x_0}{\theta - x_0}\right)^b\right]$$

Stress concentration

$$K_t = \frac{\sigma_{max}}{\sigma_0} \qquad K_{ts} = \frac{\tau_{max}}{\tau_0}$$

Stress transformation

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\phi + \tau_{xy} \cdot \sin 2\phi$$

$$\tau = -\frac{\sigma_x - \sigma_y}{2}\sin 2\phi + \tau_{xy} \cdot \cos 2\phi$$

Stresses in cylinders

$$\sigma_t = \frac{p_i r_i^2 - p_o r_o^2 - \frac{r_i^2 r_o^2}{r^2}(p_o - p_i)}{r_o^2 - r_i^2}$$

$$\sigma_r = \frac{p_i r_i^2 - p_o r_o^2 + \frac{r_i^2 r_o^2}{r^2}(p_o - p_i)}{r_o^2 - r_i^2}$$

$$\sigma_l = \frac{p_i r_i^2}{r_o^2 - r_i^2}$$

Octahedral stresses

$$\tau_{oct} = \frac{1}{3}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6\left(\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2\right)\right]^{1/2}$$

$$\sigma_{oct} = \frac{1}{3}(\sigma_x + \sigma_y + \sigma_z)$$

Rotating rings

$$\sigma_t = \rho \cdot \omega^2 \left(\frac{3+\nu}{8}\right)\left(r_i^2 + r_o^2 + \frac{r_i^2 r_o^2}{r^2} - \frac{1+3\nu}{3+\nu} r^2\right)$$

$$\sigma_r = \rho \cdot \omega^2 \left(\frac{3+\nu}{8}\right)\left(r_i^2 + r_o^2 - \frac{r_i^2 r_o^2}{r^2} - r^2\right)$$

Shear stresses in beams

$$\tau = \frac{V \cdot Q}{I \cdot t}$$

Q → the first moment of the area

Press and shrink fits

$$\sigma_{it} = -p\,\frac{R^2 + r_i^2}{R^2 - r_i^2} \qquad \sigma_{ot} = p\,\frac{r_o^2 + R^2}{r_o^2 - R^2}$$

$$p = \frac{E \cdot \delta}{R}\left[\frac{(r_o^2 - R^2)(R^2 - r_i^2)}{2 \cdot R^2 (r_o^2 - r_i^2)}\right]$$

Torque

$$T = \frac{H}{\omega} = 9550\,\frac{H}{n}$$

T → N.m; H → kW; n → rpm

Temperature effects

$$\varepsilon_x = \varepsilon_y = \varepsilon_z = \alpha(\Delta T)$$ α is the coefficient of thermal expansion

Curved members in flexure

$$\sigma = \frac{M \cdot y}{A \cdot e \cdot (r_n - y)} \qquad r_n = \frac{A}{\int \frac{dA}{r}} \qquad e = R - r_n$$

Contact stresses

1) Spherical contact

$$\sigma_1 = \sigma_2 = \sigma_x = \sigma_y = -p_{max}\left[\left(1 - \left|\frac{z}{a}\right| \tan^{-1}\frac{1}{|z/a|}\right)(1+\nu) - \frac{1}{2\left(1+\frac{z^2}{a^2}\right)}\right]$$

$$\sigma_3 = \sigma_z = \frac{-p_{max}}{1+\frac{z^2}{a^2}} \qquad p_{max} = \frac{3F}{2\pi a^2} \qquad a = \sqrt[3]{\frac{3F}{8}\frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{1/d_1 + 1/d_2}}$$

2) Cylindrical contact

$$\sigma_x = -2\nu p_{max}\left(\sqrt{1+\frac{z^2}{b^2}} - \left|\frac{z}{b}\right|\right) \qquad p_{max} = \frac{2 \cdot F}{\pi \cdot b \cdot l}$$

$$\sigma_y = -p_{max}\left(\frac{1+2\,z^2/b^2}{\sqrt{1+z^2/b^2}} - 2\left|\frac{z}{b}\right|\right) \qquad b = \sqrt{\frac{2F}{\pi \cdot l}\frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{1/d_1 + 1/d_2}}$$

$$\sigma_3 = \sigma_z = \frac{-p_{max}}{\sqrt{1+z^2/b^2}}$$

Deflection due to bending

$$\frac{M}{EI} = \frac{d^2y}{dx^2}$$

Strain energy

$$U = \underbrace{\frac{F^2 \cdot L}{2AE}}_{\text{tension compression}} + \underbrace{\frac{T^2 \cdot L}{2 \cdot G \cdot J}}_{\text{torsion}} + \underbrace{\frac{F^2 \cdot L}{2AG}}_{\text{direct shear}} + \underbrace{\int \frac{M^2 dx}{2 \cdot EI}}_{\text{bending}}$$

bending shear $\Rightarrow$ $U_s = \int \frac{C \cdot V^2 dx}{2 \cdot A \cdot G}$

Castigliano's theorem

$$\delta_i = \frac{\partial U}{\partial F_i}$$

Deflection of curved members

bending $\Rightarrow$ $U_b = \int \frac{M^2 d\theta}{2 \cdot A \cdot e \cdot E}$

Buckling

1) Euler

$$P_{cr} = \frac{C \cdot \pi^2 EI}{l^2}$$

$$I = k^2 \cdot A$$

$$\left(\frac{l}{k}\right)_1 = \left(\frac{2\pi^2 CE}{S_y}\right)^{1/2}$$

2) Johnson $\Leftarrow \left(\frac{l}{k}\right) \leq \left(\frac{l}{k}\right)_1$

$$\frac{P_{cr}}{A} = S_y - \left(\frac{S_y}{2\pi} \cdot \frac{l}{k}\right)^2 \cdot \frac{1}{C \cdot E}$$

3) Eccentric loading (secant)

$$\frac{P}{A} = \frac{S_{yc}}{1 + (e \cdot c/k^2)\sec\left[(l/2k)\sqrt{P/AE}\right]}$$

4) Struts $\Leftarrow \left(\frac{L}{k}\right) \leq \left(\frac{L}{k}\right)_2$ $\quad \left(\frac{L}{k}\right)_2 = 0.282 \left(\frac{A \cdot E}{P_{cr}}\right)^{1/2}$

$$\sigma_c = \frac{P}{A}\left(1 + \frac{e \cdot y}{k^2}\right)$$

Deflection of energy dissipative assemblies

1) Single orifice

$$F = c \cdot \dot{x}_0 \exp\left(-\frac{c g t}{W}\right) = c \cdot \dot{x}_0 - \frac{c^2 g x}{W} \qquad x_s = \frac{W \cdot \dot{x}_0}{c \cdot g}$$

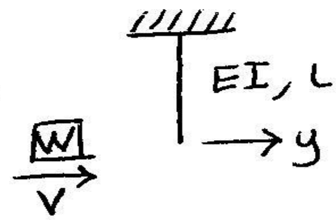
$$E_k = \frac{1}{2}\frac{W \cdot \dot{x}_0^2}{g} \qquad I = \frac{W \cdot \dot{x}_0}{g}$$

2) Multiple orifices

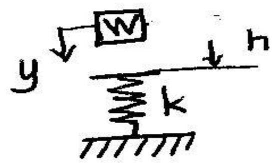
$$x_s = \frac{\dot{x}_0^2}{2 \cdot \ddot{x}_0} \qquad E_k = \frac{W \cdot \dot{x}_0^2}{2 \cdot g} \qquad I = \frac{W}{g} \cdot \dot{x}_0 \qquad F = \frac{W \cdot \ddot{x}_0}{g}$$

Suddenly-applied loading

$$y = \frac{V}{(kg/W)^{1/2}} \sin\left(\frac{kg}{W}\right)^{1/2} \cdot t \quad \Leftarrow$$



$$y = \left[\left(\frac{W}{k}\right)^2 + \frac{2Wh}{k}\right]^{1/2} \cos\left[\left(\frac{kg}{W}\right) t' - \phi\right] + h + \frac{W}{k} \Leftarrow$$



$$\phi = \frac{\pi}{2} + \tan^{-1}\left(\frac{W}{2kh}\right)^{1/2}$$

$$t' = t - t_1 \qquad t_1 = \left(\frac{2h}{g}\right)^{1/2}$$

$$F = k \cdot \delta = W + W\left[1 + \left(\frac{2hk}{W}\right)\right]^{1/2}$$

Notch sensitivity

$$q = \frac{K_f - 1}{K_t - 1}$$

Paris equation (Life prediction)

$$\frac{da}{dN} = A\,(\Delta K_I)^n$$

Maximum shear stress (Tresca) hypothesis

$\tau_{max} \geq S_y/2$ OR $\sigma_1 - \sigma_3 \geq S_y \Rightarrow$ failure occurs von Mises hypothesis

$$\sigma' = \left[\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}\right]^{1/2} = \frac{1}{\sqrt{2}}\left[(\sigma_x-\sigma_y)^2+(\sigma_y-\sigma_z)^2+(\sigma_z-\sigma_x)^2+6(\tau_{xy}^2+\tau_{yz}^2+\tau_{zx}^2)\right]^{1/2} \geq S_y$$

failure occurs

Internal-friction hypothesis

$\sigma_1-\sigma_3 = S_y$ $\qquad \frac{\sigma_1}{S_t}-\frac{\sigma_3}{S_c}=1$ $\quad \sigma_1 \geq 0 \quad \sigma_3 \leq 0 \Rightarrow$ failure occurs

$$S_{sy} = \frac{S_{yt}S_{yc}}{S_{yt}+S_{yc}}$$

Maximum normal stress hypothesis

$n\cdot\sigma_1 = S_t \qquad n\cdot\sigma_3 = -S_c$

Mohr hypothesis

$\sigma_A = \frac{S_{ut}}{n} \qquad 0<\sigma_A<S_{ut} \qquad 0<\sigma_B<S_{ut}$

$\frac{\sigma_A}{S_{ut}}-\frac{\sigma_B}{S_{uc}}=\frac{1}{n} \qquad 0<\sigma_A<S_{ut} \qquad -S_{uc}<\sigma_B<0$

Strain-life relationships

$$\frac{\Delta\varepsilon}{2} = \underbrace{\frac{\sigma_F'}{E}(2N)^b}_{\text{high cycle fatigue}} + \underbrace{\varepsilon_F'(2N)^c}_{\text{low cycle fatigue}}$$

Endurance limit

$$S_e = k_a k_b k_c k_d k_e S_e'$$

Fluctuating stresses

$$\sigma_m = \frac{\sigma_{max}+\sigma_{min}}{2}$$

$$\sigma_a = \left|\frac{\sigma_{max}-\sigma_{min}}{2}\right|$$

Soderberg line

$$\frac{\sigma_a}{S_e}+\frac{\sigma_m}{S_y}=\frac{1}{n}$$

Langer first-cycle-yielding locus

$$\frac{S_a}{S_{yt}}+\frac{S_m}{S_{yt}}=1$$

Modified Goodman line

$$\frac{\sigma_a}{S_e}+\frac{\sigma_m}{S_{ut}}=\frac{1}{n}$$

Gerber locus

$$\frac{n\sigma_a}{S_e}+\left(\frac{n\sigma_m}{S_{ut}}\right)^2=1$$

DE-elliptic locus

$$\left(\frac{n\sigma_a}{S_e}\right)^2+\left(\frac{n\sigma_m}{S_y}\right)^2=1$$

Module

$m = d/N$ $\qquad$ d is pitch diameter

$p = \frac{\pi\cdot d}{N}$ $\qquad$ p is circular pitch

Helical gears

$p_n = p_t\cdot\cos\psi \qquad p_x = \frac{p_t}{\tan\psi}$

$$\cos\psi = \frac{\tan\phi_n}{\tan\phi_t}$$

ψ: helix angle $\quad \phi$: pressure angle

$$N' = \frac{N}{\cos^3\psi}$$ → virtual number of teeth

Straight bevel gears

$$\tan\gamma = \frac{N_P}{N_G} \qquad \tan\Gamma = \frac{N_G}{N_P}$$

Worm gears

$$d_G = \frac{N_G \cdot p_t}{\pi}$$

$$\frac{C^{0.875}}{3} \leq d_W \leq \frac{C^{0.875}}{1.7}$$

$$\tan\lambda = \frac{L}{\pi d_W}$$

Gear trains

$$n_L = e \cdot n_F$$

$$e = \frac{\text{product of driving tooth numbers}}{\text{product of driven tooth numbers}}$$

Planetary gear trains

$$e = \frac{n_L - n_A}{n_F - n_A}$$

Force analysis – helical gearing

$$W_r = W \cdot \sin\phi_n$$

$$W_t = W \cdot \cos\phi_n \cdot \cos\psi$$

$$W_a = W \cdot \cos\phi_n \cdot \sin\psi$$

$$T = \frac{d}{2} \cdot W_t$$

Force analysis – bevel gearing

$$W_t = \frac{T}{r_{av}} \qquad W_r = W_t \cdot \tan\phi \cos\gamma$$

$$W_a = W_t \cdot \tan\phi \sin\gamma$$

Force analysis – worm gearing

$$W^x = W \cdot \cos\phi_n \cdot \sin\lambda$$

$$W^y = W \cdot \sin\phi_n$$

$$W^z = W \cdot \cos\phi_n \cdot \cos\lambda$$

$$W_{Wt} = -W_{Ga} = W^x$$

$$W_{Wr} = -W_{Gr} = W^y$$

$$W_{Wa} = -W_{Gt} = W^z$$

$$\eta = \frac{\cos\phi_n - f \cdot \tan\lambda}{\cos\phi_n + f \cdot \cot\lambda}$$ → efficiency (worm gearing)

Choosing module

1) Pitting

$$m_n = 0.9 \sqrt[3]{\frac{K_A \cdot K_V \cdot T_1 \cdot E\,(i+1)\cos^4\psi}{N_1^2 \cdot P_{em}^2 \cdot i \cdot \varepsilon_\alpha \cdot F_W}}$$

2) Fracture

$$m_n = 0.6 \sqrt[3]{\frac{K_A \cdot K_V \cdot T_1 \cdot \gamma \cdot \cos\psi}{N_1 \cdot \sigma_{em} \cdot \varepsilon_\alpha \cdot F_W}}$$

Power screws

$$T = \frac{F \cdot d_m}{2}\left(\frac{l + \pi \mu d_m}{\pi d_m - \mu l}\right) + \frac{F \cdot \mu_c d_c}{2}$$ ⇐ raising the load

$$T = \frac{F \cdot d_m}{2}\left(\frac{\pi \cdot \mu d_m - l}{\pi d_m + \mu l}\right) + \frac{F \cdot \mu_c \cdot d_c}{2}$$ ⇐ lowering the load

Tension connections

$$k_b = \frac{A_d A_t E}{A_d \cdot L_t + A_t \cdot L_d} \qquad k_m = \frac{\pi \cdot E \cdot d \cdot \tan\alpha}{2 \cdot \ln \dfrac{(L \cdot \tan\alpha + d_w - d)(d_w + d)}{(L \cdot \tan\alpha + d_w + d)(d_w - d)}}$$

$$T = (0.20) \cdot F_i \cdot d$$

$$F_i = \begin{cases} 0.75\ F_p & \text{for reused connections} \\ 0.90\ F_p & \text{for permanent connections} \end{cases}$$

Bearing life

$$F_e \cdot L^{1/a} = C$$ $a = 3$ for ball bearings, $a = 10/3$ " roller "

$$C_{10} \cdot L_{10}^{1/a} = F \cdot L^{1/a}$$

$$C_{10} = F_D \cdot \left[\frac{x_D}{x_o + (\theta - x_o)(\ln 1/R_D)^{1/b}}\right]^{1/a}$$

$$R = \exp\left[-\left\{\frac{\left[x_D - x_o\left(\frac{C'_{10}}{f_A \cdot F_D}\right)^{a}\right]}{(\theta - x_o)\left(\frac{C'_{10}}{f_A \cdot F_D}\right)^{a}}\right\}^{b}\right]$$

C'_{10}: basic load rating

Journal bearings

$$S = \left(\frac{r}{c}\right)^2 \left(\frac{\mu \cdot N}{P}\right)$$

Welded, brazed, bonded joints

$$I = (0.707)\, h \cdot I_u$$

Mechanical springs

$$\tau = \frac{8 \cdot F \cdot D}{\pi \cdot d^3} + \frac{4F}{\pi d^2} \qquad C = \frac{D}{d} \qquad K_s = \frac{2C+1}{2C} \qquad K_B = \frac{4C+2}{4C-3}$$

$$L_0 < \frac{\pi . D}{\alpha} \left[\frac{2[E-G]}{2G+E} \right]^{1/2}$$

$$\omega = \pi \sqrt{\frac{k.g}{W}}$$

Clutches, brakes

$$dN = p.b.r.d\theta$$

$$p = \frac{p_a}{\sin\theta_a} . \sin\theta$$

Internal expanding rim clutches and brakes

$$M_f = \int_{\theta_1}^{\theta_2} f\, dN\, (r - a.\cos\theta) \qquad M_N = \int_{\theta_1}^{\theta_2} dN\, (a.\sin\theta)$$

for clockwise $\quad F = \dfrac{M_N - M_f}{c} \qquad T = \displaystyle\int_{\theta_1}^{\theta_2} f.r.dN$

$$R_x = \int_{\theta_1}^{\theta_2} dN.\cos\theta - \int_{\theta_1}^{\theta_2} f\, dN \sin\theta - F_x$$

$$R_y = \int_{\theta_1}^{\theta_2} dN.\sin\theta + \int_{\theta_1}^{\theta_2} f\, dN \cos\theta - F_y$$

for counterclockwise $\quad F = \dfrac{M_N + M_f}{c}$

$$R_x = \int_{\theta_1}^{\theta_2} dN.\cos\theta + \int_{\theta_1}^{\theta_2} f\, dN \sin\theta - F_x$$

$$R_y = \int_{\theta_1}^{\theta_2} dN \sin\theta - \int_{\theta_1}^{\theta_2} f\, dN \cos\theta - F_y$$

External contracting rim clutches and brakes

$$M_f = \frac{f.p_a.b.r}{\sin\theta_a} \int_{\theta_1}^{\theta_2} \sin\theta\, (r - a.\cos\theta)\, d\theta$$

$$M_N = \frac{p_a.b.r.a}{\sin\theta_a} \int_{\theta_1}^{\theta_2} \sin^2\theta\, d\theta$$

for clockwise $\quad F = \dfrac{M_N + M_f}{c}$

$$R_x = \int dN.\cos\theta + \int f\, dN \sin\theta - F_x \qquad R_y = \int f\, dN \cos\theta - \int dN \sin\theta + F_y$$

for counterclockwise

$$F = \frac{M_N - M_f}{c} \qquad R_x = \int dN\cos\theta - \int f\,dN\sin\theta - F_x$$

$$R_y = -\int f\,dN\cos\theta - \int dN\sin\theta + F_y$$

Band type clutches and brakes

$$\frac{P_1}{P_2} = e^{f\cdot\phi} \qquad T = (P_1 - P_2)\cdot\frac{D}{2}$$

$$dN = p\cdot b\cdot r\cdot d\theta = P\cdot d\theta \Rightarrow p = \frac{P}{b\cdot r}$$

Axial clutches and brakes

$$\left.\begin{aligned} F &= \frac{\pi\cdot p_a\cdot d}{2}(D-d) \\ T &= \frac{\pi\cdot p_a\cdot d}{8}(D^2-d^2) \end{aligned}\right\} \text{uniform wear}$$

$$\left.\begin{aligned} F &= \frac{\pi\cdot p_a}{4}(D^2-d^2) \\ T &= \frac{2\cdot\pi\cdot f\cdot p}{24}(D^3-d^3) \end{aligned}\right\} \text{uniform pressure}$$

$$p = p_a$$

Cone clutches and brakes

$$\left.\begin{aligned} F &= \frac{\pi\cdot p_a\cdot d}{2}(D-d) \\ T &= \frac{F\cdot f}{4\cdot\sin\alpha}(D+d) \end{aligned}\right\} \text{uniform wear}$$

$$F = \frac{\pi\cdot p_a}{4}(D^2-d^2)$$

$$T = \frac{F\cdot f}{3\cdot\sin\alpha}\,\frac{D^3-d^3}{D^2-d^2}$$

Flexible mechanical elements

$$\frac{F_1 - F_c}{F_2 - F_c} = \exp(f\cdot\phi)$$

$$F_c = m\cdot r^2\cdot\omega^2$$

$$F_1 = F_i + F_c + T/D$$

$$F_2 = F_i + F_c - T/D$$