A shaft is a rotating member used to transmit power or motion.

An axle is a nonrotating member which carries no torque and is used to support rotating wheels, pulleys, etc.

Design considerations ⇒ Deflection and rigidity (bending deflection, torsional deflection, slope at bearings and shaft-supported elements, shear deflection due to transverse loading of short shafts); Stress and strength (static strength, fatigue strength, reliability)

Torque-transfer elements ⇒ keys, splines, setscrews, pins, press or shrink fits, tapered fits.

Locational-devices (for accurate axial location of the device) ⇒ cotter and washer, nut and washer, sleeve, shaft shoulder, ring and groove, setscrew, split hub or tapered two-piece hub, collar and screw, pins.

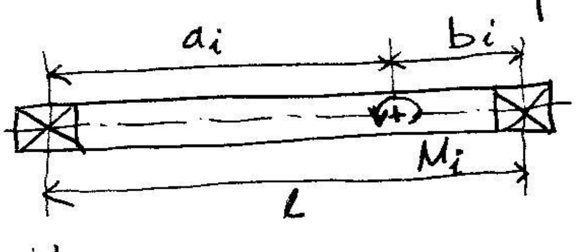
Geometric Constraints

for the left-bearing slope constraint active

$$d = \left| \frac{32n}{3\pi E l \Sigma\theta} \left\{ \left[\sum F_i b_i (b_i^2 - l^2) + \sum M_i (3a_i^2 - 6a_i l + 2l^2) \right]_H^2 + \left[\sum F_i b_i (b_i^2 - l^2) + \sum M_i (3a_i^2 - 6a_i l + 2l^2) \right]_V^2 \right\}^{1/2} \right|^{1/4}$$

for the right-bearing constraint active

$$d = \left| \frac{32n}{3\pi E l \Sigma\theta} \left\{ \left[\sum F_i a_i (l^3 - a_i^3) + \sum M_i (3a_i^2 - l^2) \right]_H^2 + \left[\sum F_i a_i (l^2 - a_i^2) + \sum M_i (3a_i^2 - l^2) \right]_V^2 \right\}^{1/2} \right|^{1/4}$$



$\Sigma\theta$ ⇒ absolute value of the allowable slope at the bearing

The tabular method for stepped shafts,

$$y = \int_0^x \int_0^x \frac{M}{EI}\,dx\,dx + C_1 \cdot x + C_2$$

$$\frac{dy}{dx} = \int_0^x \frac{M}{EI}\,dx + C_1$$

$$C_1 = \frac{\int_0^{x_a}\int_0^{x_a} M/(EI)\,dx\,dx - \int_0^{x_b}\int_0^{x_b} M/(EI)\,dx\,dx}{x_a - x_b}$$

$$C_2 = \frac{x_b \int_0^{x_a}\int_0^{x_a} M/(EI)\,dx\,dx - x_a \int_0^{x_b}\int_0^{x_b} M/(EI)\,dx\,dx}{x_a - x_b}$$

$x_a, x_b \Rightarrow$ bearing locations

$$d_{new} = d_{old} \left| \frac{n_d \cdot y_{old}}{y_{all}} \right|^{1/4} \qquad d_{new} = d_{old} \left| \frac{n_d (dy/dx)_{old}}{(\text{slope})_{all}} \right|^{1/4}$$

$y_{all} \Rightarrow$ allowable deflection
(any entry is too large in absolute magnitude, a new diameter can be found)

$(\text{slope})_{all} \Rightarrow$ allowable slope
(any slope is too large in absolute magnitude, a new diameter can be found)

note the largest d_{new}/d_{old} ratio, then multiply all diameters by this ratio

Table 18–1
Form for Tabulation Method for Shaft Transverse Deflection Due to Bending Moment

	Moment M	Distance x	Diameter d	Modulus E	$\frac{M}{EI}$	$\int_0^x \frac{M}{EI}\,dx$	$\int_0^x \left(\int_0^x \frac{M}{EI}\,dx\right) dx$	Deflection y	Slope dy/dx
1						0	0		
2									
3									

Strength Constraints

$$\sigma_{xa} = \left| \frac{32 \cdot M_a}{\pi d^3} \right| \qquad \tau_{xym} = \frac{16 \cdot T_m}{\pi \cdot d^3}$$

$$\sigma_a' = \sigma_{xa} \qquad \sigma_m' = \sqrt{3}\, \tau_{xym}$$

Fatigue - Strength Loci for Shafts

Locus Name	Basic Formula
Soderberg	$\frac{n\sigma_a}{S_e} + \frac{n\sigma_m}{S_y} = 1$
Goodman	$\frac{n\sigma_a}{S_e} + \frac{n\sigma_m}{S_{ut}} = 1$
Gerber	$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1$
ASME elliptic	$\left(\frac{n\sigma_a}{S_e}\right)^2 + \left(\frac{n\sigma_m}{S_y}\right)^2 = 1$
Bagci	$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_y}\right)^4 = 1$

Yielding (Langer) $\frac{n}{S_y}(\sigma_a + \sigma_m) = 1$

Shaft Diameter Equation for the DE - Gerber Criterion

$$\sigma_a' = (\sigma_{xa}^2 + 3\tau_{xya}^2)^{1/2} = \frac{16}{\pi d^3}\sqrt{4(K_f M_a)^2 + 3(K_{fs} T_a)^2}$$

$$\sigma_m' = (\sigma_{xm}^2 + 3\tau_{xym}^2)^{1/2} = \frac{16}{\pi d^3}\sqrt{4(K_f M_m)^2 + 3(K_{fs} T_m)^2}$$

Critical Speeds

$$\omega_1 = \left(\frac{\pi}{L}\right)^2 \sqrt{\frac{EI}{m}} = \left(\frac{\pi}{l}\right)^2 \sqrt{\frac{gEI}{A \cdot \gamma}}$$

for an ensemble of attachments

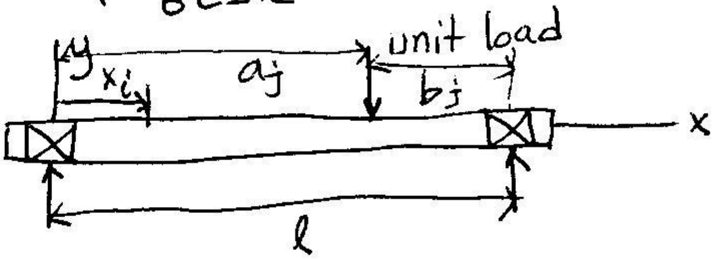
$$\omega_1 = \sqrt{\frac{g \cdot \Sigma w_i \cdot y_i}{\Sigma w_i \cdot y_i^2}}$$

$w_i \Rightarrow$ weight of i th attachment
$y_i \Rightarrow$ deflection at the i th body location

(Rayleigh's equation)

the use of influence coefficients

$$\delta_{ij} = \begin{cases} \dfrac{b_j \cdot x_i}{6EI\ell}\left(\ell^2 - b_j^2 - x_i^2\right) & x_i \le a_i \\[2ex] \dfrac{a_j(\ell - x_i)}{6EI\ell}\left(2\ell x_i - a_j^2 - x_i^2\right) & x_i > a_i \end{cases}$$



$$\delta_{ij} = \delta_{ji}$$

an influence coefficient is the transverse deflection at location i on a shaft due to a unit load at location j on the shaft

$$y_1 = F_1\delta_{11} + F_2\delta_{12} + F_3\delta_{13}$$
$$y_2 = F_1\delta_{21} + F_2\delta_{22} + F_3\delta_{23}$$
$$y_3 = F_1\delta_{31} + F_2\delta_{32} + F_3\delta_{33}$$

F_i is because of W_i or $m_i\omega^2 y_i$

$$\begin{vmatrix} m_1\delta_{11} - 1/\omega^2 & m_2\delta_{12} & m_3\delta_{13} \\ m_1\delta_{21} & (m_2\delta_{22} - 1/\omega^2) & m_3\delta_{23} \\ m_1\delta_{31} & m_2\delta_{32} & (m_3\delta_{33} - 1/\omega^2) \end{vmatrix} = 0$$

$$\frac{1}{\omega_1^2} = \sum_{i=1}^{n} \frac{1}{\omega_{ii}^2} \Leftarrow \text{(neglect } 1/\omega_2^2,\ 1/\omega_3^2,\ \text{etc.)}$$

Dunkerley's approximation

Consider three loads P, Q, and R at positions 1, 2, and 3 on the shaft,

$$\omega_{11}^2 = \frac{g}{P\cdot\delta_{11}} \qquad \omega_{22}^2 = \frac{g}{Q\cdot\delta_{11}} \qquad \omega_{33}^2 = \frac{g}{R\cdot\delta_{11}}$$

$$\frac{1}{\omega_1^2} = \frac{1}{\omega_{11}^2} + \frac{1}{\omega_{22}^2} + \frac{1}{\omega_{33}^2} = \frac{P+Q+R}{g}\cdot\delta_{11}$$