Flexible Mechanical Elements

Belts

flat belts ⇒ crowned pulleys

round and V belts ⇒ grooved pulleys or sheaves

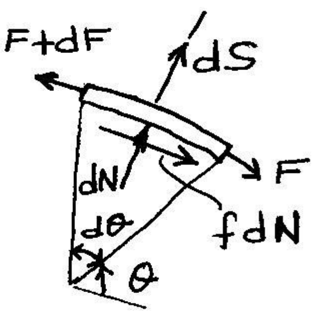
timing belts ⇒ toothed wheels or sprockets

Open Belt ⇒ $\theta_s = \pi - 2 \cdot \sin^{-1} \frac{D-d}{2C} = \theta_d \quad \theta_L = \pi + 2 \cdot \sin^{-1} \frac{D-d}{2C} = \theta_D$

$$L = \sqrt{4C^2 - (D-d)^2} + \frac{1}{2} (D \cdot \theta_L + d\,\theta_s)$$

Crossed Belt ⇒ $\theta = \pi + 2 \cdot \sin^{-1} \frac{D+d}{2C}$

$$L = \sqrt{4C^2 - (D+d)^2} + \frac{\theta}{2} (D+d)$$



$$dS = (m r d\theta) r \omega^2 = m r^2 \omega^2 d\theta = m V^2 d\theta = F_c d\theta$$

$$\left.\begin{aligned} dN &= F d\theta - dS \\ dF &= F dN \end{aligned}\right\} \frac{dF}{d\theta} - f \cdot F = - f \cdot m \cdot r^2 \cdot \omega^2$$

$$F = (F_2 - m r^2 \omega^2) \exp(f\theta) + m r^2 \omega^2$$

$$F\big|_{\theta=\phi} = F_1 \quad \Rightarrow \quad \frac{F_1 - m \cdot r^2 \cdot \omega^2}{F_2 - m \cdot r^2 \cdot \omega^2} = \frac{F_1 - F_c}{F_2 - F_c} = \exp(f \cdot \phi)$$

$$F_1 - F_2 = (F_1 - F_c) \cdot \frac{\exp(f \cdot \phi) - 1}{\exp(f \cdot \phi)}$$

$$F_c = \frac{w}{g} V^2$$

(w → weight of a meter of belt)

tight side: $F_1 = F_i + F_c + \Delta F'$ $\qquad$ loose side: $F_2 = F_i + F_c - \Delta F'$

$$F_1 = F_i + F_c + T/D \qquad F_2 = F_i + F_c - T/D$$

$$F_1 - F_2 = \frac{2T}{D} = \frac{T}{D/2} \qquad F_i = \frac{F_1 + F_2}{2} - F_c = \frac{T}{D} \frac{\exp(f\phi)+1}{\exp(f\phi)-1}$$

$$F_1 = F_c + F_i \frac{2\exp(f\phi)}{\exp(f\phi)+1} \qquad F_2 = F_c + F_i \frac{2}{\exp(f\phi)+1}$$

$$H = \frac{(F_1 - F_2) \cdot V}{1000} \text{ kW}$$

H ← transmitted power

$$H_d = H_{nom} \cdot K_s \cdot n_d$$

$$(F_1)_a = b F_a C_p C_v$$

C_p ← pulley correction factor

C_v ← velocity correction factor

F_a ← allowable tension (N/m)

Tables 17-2, ..., 5 for some useful information about flat belts.

Flat Metal Belts

$$\sigma_b = \frac{E \cdot t}{(1-\nu^2) \cdot D} = \frac{E}{(1-\nu^2)(D/t)}$$

$(\sigma)_1 = F_1/(b \cdot t)$ $\quad (\sigma_2) = F_2/(b \cdot t)$

the largest tensile stress $(\sigma_b)_1 + F_1/(b \cdot t)$

the smallest " " $(\sigma_b)_2 + F_2/(b \cdot t)$

Tables 17-6, ..., 17-8 for some useful information.

V-Belts

$$L_p = 2C + 1.57\,(D+d) + (D-d)^2/(4C)$$

L_p ← pitch length

$$C = 0.25 \left\{ \left[\frac{\pi}{2}(D+d) - L_p\right] + \sqrt{\left[\frac{\pi}{2}(D+d) - L_p\right]^2 - 2(D-d)^2} \right\}$$

Tables 17-9, ..., 17-17 for some useful information.

Belting equations ⇒ Table 17-18.

Timing Belts

Table 17-19 ⇒ pitch values for timing belts.

Belting equations ⇒ Table 17-18.

Roller Chains

Table 17-20 ⇒ standard roller chains

$$\sin\frac{\gamma}{2} = \frac{P/2}{D/2} \Rightarrow D = \frac{P}{\sin(\gamma/2)} \qquad \gamma = \frac{360}{N} \Rightarrow D = \frac{P}{\sin(180/N)}$$

P ← pitch

D ← pitch diameter of sprocket

$V = \frac{N \cdot p \cdot n}{1000}$ ← n: sprocket speed (rev/min)

V: Chain velocity

maximum exit velocity of the chain

$$V_{max} = \frac{\pi \cdot D \cdot n}{1000} = \frac{\pi n p}{1000 \cdot \sin(\gamma/2)}$$

minimum exit velocity of the chain

$$d = D \cdot \cos\frac{\gamma}{2} \Rightarrow V_{min} = \frac{\pi d n}{1000} = \frac{\pi n p}{1000} \cdot \frac{\cos(\gamma/2)}{\sin(\gamma/2)}$$

$$\gamma/2 = 180/N \Rightarrow \frac{\Delta V}{V} = \frac{V_{max} - V_{min}}{V} = \frac{\pi}{N}\left[\frac{1}{\sin(180/N)} - \frac{1}{\tan(180/N)}\right]$$

$\frac{\Delta V}{V}$: chordal speed variation

Nominal power H_1 (link-plate limited)

$$H_1 = 0.003\, N_1^{1.08}\, n_1^{0.9} \left(\frac{p}{25.4}\right)^{(3 - 0.07(p/25.4))} \text{ kW}$$

Nominal power H_2 (roller limited)

$$H_2 = \frac{746\, K_r\, N_1^{1.5} (p/25.4)^{0.8}}{n_1^{1.5}}$$

$$\frac{L}{p} = \frac{2C}{p} + \frac{N_1 + N_2}{2} + \frac{(N_2 - N_1)^2}{4\pi^2 C/p}$$

$\frac{L}{p}$: approximate length in pitches

$$\frac{C}{p} = \frac{1}{4}\left[-A + \sqrt{A^2 - 8\left(\frac{N_2 - N_1}{2\pi}\right)^2}\right]$$

$$A = \frac{N_1 + N_2}{2} - \frac{L}{p}$$

$H_a = K_1 K_2 H_{tab}$ ← H_a: allowable power

$H_d = H_{nom} \cdot K_s \cdot n_d$ ← H_d: transmitted power

Wire Rope

$$M = EI/r = \sigma I/c \Rightarrow \sigma = Ec/r = E_r \frac{d_w}{D}$$

d_w: wire diameter; D: sheave diameter

$F_b = \sigma \cdot A_m$

$n = F_u / F_t$ (F_u: ultimate wire load; F_t: largest working tension)

$p = \frac{2F}{d \cdot D}$ ← p: pressure of the rope

$$F_f = \frac{(p/S_u)\, S_u \cdot d \cdot D}{2}$$

F_f: fatigue tension

$$n = \frac{F_f - F_b}{F_t}$$

n: factor of safety in fatigue