Clutches, Brakes, Couplings, and Flywheels

The actuating force
The torque transmitted
The energy loss
The temperature rise

Rim types with internal expanding shoes, Rim types with external contracting shoes, Band types, Disk or axial types, Cone types, Miscellaneous types.

Rudiments of Brake Analysis

Estimate, model, or measure the pressure distribution on the friction surfaces.

Find a relationship between the largest pressure and the pressure at any point.

Use the conditions of static equilibrium to find the braking force or torque and the support reactions.

$$w_2 \int_0^{w_1} p(u)\,du = p_{av} \cdot w_1 w_2$$

$$\int_0^{w_1} p(u)\,u\,du = \bar{u} \int_0^{w_1} p(u)\,du = p_{av} \cdot w_1 \cdot \bar{u}$$

$$\sum F_x = R_x \mp \int_0^{w_1} f\,w_2\,p(u)\,du = R_x \mp f w_2 \int_0^{w_1} p(u)\,du = R_x \mp f\,p_{av} w_1 w_2 = 0$$

$R_x = \pm f \cdot p_{av} \cdot w_1 \cdot w_2$

+ ⇒ for leftward motion of floor
− ⇒ " rightward " " "

$\sum F_y = 0 \Rightarrow R_y = F - p_{av} \cdot w_1 \cdot w_2$ ⇒ for either direction

$$\sum M_A = 0 \Rightarrow F = \frac{w_2}{b}\left[\int_0^{w_1} p(u)(c+u)\,du \mp a f \cdot \int_0^{w_1} p(u)\,du\right]$$

$F \leq 0 \Rightarrow f_{cr} \geq \dfrac{c+\bar{u}}{a}$ (self-locking)

Internal Expanding Rim Clutches and Brakes

$$p = \frac{p_a}{\sin\theta_a} \cdot \sin\theta$$

$$dN = p \cdot b \cdot r \cdot d\theta = \frac{p_a \cdot b \cdot r \cdot \sin\theta \, d\theta}{\sin\theta_a}$$

$$M_f = \int_{\theta_1}^{\theta_2} f \, dN \, (r - a \cdot \cos\theta)$$

$$M_N = \int_{\theta_1}^{\theta_2} dN \, (a \cdot \sin\theta)$$

for clockwise rotation;

$$F = \frac{M_N - M_f}{c} \qquad M_N = M_f \Rightarrow \text{self-locking}$$

$$T = \int_{\theta_1}^{\theta_2} f \cdot r \cdot dN$$

$$R_x = \int_{\theta_1}^{\theta_2} dN \cos\theta - \int_{\theta_1}^{\theta_2} f \, dN \sin\theta - F_x$$

$$R_y = \int_{\theta_1}^{\theta_2} dN \sin\theta + \int_{\theta_1}^{\theta_2} f \, dN \cos\theta - F_y$$

for counterclockwise rotation;

$$F = \frac{M_N + M_f}{c}$$

$$R_x = \int_{\theta_1}^{\theta_2} dN \cos\theta + \int_{\theta_1} f \, dN \sin\theta - F_x$$

$$R_y = \int_{\theta_1}^{\theta_2} dN \sin\theta - \int_{\theta_1}^{\theta_2} f \, dN \cos\theta - F_y$$

In these equations, the reference system has its origin at the center of the drum, positive x axis is taken through the hinge pin, the positive y axis is in the direction of the shoe.

External Contracting Rim Clutches and Brakes

$$M_f = \frac{f \cdot p_a \cdot b \cdot r}{\sin\theta_a} \int_{\theta_1}^{\theta_2} \sin\theta \,(r - a \cdot \cos\theta)\, d\theta$$

$$M_N = \frac{p_a \cdot b \cdot r \cdot a}{\sin\theta_a} \int_{\theta_1}^{\theta_2} \sin^2\theta \, d\theta$$

for clockwise;

$$F = \frac{M_N + M_f}{c}$$

$$R_x = \int dN \cdot \cos\theta + \int f\, dN \sin\theta - F_x$$

$$R_y = \int f\, dN \cos\theta - \int dN \sin\theta + F_y$$

for counterclockwise;

$$F = \frac{M_N - M_f}{c}$$

$$R_x = \int dN \cdot \cos\theta - \int f\, dN \sin\theta - F_x$$

$$R_y = -\int f\, dN \cos\theta - \int dN \sin\theta + F_y$$

Band-Type Clutches and Brakes

$$\left.\begin{array}{l} dN = P\,d\theta \\ dP - f\,dN = 0 \end{array}\right\rangle \int_{P_2}^{P_1} \frac{dP}{P} = f \int_0^{\phi} d\theta \Rightarrow \ln\frac{P_1}{P_2} = f \cdot \phi$$

$$\frac{P_1}{P_2} = e^{f \cdot \phi} \qquad T = (P_1 - P_2)\frac{D}{2}$$

$$dN = p \cdot b \cdot r \cdot d\theta = P\,d\theta \Rightarrow p = \frac{P}{b \cdot r} = \frac{2P}{b \cdot D}$$

$$p_a = \frac{2 \cdot P_1}{b \cdot D}$$

Frictional-Contact Axial Clutches

Uniform Wear;

$$dF = 2\pi p r dr \qquad p \cdot r = p_a \cdot \frac{d}{2}$$

$$F = \int_{d/2}^{D/2} 2\pi p r dr = \pi p_a d \int_{d/2}^{D/2} dr = \frac{\pi \cdot p_a \cdot d}{2} (D-d)$$

$$T = \int_{d/2}^{D/2} 2\pi f p r^2 dr = \frac{\pi \cdot p_a \cdot d}{8} (D^2 - d^2) \quad \text{OR} \quad T = \frac{F \cdot f}{4} (D+d)$$

Uniform Pressure;

$$F = \frac{\pi \cdot p_a}{4} (D^2 - d^2)$$

$$T = 2\pi f p \int_{d/2}^{D/2} r^2 dr = \frac{2\pi f p}{24} (D^3 - d^3) \qquad p = p_a$$

$$T = \frac{F \cdot f}{3} \frac{D^3 - d^3}{D^2 - d^2}$$

Disk Brakes

$$F = \int_{\theta_1}^{\theta_2} \int_{r_i}^{r_o} p r dr d\theta = (\theta_2 - \theta_1) \int_{r_i}^{r_o} p r dr$$

$$T = \int_{\theta_1}^{\theta_2} \int_{r_i}^{r_o} f p r^2 dr d\theta = (\theta_2 - \theta_1) \cdot f \cdot \int_{r_i}^{r_o} p r^2 dr$$

$$r_e = \frac{T}{f \cdot F} \qquad \bar{r} = \frac{M_x}{F} = \frac{(\cos\theta_1 - \cos\theta_2)}{\theta_2 - \theta_1} \cdot r_e$$

r_e → equivalent radius

Uniform wear; $p = p_a \cdot r_i / r$

$$F = (\theta_2 - \theta_1) p_a \cdot r_i (r_o - r_i)$$

$$T = (\theta_2 - \theta_1) f p_a r_i \int_{r_i}^{r_o} r dr = \frac{1}{2} (\theta_2 - \theta_1) f p_a r_i (r_o^2 - r_i^2)$$

$$r_e = \frac{r_o + r_i}{2} \qquad \bar{r} = \frac{\cos\theta_1 - \cos\theta_2}{\theta_2 - \theta_1} \cdot \frac{r_o + r_i}{2}$$

Uniform pressure;

$$p = p_a$$

$$F = (\theta_2-\theta_1)\cdot p_a \int_{r_i}^{r_o} r\,dr = \frac{1}{2}(\theta_2-\theta_1)\,p_a\,(r_o^2-r_i^2)$$

$$T = (\theta_2-\theta_1)\,f\,p_a \int_{r_i}^{r_o} r^2\,dr = \frac{1}{3}(\theta_2-\theta_1)\,f\,p_a\,(r_o^3-r_i^3)$$

$$r_e = \frac{2}{3}\,\frac{r_o^3-r_i^3}{r_o^2-r_i^2} \qquad \bar{r} = \frac{2}{3}\,\frac{r_o^3-r_i^3}{r_o^2-r_i^2}\cdot\frac{\cos\theta_1-\cos\theta_2}{\theta_2-\theta_1}$$

Circular (Button or Puck) Pad Caliper Brake

$$r_e = \delta e \qquad F = \pi R^2 p_{av} \qquad T = f\cdot F\cdot r_e$$

Tables 16-1,2,3 some useful information.

Cone Clutches and Brakes

uniform wear;

$$p = p_a\frac{d}{2r} \qquad F = \int p\,dA\sin\alpha = \frac{\pi\cdot p_a\cdot d}{2}(D-d)$$

$$T = \int r f p\,dA = \frac{\pi\cdot f\cdot p_a d}{8\cdot\sin\alpha}(D^2-d^2) = \frac{F\cdot f}{4\cdot\sin\alpha}(D+d)$$

uniform pressure;

$$p = p_a$$

$$F = \int p_a\,dA\sin\alpha = \frac{\pi p_a}{4}(D^2-d^2)$$

$$T = \int r f p_a\,dA = \frac{\pi f p_a}{12\sin\alpha}(D^3-d^3) = \frac{F f}{3\cdot\sin\alpha}\,\frac{D^3-d^3}{D^2-d^2}$$

Energy Considerations

$$I_1\cdot\ddot{\theta}_1 = -T \qquad I_2\cdot\ddot{\theta}_2 = T \quad\Rightarrow\quad \dot{\theta}_1 = -\frac{T}{I_1}\cdot t+\omega_1 \quad \dot{\theta}_2 = \frac{T}{I_2}\cdot t+\omega_2$$

$$\dot{\theta} = \dot{\theta}_1-\dot{\theta}_2 = \omega_1-\omega_2 - T\left(\frac{I_1+I_2}{I_1\cdot I_2}\right)\cdot t$$

$$\dot{\theta} = 0 \Rightarrow t_1 = \frac{I_1\cdot I_2(\omega_1-\omega_2)}{T(I_1+I_2)} \Leftarrow$$ time for clutching operation

$$u = T\cdot\dot{\theta}$$

$\leftarrow$ rate of energy dissipation

$$E = \int_0^{t_1} u\,dt = \frac{I_1 I_2(\omega_1-\omega_2)^2}{2(I_1+I_2)}$$

Temperature Rise

$$\Delta T = \frac{E}{C \cdot m}$$

specific heat capacity

mass of clutch or brake parts

Friction Materials

Woven-cotton lining, molded-asbestos lining, molded-asbestos pads, sintered-metal pads, cermet pads.

Table 16-3,4.