Mechanical Springs

Wire springs: Helical springs of round or square wire; they resist tensile, compressive or torsional loads.
Flat springs: Cantilever and elliptical types.
Special-shaped springs:

Stresses in Helical Springs

$$\tau_{max} = \pm \underbrace{\frac{T \cdot r}{J}}_{\text{torsion}} + \underbrace{\frac{F}{A}}_{\text{direct shear}} \qquad T = \frac{FD}{2} \quad r = \frac{d}{2} \quad J = \frac{\pi d^4}{32} \quad A = \frac{\pi d^2}{4}$$

$$\tau = \frac{8FD}{\pi \cdot d^3} + \frac{4F}{\pi d^2} \qquad C = \frac{D}{d}$$

coil curvature

$$\tau = K_s \cdot \frac{8FD}{\pi d^3} \qquad K_s = \frac{2C+1}{2C}$$

shear-stress augmentation factor

The Curvature Effect

The curvature of wire increases the stress on the inside of the spring.

$$K_W = \frac{4C-1}{4C-4} + \frac{0.615}{C} \qquad K_B = \frac{4C+2}{4C-3}$$

Wahl factor

Bergsträsser factor

K_B is preferred.

$$K_c = \frac{K_B}{K_s}$$

curvature correction factor

Deflection of Helical Springs

Using Castigliano's theorem,

$$U = \frac{T^2 L}{2GJ} + \frac{F^2 L}{2AG} \qquad T = \frac{FD}{2} \quad l = \pi D N \quad J = \frac{\pi d^4}{32} \quad A = \frac{\pi d^2}{4}$$

$$U = \frac{4F^2 D^3 N}{d^4 G} + \frac{2F^2 DN}{d^2 G}$$

$$C = D/d$$

$$y = \frac{\partial U}{\partial F} = \frac{8FD^3N}{d^4G} + \frac{4FDN}{d^2G} \Rightarrow y = \frac{8FD^3N}{d^4G}\left(1 + \frac{1}{2C^2}\right) \approx \frac{8FD^3N}{d^4G}$$

$$k = \frac{F}{y} = \frac{d^4 G}{8D^3N}$$

Extension Springs

Stress-augmentation factors for the critical points (because of hook ends)

$$(K)_A = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1-1)} \qquad C_1 = \frac{2r_1}{d}$$

hook radius

$$(K)_B = \frac{4C_2 - 1}{4C_2 - 4} \qquad C_2 = \frac{2r_2}{d}$$

hook radius

lower radius for improved design

$$L_B = d(N_a + 1)$$

number of active coils

Compression Springs

$$L_s = (N_t - a)\cdot d \qquad a \simeq 0.75$$

look at Table 10-2

$$Q = N_t - N_a \qquad L_s = (N_t - 1)\cdot d = (N_a + Q - 1)\,d = (N_a + Q')\,d$$

dead turns

$$L = \pi (N_a + Q)\cdot D \qquad V = \pi^2 d^2 (N_a + Q)\cdot D/4$$

the length of wire

volume of wire

Stability

$$y_{cr} = L_o \cdot C_1' \left[1 - \left(1 - \frac{C_2'}{\lambda_{eff}^2}\right)^{1/2}\right] \qquad \lambda_{eff} = \frac{\alpha \cdot L_o}{D}$$

end condition (Table 10-3)

effective slenderness ratio

$$C_1' = \frac{E}{2(E-G)} \qquad C_2' = \frac{2\pi^2(E-G)}{2G+E}$$

$$\frac{C_2'}{\lambda_{eff}^2} < 1 \Rightarrow \text{absolute stability}$$

$$\Downarrow$$

$$L_o < \frac{\pi D}{\alpha}\left[\frac{2(E-G)}{2G+E}\right]^{1/2}$$

For steels $L_o < 2.63 \frac{D}{\alpha}$

For squared and ground ends

$\alpha = 0.5 \quad L_o \leq 5.26\,D$

Spring Materials

Manufactured by hot or cold-working processes.
Plain carbon steels, alloy steels, corrosion-resisting steels, phosphor bronze, spring brass, beryllium copper, nickel alloys (Table 10-4).

$$S_{ut} = \frac{A}{d^m}$$ (tensile strength depends on wire size)

Table 10-5

$$0.35\, S_{ut} \leq S_{sy} \leq 0.52\, S_{ut}$$

Table 10-6 --- 10-13 (useful information about springs)

Helical Compression Springs for Static Service

Figure 10-9 summarizes useful information.

Critical Frequency of Helical Springs

The physical dimensions of the spring are not such as to create a natural vibratory frequency close to the frequency of the applied force; otherwise, resonance may occur, resulting in damaging stresses, since the internal damping of spring materials is quite low.

The governing equation for a spring placed between two flat and parallel plates is the wave equation

weight of spring

$$\frac{\partial^2 u}{\partial y^2} = \frac{W}{k \cdot g \cdot L^2} \frac{\partial^2 u}{\partial t^2}$$

spring rate

length of spring between plates

$$\omega = m\pi \sqrt{\frac{k \cdot g}{W}} \qquad f = \frac{\omega}{2\pi} = \frac{m}{2} \sqrt{\frac{k \cdot g}{W}}$$

$m = 1 \Rightarrow$ fundamental frequency

f should be 15 to 20 times the frequency of the force or motion.