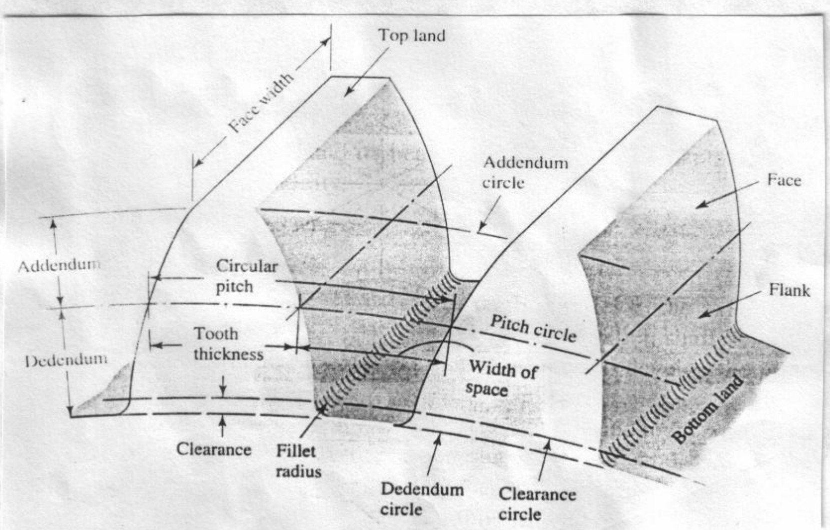
Gears - General

Types of gears

- Spur gears have teeth parallel to the axis of rotation.
- Spur gears are used to transmit motion from one shaft to another, parallel, shaft.
- Helical gears have teeth inclined to the axis of rotation.
- Sometimes helical gears are used to transmit motion between nonparallel shafts.
- Bevel gears have teeth formed on conical surfaces and are used mostly for transmitting motion between intersecting shafts.
- Hypoid gears, worm gear. (Nonparallel and nonintersecting shafts)

Nomenclature



pitch circle: taksimat dairesi

addendum circle: diş başı dairesi

dedendum circle: diş dibi dairesi

base circle: temel daire

- The circular pitch p is the distance, measured on the pitch circle, measured from a point on one tooth to a corresponding point on an adjacent tooth.
- The module m is the ratio of the pitch diameter to the number of teeth.
- The diametral pitch P is the ratio of the number of teeth on the gear to the pitch diameter.

- The addendum a is the radial distance between the top land and the pitch circle.
- The deden dum b is the radial distance from the bottom land to the pitch circle.
- The clerance circle is a circle that is tangent to the addendum circle of the mating gear.
- Some useful relations,

$P = \dfrac{N}{d}$ — N → number of teeth; d → pitch diameter; P → diametral pitch

$m = \dfrac{d}{N}$ — d → pitch diameter (mm); m → module (mm)

$p = \dfrac{\pi d}{N} = \pi m$ — p → circular pitch

- Tooth system is a standard which specifies the relationships involving addendum, dedendum, working depth, tooth thickness, and pressure angle. (Tables 13-1 ~ 13-5)

<u>Design of spur gears</u>

- Choose an appropriate module (m), (Use fatigue, fracture mechanic, strength of materials relations, and standard sizes) → (Table 13-2)
- Calculate addendum and dedendum (Table 13-1)

addendum ⇒ $a = 1 \cdot m$ dedendum ⇒ $b = (1.2 \sim 1.25) \cdot m$

- Calculate the pitch diameter, d; and the diameters of addendum and dedendum circles, d_a and d_b

$$d = N \cdot m \qquad d_a = d + 2 \cdot a = m\,(N+2)$$
$$d_b = d - 2.5\,b = m\,(N - 2.5)$$

- Distance between the axes,

$$\frac{d_P + d_G}{2} = m\,\frac{N_P + N_G}{2}$$

d_P → pitch diameter of pinion; d_G → pitch diameter of gear

Choose $N_P \cong 15 \sim 25$

N_P should be greater than 13

(Table 13-6)

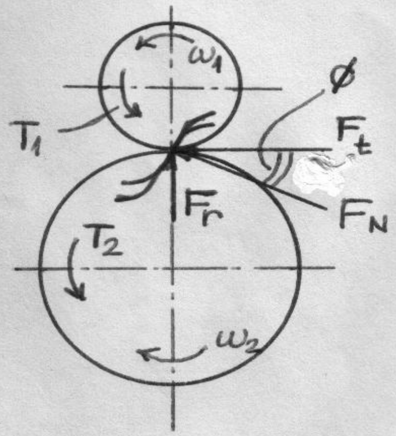
- Calculate the face width, F
  for this choose a width ratio (the width ratio depends on the material) $\Rightarrow$ $2 \sim 14$

  $$F = F_W \cdot p = F_W \, \pi . m$$

  F_W → width ratio

- To produce a constant angular-velocity ratio during meshing (conjugate action), involute profile is used for gear teeth.

- Force analysis $\quad T_1 = 9550 \dfrac{H}{n}$



$$T_1 = F_t \cdot \frac{d_1}{2}$$

$$T_2 = F_t \cdot \frac{d_2}{2}$$

$$F_t = F_N \cdot \cos\phi$$

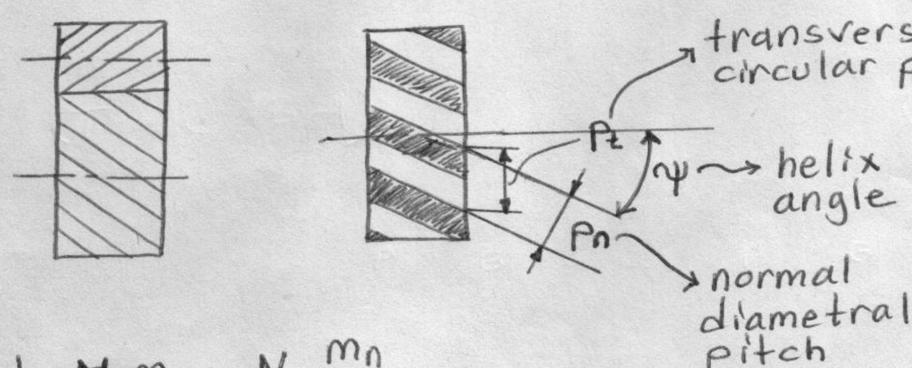
$$F_r = F_N \cdot \sin\phi$$

$$F_t = W_t \qquad F_r = W_r \qquad F_N = W$$

for appropriate ϕ Look at Table 13-1

$\phi \cong 20°$ (for spur gears)

Design of helical gears



$$P_n = P_t \cdot \cos\psi$$

$$\frac{P_n}{\pi} = m_n \rightarrow \text{normal module}$$

$$P_t = \frac{P_n}{\cos\psi}$$

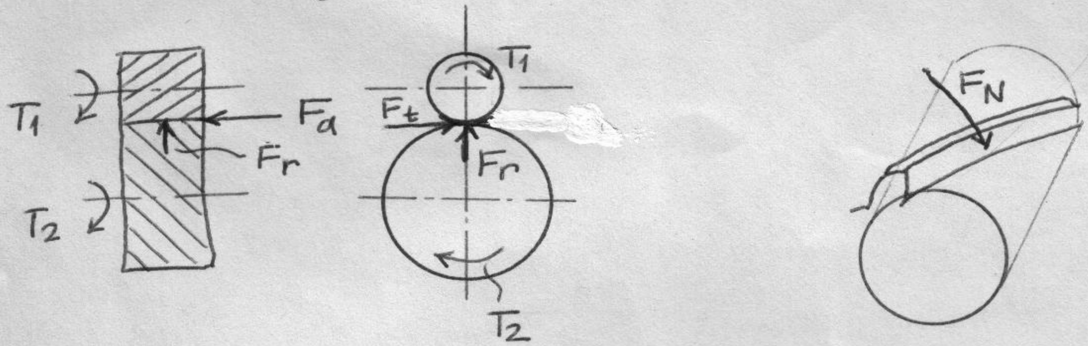
$$d = N . m_a = N \frac{m_n}{\cos\psi}$$

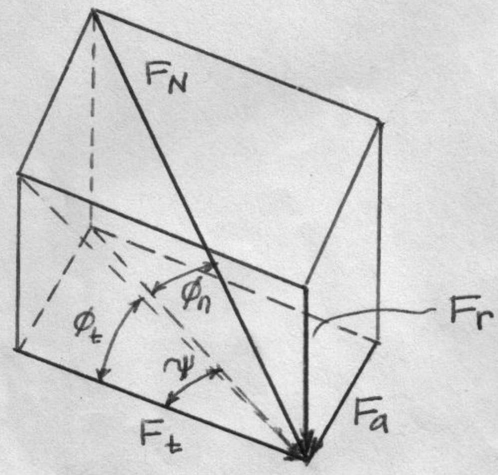
$$d_a = d + 2m_n = m_n \cdot \left(\frac{N}{\cos\psi} + 2\right) \qquad d_b = d - 2.5\,m_n = m_n\left(\frac{N}{\cos\psi} - 2.5\right)$$

distance between the axes, (center distance) $\quad \dfrac{d_P + d_G}{2} = m_n \cdot \left(\dfrac{N_P + N_G}{2 . \cos\psi}\right)$

$N' = \dfrac{N}{\cos^3\psi}$ → virtual number of teeth (eşdeğer diş sayısı)

- Force analysis





$$F_r = F_N \cdot \sin\phi_n$$
$$F_t = F_N \cdot \cos\phi_n \cdot \cos\psi$$
$$F_a = F_N \cdot \cos\phi_n \cdot \sin\psi$$

$F_r = W_r \qquad F_N = W$

$F_t = W_t$

$F_a = W_a$

Choosing module (m) according to failure criteria

- Pitting is a surface fatigue failure due to many repetitions of high contact stresses

$$m_n = 0.9 \sqrt[3]{\frac{K_A \cdot K_v \cdot T_1 \cdot E \cdot (i+1)\cos^4\psi}{N_1^2 \cdot P_{em}^2 \cdot i \cdot \varepsilon_p \cdot F_w}} \quad (mm)$$

- Fracture of a tooth

$$m_n = 0.6 \cdot \sqrt[3]{\frac{K_A \cdot K_v \cdot T_1 \cdot \gamma \cdot \cos\psi}{N_1 \cdot \sigma_{em} \cdot \varepsilon_\alpha \cdot F_w}} \quad (mm)$$

ε_α : profile ratio

K_A : Impact or contact load factor

K_v : Dynamic load factor

T_1 : Torque on pinion (N.mm)

N_1 : Number of teeth of pinion

γ : Form factor $\qquad i = \frac{\omega_1}{\omega_2} = \frac{N_2}{N_1}$

E : Young's modulus (N/mm^2)

σ_{em} : Tensile (bending) strength (N/mm^2)

P_{em} : Hertz pressure (N/mm^2) (to resist pitting) conjugate action: produce a constant angular-velocity ratio during meshing

involute profile (evolvent eğrisi)

$V = |r_1 \omega_1| = |r_2 \omega_2|$

contact ratio (profil kavrama oranı), $m_c = \frac{q_t}{p} = \frac{L_{ab}}{p \cdot \cos\phi}$

interference (alttan kesme): the contact of portions of tooth profiles which are not conjugate. To avoid interference use the tooth numbers given in Table 13-6.

The forming of gear teeth

Sand casting, shell molding, investment casting, permanent-mold casting, die casting, centrifugal shell casting

Powder-metallurgy process, extrusion (aluminum)

Form cutters, generating cutters (steel, large load carrying gears)

Cold rolling, cold forming (high-quality generated profile)

Teeth may be machined by milling, shaping, hobbing; they may be finished by sharing, burnishing, grinding, lapping.

Straight Bevel Gears (Konik Dişli Çark)

$\tan\gamma = \frac{N_P}{N_G}$ $\quad \tan\Gamma = \frac{N_G}{N_P}$ $\quad \gamma, \Gamma$: pitch angle of pinion and gear, respectively.

Worm Gears (Sonsuz Vida)

$d_G = \frac{N_G \cdot p_t}{\pi}$ $\qquad L = p_x \cdot N_W$

$\frac{C^{0.875}}{3} \le d_W \le \frac{C^{0.875}}{1.7}$ $\qquad \tan\lambda = \frac{L}{\pi \cdot d_W}$

C: the center distance

Gear Trains

$$n_3 = \left| \frac{N_2}{N_3} n_2 \right| = \left| \frac{d_2}{d_3} n_2 \right|$$

pinion

gear

$$n_6 = -\frac{N_2}{N_3}\frac{N_3}{N_4}\frac{N_5}{N_6} n_2$$

$\Rightarrow$ gear 3 is an idler

2,3,5 drivers

3,4,6 driven members

$$e = \frac{\text{product of driving tooth numbers}}{\text{product of driven tooth numbers}}$$

$n_L = e \cdot n_F$ → speed of first gear

→ speed of last gear

Planetary, or epicyclic, gear trains: Some of the gear axes rotate about others.

$$e = \frac{n_L - n_A}{n_F - n_A}$$

→ r/min of last gear in planetary train

→ r/min of arm

→ r/min of first gear in planetary train

Force Analysis - Bevel Gearing

$W_t = \dfrac{T}{r_{av}}$ $\quad W_r = W_t \cdot \tan\phi \cdot \cos\gamma \quad W_a = W_t \cdot \tan\phi \cdot \sin\gamma$

Force Analysis - Worm Gearing

$W^x = W \cdot \cos\phi_n \cdot \sin\lambda \qquad W_{Wt} = -W_{Ga} = W^x$

$W^y = W \cdot \sin\phi_n \qquad W_{Wr} = -W_{Gr} = W^y$

$W^z = W \cdot \cos\phi_n \cdot \cos\lambda \qquad W_{Wa} = -W_{Gt} = W^z$

$$\eta = \frac{\cos\phi_n - f \cdot \tan\lambda}{\cos\phi_n + f \cdot \cot\lambda}$$

→ efficiency