Steady Loading

- Strength is a property or characteristic of a material or of a mechanical element.
- A static load is a stationary force or moment acting on a member.
- Examine the relations between strength of a part and its anticipated static loading to sdect the optimum material and dimensions
- Two approaches,
  - The deterministic, or factor-of-safety, approach. The maximum stress or stresses in a part are kept below the minimum strength by a suitable design factor or margin of safety, to ensure that the part will not fail.
  - The stochastic, or reliability, approach. This method involves the selection of materials, processing, and dimensions such that the probability of failure is always less than a preselected value.

Static strength

- We usually need test data before manufacturing. Tests should be in the same conditions with those of service. Bending, bending and torsion, tension, heat, etc.
- Test increases cost. If failure of the part would endanger human life, or the part is made in extremely large quantities, an elaborate testing program is justified during design.

Stress concentration

- No need to use K_f for ductile materials.
- If the material is brittle, then K_t ("full value" of K_f) should be used in computing the stress.

Failure theories

- The maximum-normal-stress theory
  - Failure occurs whenever one of the three principal stresses equals the strength.

$$\sigma_1 = S_t \quad \text{or} \quad \sigma_3 = -S_c\,, \quad \sigma_1 > \sigma_2 > \sigma_3$$

- The maximum-normal-strain theory (Saint Venant's theory)
  - Failure occurs when the largest of the three principal strains becomes equal to the strain corresponding to the yield strength.

$$\sigma_1 - \nu(\sigma_2 + \sigma_3) = \pm S_y \qquad \sigma_2 - \nu(\sigma_3 + \sigma_1) = \pm S_y$$

$$\sigma_3 - \nu(\sigma_1 + \sigma_2) = \pm S_y$$

Ductile Materials

Maximum Shear Stress (Tresca or Guest) Hypothesis

Yielding begins whenever the maximum shear stress in any element becomes equal to the maximum shear stress in a tension-test specimen of the same material when that specimen begins to yield.

$\sigma_1 > \sigma_2 > \sigma_3 \qquad \tau_{max} \geq \frac{S_y}{2}$ or $\sigma_1 - \sigma_3 \geq S_y$

$S_{sy} = 0.50\, S_y$

$\sigma_1 = \sigma_1' + \sigma_1'' \qquad \sigma_1'' = \sigma_2'' = \sigma_3'' \Rightarrow$ hydrostatic components

$\sigma_2 = \sigma_2' + \sigma_2'' \qquad = \sigma_{av}$

$\sigma_3 = \sigma_3' + \sigma_3''$

If $\sigma_1' = \sigma_2' = \sigma_3' = 0$, then

$$\left.\begin{aligned} \tau_{1/2} &= \frac{\sigma_1 - \sigma_2}{2} = 0 \\ \tau_{2/3} &= \frac{\sigma_2 - \sigma_3}{2} = 0 \\ \tau_{1/3} &= \frac{\sigma_1 - \sigma_3}{2} = 0 \end{aligned}\right\} \text{no yielding}$$

Strain-Energy Hypotheses

Failure by yielding occurs when the total strain energy in a unit volume reaches or exceeds the strain energy in the same volume corresponding to the yield strength in tension or in compression.

$$u_s = \frac{S_y^2}{2E} \qquad u_\sigma = \frac{1}{2E}\left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\right]$$

$$\sigma_{av} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

$$u_v = \frac{1-2\nu}{6E}\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + 2\sigma_1\sigma_2 + 2\sigma_2\sigma_3 + 2\sigma_3\sigma_1\right)$$

$$u_d = u_\sigma - u_v = \frac{1+\nu}{3E}\left[\frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2}{2}\right]$$

$$u_d = \frac{1+\nu}{3E} \cdot \sigma'^2$$ (for one dimensional case)

$$\sigma' = \left[\frac{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2}{2}\right]^{1/2} \geq S_y \Rightarrow \text{yielding}$$

The distorsion energy hypothesis is also called

- The shear energy hypothesis
- The von Mises–Hencky hypothesis
- The octahedral-shear-stress hypothesis

$$\sigma' = \frac{1}{\sqrt{2}}\left[(\sigma_x-\sigma_y)^2 + (\sigma_y-\sigma_z)^2 + (\sigma_z-\sigma_x)^2 + 6\,(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2\right]^{1/2}$$

Internal-Friction Hypothesis

Tension, compression and shear tests.

$$\sigma_1 - \sigma_3 = S_y$$

$$\tau_{max}^2 = a\cdot\sigma_n^2 + b$$

$$S_{sy} = \frac{S_{yt}\cdot S_{yc}}{S_{yt} + S_{yc}}$$

$$\frac{\sigma_1}{S_t} - \frac{\sigma_3}{S_c} = 1 \qquad \sigma_1 \geq 0 \quad \sigma_3 \leq 0$$

Criticism of Hypotheses

The distortion-energy hypothesis is the best available.

For ductile materials with unequal yield strengths, S_{yt} in tension and S_{yc} in compression, the Mohr hypothesis is the best available.

Interference-General

$$R = 1 - \int_0^1 R_2\, dR_1$$

R: Relability

R_1: Reliability of stress distribution

R_2: Reliability of strength distribution

- Coulomb – Mohr

$\sigma_A = \frac{S_{ut}}{n} \quad \sigma_A \geq 0, \ \sigma_B \geq 0$

$\frac{\sigma_A}{S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{1}{n} \quad \sigma_A \geq 0, \ \sigma_B < 0$

$n=1$ (to predict failure)
↳ factor of safety

- Modified – Mohr

$\sigma_A = \frac{S_{ut}}{n} \quad \sigma_A \geq 0, \ \sigma_B \geq 0$

$\sigma_A = \frac{S_{ut}}{n} \quad \sigma_B \geq -S_{ut}$

$n=1$

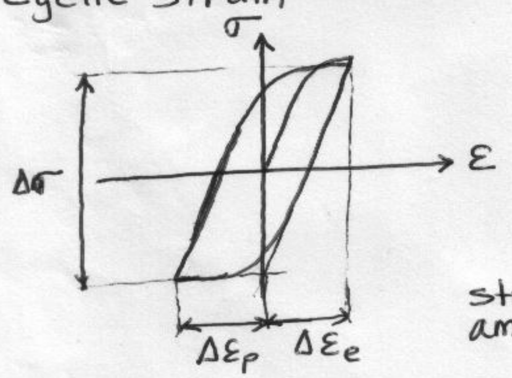
$\sigma_A - \frac{S_{ut}\sigma_B}{S_{uc}-S_{ut}} = \frac{S_{uc}S_{ut}}{n(S_{uc}-S_{ut})} \quad \sigma_B < -S_{ut} \quad \sigma_A \geq 0 \ \sigma_B < 0$

Variable Loading

- A rotating shaft, wings, fuselages, etc.
- The level of stress is fluctuating. These stresses also are called repeated, alternating, or fluctuating stresses.
- Failure after the stresses have been repeated a very large number of times. Therefore it is called a fatigue failure.
- A fatigue failure begins with a small crack?
- A fatigue failure doesn't give warning!

The strain-life theory of fatigue failure

- Cyclic strain



$\Delta\varepsilon = \Delta\varepsilon_e + \Delta\varepsilon_p$

$\frac{\Delta\varepsilon}{2} = \underbrace{\frac{\sigma_F'}{E}(2N)^b + \varepsilon_F'(2N)^c}_{\text{Manson-Coffin relationship}}$

$\frac{\Delta\varepsilon}{2}$ → strain amplitude

σ_F' : fatigue strength coefficient

ε_F' : fatigue ductility coefficient

for the coefficients and exponents
Table 7-1, page 274

Strain life fatigue failure model (continued)

Low-cycle life $\Rightarrow$ $\frac{\Delta \varepsilon_p}{2} = \varepsilon_F' (2N_f)^c$

High-cycle life $\Rightarrow \sigma_a = \sigma_F' (2N_f)^b$

Transition life N_T between low-cycle and high cycle domains $\Rightarrow$ $N_T = \frac{1}{2}\left(\frac{\sigma_F'}{E \cdot \varepsilon_F'}\right)^{1/(c-b)}$

General $\Rightarrow \sigma_a = S_a = \sigma_F'(2N_f)^b + E\,\varepsilon_F'\,(2N_f)^c$

$$\frac{\Delta \varepsilon}{2} = \frac{\Delta \varepsilon_e}{2} + \frac{\Delta \varepsilon_p}{2} = \left(\frac{\sigma_F'}{E}\right)(2N_f)^b + \varepsilon_F'(2N_f)^c$$

$\sigma_F', \varepsilon_F'$: True stress and true strain

S : Nominal engineering strength

In the absence of $\sigma_F', \varepsilon_F', b,$ and c, Collin suggests

$\sigma_F' = \sigma_f$ (true stress at fracture)

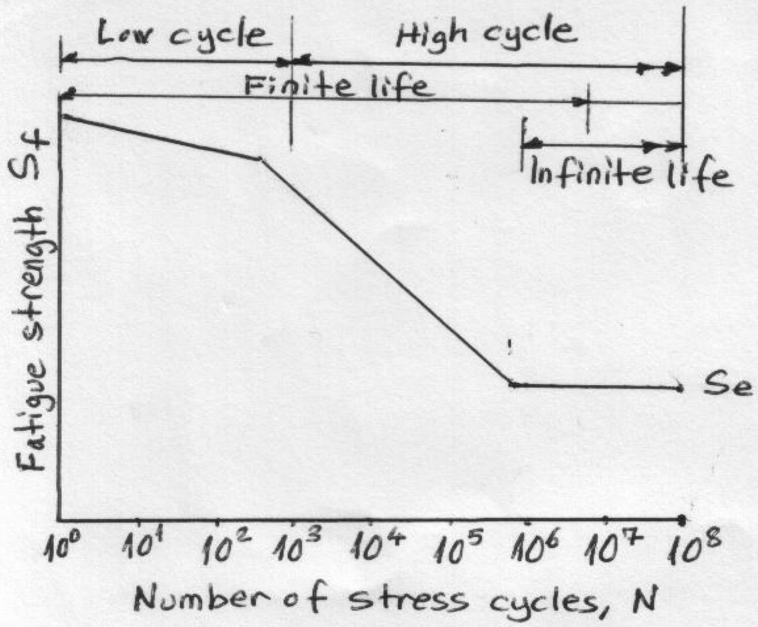
$\varepsilon_F' = \varepsilon_f$ (true fracture ductility)

$c = -0.6$

$$b = -\frac{\log(2\sigma_f/S_u)}{\log(2N_e)} = -0.16 \log(2\sigma_f S_u) = -0.16 \log \cdot \left[2m^{-m} \exp(m)\right]$$

Stress-Life definitions

- A number of tests are necessary to establish the fatigue strength of a material.
- S-N diagram



One cycle means a single application and removal of a load and then another application and removal of the load in the opposite direction.

$N \leq 1000 \Rightarrow$ Low-cycle fatigue

$N > 1000 \Rightarrow$ High-cycle fatigue

S_e : Endurance limit

The endurance Limit

- 40% ~ 60% of the tensile strength for steels.
- The dispersion is not because of a dispersion in the tensile strength!
- Aluminum alloys don't have endurance limits.

The fatigue strength

- $S_f = aN^b$ (S-N Line)
- In low cycle fatigue

$$(S_f)_{10^3} = a(10^3)^b = a(10)^{3b} = f S_{ut}$$

$$f = \frac{a}{S_{ut}}(10)^{3b}$$

- In high cycle fatigue, the strain is predominantly elastic

$$f = \frac{a}{S_{ut}}(10)^{3b} = \frac{2^b \sigma_f'}{S_{ut}}\left(\frac{\sigma_f'}{S_e}\right)$$

σ_f' → fatigue strength coefficient

- $\log S_f = \log a + b \log N$

$$a = \frac{(0.9 S_{ut})^2}{S_e} \qquad b = -\frac{1}{3}\log\frac{0.9 S_{ut}}{S_e}$$

Endurance Limit modifying factors

$$S_e = k_a k_b k_c k_d k_e S'_e$$

S'_e: endurance limit of test specimen

S_e: endurance limit of mechanical element
k_a: surface factor
k_b: size factor
k_c: load factor
k_d: temperature factor
k_e: miscellaneous effects factor

- The surface of the test specimen is highly polished

  $k_a = a S_{ut}^{b}$

- k_b for bending and torsion

  $k_b = \left(\frac{d}{0.3}\right)^{-0.1133}$ in $\quad 0.11 \leq d \leq 2$ in

  $k_b = \left(\frac{d}{7.62}\right)^{-0.1133}$ mm $\quad 2.79 \leq d \leq 51$ mm

  for larger sizes k_b varies from 0.60 to 0.75

  k_b for tension

  $k_b = 1$ (no size effect for tension)

- k_c load factor

0.923	axial loading	$S_{ut} \leq 220$ kpsi (1520 MPa)
1	" "	$S_{ut} > 220$ " (" ")
1	bending	
0.577	torsion and shear	

- k_d temperature factor
  - Below room temperature, brittle fracture is a strong possibility
  - For higher temperatures, yield strength drops off rapidly

  $k_d = \frac{S_T}{S_{RT}}$

  S_T: tensile strength at operating temperature
  S_{RT}: tensile strength at room temperature

- k_e miscellaneous-effects factor
  - Compressive residual stresses increase the endurance limit.
  - Tensile residual stresses decrease the endrance limit.

- Corrosion has negative effect on the fatigue resistance.
- Chromium, nickel, cadmium plating reduces the endurance Limit.
- Metal spraying reduces the endurance Limit.
- Cyclic frequency (with low frequency, high temperature) reduces the life.
- Microscopic motion of tightly fitting parts or structures (bolted joints, bearing-race fits, wheel hubs) reduces the life.
- Stress concentration

$$k_e = \frac{1}{K_f}$$

Fluctuating stresses

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} \qquad \sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

σ_m → midrange stress

σ_a → stress amplitude

$$R = \frac{\sigma_{min}}{\sigma_{max}} \qquad A = \frac{\sigma_a}{\sigma_m}$$

→ stress ratio

Fatigue strength under fluctuating stresses

- Soderberg equation

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{yt}} = \frac{1}{n}$$

S_{yt} → yield strength

- Modified Goodman relation

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}$$

- Gerber equation

$$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1$$

Cumulative fatigue damage

σ_1 for n_1 cycles
σ_2 for n_2 cycles
$\vdots$

Miner's rule

$$\frac{n_1}{N_1}+\frac{n_2}{N_2}+\cdots+\frac{n_i}{N_i}=C \qquad 0.7 \leq C \leq 2.2$$

N_1 → life corresponding to σ_1

The fracture-mechanics approach

- The growth of a fatigue crack

$$\frac{da}{dN}=\frac{C(\Delta K-\Delta K_{th})^m}{(1-R)K_c-\Delta K} \qquad \Delta K=\underbrace{\sigma_r}_{\text{stress range}}(\pi a)^{1/2}\left(\frac{K_I}{K_o}\right)$$

C, m : empirical constants (in tables)
ΔK_{th} : threshold value

The Design of Screws, Fasteners, and Connections

Threads Standards and Definitions

- The terminology of screw threads
  - The pitch is the distance between adjacent thread forms measured parallel to the thread axis.
  - The major diameter d is the largest diameter of a screw thread.
  - The minor diameter d_r or d_1 is the smallest diameter of a screw thread.
  - The lead l is the distance the nut moves parallel to the screw axis when the nut is given one turn.
- The thread size is specified
  - by giving the pitch p for metric sizes

nominal major diameter d	pitch p	tensile stress area A_t	minor diameter area A_r
12	1.75	84.3	76.3

(Table 8-1, page 327)

M12×1.75

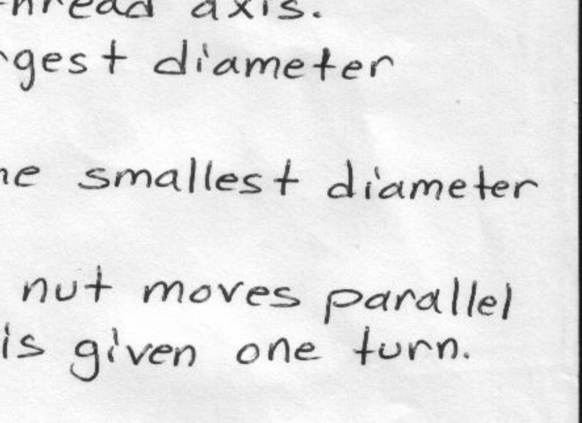


FIGURE 8-1
Terminology of screw threads. Sharp vee threads shown for clarity; the crests and roots are actually flattened or rounded during the forming operation.

The endurance limit (continued)

$\phi = S'_e/\bar{S}_{ut}$ (fatigue ratio)

$$S'_e = \begin{cases} 0.506\,\bar{S}_{ut}\,\mathbf{LN}(1, 0.138) \text{ kpsi or MPa} & \bar{S}_{ut} \le 212 \text{ kpsi } (1460 \text{ MPa}) \\ 107\,\mathbf{LN}(1, 0.139) \text{ kpsi} & \bar{S}_{ut} > 212 \text{ kpsi} \\ 740\,\mathbf{LN}(1, 0.139) \text{ MPa} & \bar{S}_{ut} > 1460 \text{ MPa} \end{cases}$$

$\bar{S}_{ut}$: mean ultimate tensile strength

Table E-24, E-24b (endurance limit and fatigue strength information of some classes of cast irons and aluminum alloys)

Table 7-13, 7-14 (SI) endurance limits and modifying factors

Table 7-15, 7-16 DE-Gerber and Langer failure loci and DE-Elliptic and Langer failure loci

Combinations of Loading Modes (follow procedure given ~~#~~ in this section and ~~~~ study example 7-14)

Stochastic Failure Loci Under Fluctuating Stresses

The Fracture-Mechanics Approach (continued)

$$\frac{da}{dN} = C(\Delta K_I)^m \quad \text{(simplified)}$$

$$\int_{a_0}^{a_f} da = \int_0^{N_f} C(\Delta K_I)^m\, dN$$

The Design Factor in Fatigue

$$\bar{n} = \exp\left[-z\sqrt{\ln(1+C_n^2)} + \ln\sqrt{1+C_n^2}\right] \qquad C_n = \sqrt{\frac{C_S^2 + C_\sigma^2}{1 + C_\sigma^2}}$$

$C_s \to C_{se}$ (fully reversed)

$C_s \to C_{sa}$ (otherwise)

$C_{se} = (C_{ka}^2 + C_{kc}^2 + C_{kd}^2 + C_{ke}^2 + C_\phi^2) \to C_{se'}^2$