Deflection and Stiffness

no bending, deflection or twist $\Rightarrow$ rigid

large movement $\Rightarrow$ flexible

Spring rates

Spring: mechanical element which exerts a force when deformed

Figure 3-1 a: Simply supported beam

Figure 3-1 b: Beam supported on two cylinders

Figure 3-1 c: Dish-shaped round disk

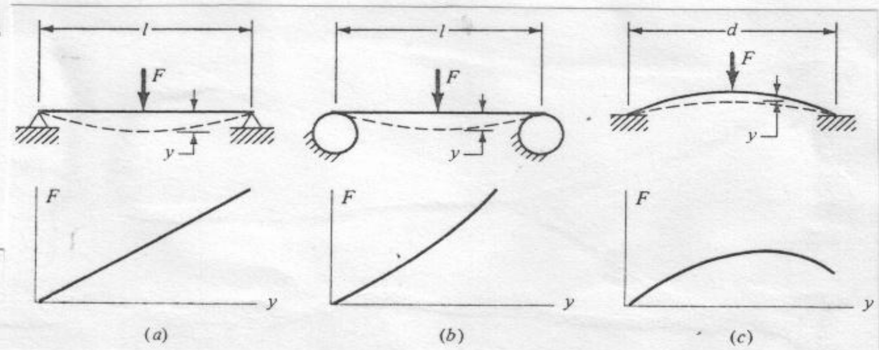


FIGURE 3-1

(*a*) A linear spring; (*b*) a stiffening spring; (*c*) a softening spring.

$$k(y) = \lim_{\Delta y \to 0} \frac{\Delta F}{\Delta y} = \frac{dF}{dy}$$

for the linear spring $k = F/y$ → spring constant

Tension, Compression, and Torsion

for tension $\delta = \dfrac{F.L}{A.E}$

incase of compression there is a possibility of buckling

spring constant of an axially loaded bar $k = \dfrac{AE}{L}$

angular deflection $\theta = \dfrac{TL}{GJ}$ (in radians)

torsional spring rate $k = \dfrac{T}{\theta} = \dfrac{GJ}{L}$

Flexure

$$\frac{1}{\rho} = \frac{M}{EI} \quad \text{and} \quad \frac{1}{\rho} = \frac{d^2y/dx^2}{[1+(\underbrace{dy/dx}_{\theta:\text{slope}})^2]^{3/2}}$$

$$\frac{M}{EI} = \frac{d^2y}{dx^2} \qquad \frac{V}{EI} = \frac{d^3y}{dx^3} \qquad \frac{q}{EI} = \frac{d^4y}{dx^4}$$

The area-moment method (Mohr method)

$$\underbrace{\frac{d^2y}{dx^2} = \frac{M}{EI} \qquad \frac{d^2M}{dx^2} = q}_{\text{similarity between two equations}}$$

similarities

y	M
$\theta = \dfrac{dy}{dx}$	$V = \dfrac{dM}{dx}$
$\dfrac{M}{EI} = \dfrac{d^2y}{dx^2}$	$q = \dfrac{d^2M}{dx^2}$

Finding deflections by numerical integration

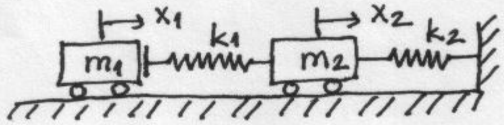
$$\phi_1 = \int_{x_0}^{x_1} f(x)\,dx = \frac{y_0+y_1}{2}(x_1-x_0)+\phi_0$$

$$\phi_2 = \int_{x_0}^{x_2} f(x)\,dx = \frac{y_1+y_2}{2}(x_2-x_1)+\phi_1$$

$$\phi_i = \frac{y_{i-1}+y_i}{2}(x_i-x_{i-1})+\phi_{i-1}$$

Shock and impact

- Impact: collision of two masses with initial relative velocity
- design of coining, stamping, and forming includes impact analysis
- in some cases impact occurs because of excessive deflections, clearances between parts
- shock is suddenly applied force or disturbance
- impact model



$$m_1\ddot{x}_1 + k_1(x_1-x_2) = 0$$
$$m_2\ddot{x}_2 + k_2x_2 - k_1(x_1-x_2) = 0$$

Strain energy

- external work is transformed into strain or potential energy

  $U = \frac{F}{2}y = \frac{F^2}{2k}$

- tension and compression $\quad k = \frac{AE}{l} \Rightarrow U = \frac{F^2 l}{2AE}$
- torsion $\quad U = \frac{T^2 l}{2GJ}$
- direct shear $\quad U = \frac{F^2 l}{2AG}$
- bending $\quad U = \int \frac{M^2 dx}{2EI}$
- strain energy stored in a unit volume

$u = \frac{F^2 l}{2AE} / lA = \frac{\sigma^2}{2E}$ tension and compression

$u = \frac{\tau^2}{2G}$ direct shear

$u = \frac{\tau_{max}^2}{4G}$ torsion

a high stress in a material with a low modulus of elasticity, or rigidity,

$\Downarrow$

greatest amount of energy storage

- better result for shear loading (shear in bending)

$$U = \int \frac{C V^2 dx}{2AG}$$

$C \rightarrow$ correction factor

Rectangular	1.50
Circular	1.33
Tubular, round	2.00
Box sections	1.00

Castigliano's theorem

- when forces act on elastic systems subject to small displacements, the displacement corresponding to any force is equal to the partial derivative of the total strain energy with respect to that force

$$\delta_i = \frac{\partial U}{\partial F_i} \qquad \text{for example,} \quad \delta = \frac{\partial}{\partial F}\left(\frac{F^2 L}{2AE}\right) = \frac{FL}{AE}$$

Statically indeterminate problems

- if the laws of statics are not sufficient to determine all the unknown forces and moments, this is called statically indeterminate problem
- for example, the nested helical spring

$$\Sigma F = F - F_1 - F_2 = 0$$

two unknowns
one equation

let's use the deformation relation

$$\delta_1 = \delta_2 = \delta \Rightarrow \frac{F_1}{k_1} = \frac{F_2}{k_2}$$

Deflection of curved members

- machine frames, springs, clips, fasteners
- Castigliano's theorem is useful for the analysis of curved members

Compression members

- Under compression, if the failure is because of the bending, it is called column.
- They can be categorized as
    1. Long columns with central loading
    2. Intermediate-length columns with central loading
    3. Columns with eccentric loading
    4. Struts or short columns with eccentric loading

Long columns with central loading

$$M = -Py \qquad \frac{d^2y}{dx^2} = -\frac{P}{EI}y \Rightarrow \underbrace{\frac{d^2y}{dx^2} + \frac{P}{EI}y = 0}$$

$$P_{cr} = \frac{\pi^2 EI}{l^2} \quad \text{or} \quad \frac{P_{cr}}{A} = \frac{\pi^2 E}{(l/k)^2}$$

differential equation for simple harmonic motion

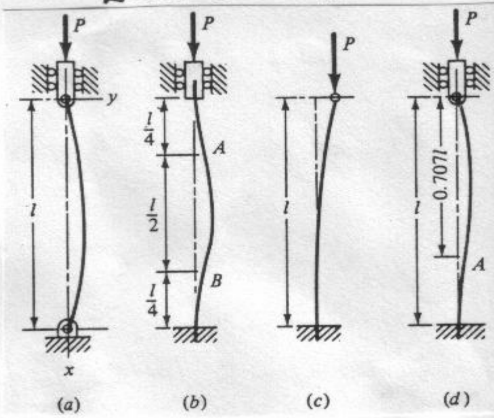


FIGURE 3-20
(a) Both ends rounded or pivoted; (b) both ends fixed; (c) one end free, one end fixed; (d) one end rounded and guided and one end fixed.

- for different end conditions

end-condition constant

$$P_{cr} = \frac{C\pi^2 EI}{l^2} \quad \text{or} \quad \frac{P_{cr}}{A} = \frac{C\pi^2 E}{(L/k)^2}$$

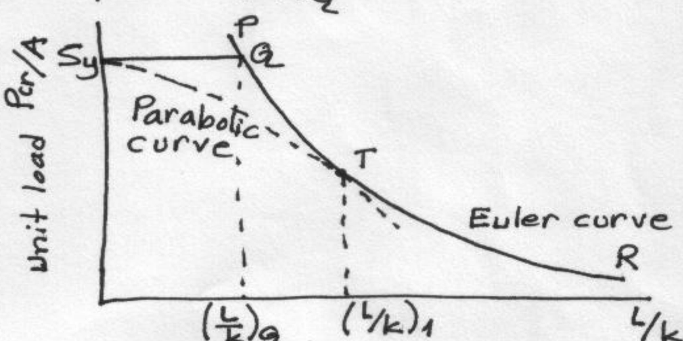
	C (Theoretical)	C (Recommended)	C (Conservative)
Fixed-free	1/4	1/4	1/4
Rounded-rounded	1	1	1
Fixed-rounded	2	1.2	1
Fixed-fixed	4	1.2	1

- a column failure is sudden, total, and unexpected
- neither simple compression nor Euler for $\sim (L/k)_Q$

Euler for $(L/k) > (L/k)_1$

Other methods for $(L/k) \leq (L/k)_1$

many time assume $T \Leftarrow \frac{P_{cr}}{A} = \frac{S_y}{2}$



Intermediate-length columns with central loading

- The parabolic or J. B. Johnson formula for machine, automotive, aircraft, and structural-steel construction fields

$$\frac{P_{cr}}{A} = S_y - \left(\frac{S_y}{2\pi}\frac{L}{k}\right)^2 \frac{1}{CE} \qquad \frac{L}{k} \leq \left(\frac{L}{k}\right)_1$$

Columns with eccentric loading

- deviations from an ideal column (load eccentricities, crookedness)

$$\Sigma M_0 = M + Pe + Py = 0$$

$$\sigma_c = \frac{P}{A}\left[1 + \frac{ec}{k^2}\sec\left(\frac{L}{2k}\sqrt{\frac{P}{EA}}\right)\right]$$

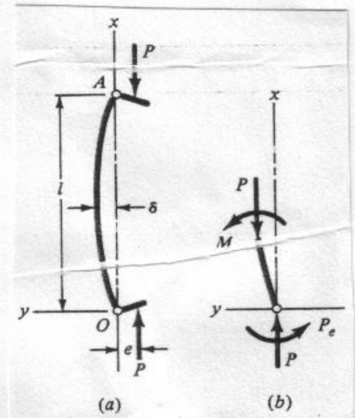


FIGURE 3-22
Notation for an eccentric column.

Struts, or short compression members

- a strut is a short compression member

$$\sigma_c = \frac{P}{A} + \frac{My}{I} = \frac{P}{A} + \frac{PeyA}{IA} = \frac{P}{A}\left(1 + \frac{ey}{k^2}\right)$$

- Limiting slenderness ratio

$$\left(\frac{L}{k}\right)_2 = 0.282 \left(\frac{AE}{P_{cr}}\right)^{1/2}$$

$$\frac{L}{k} \leq \left(\frac{L}{k}\right)_2$$

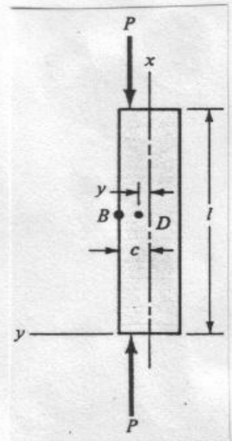


FIGURE 3-24

Statistical Considerations

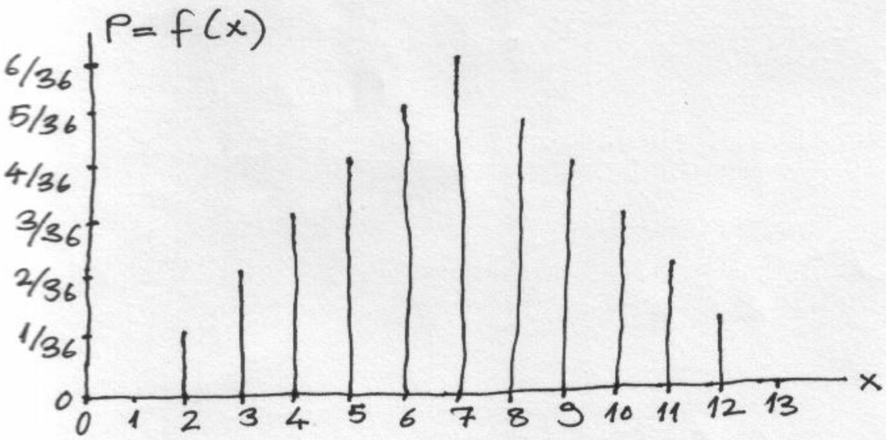
- Lifes of products such as automobiles, watches, lawnmowers, washing machines are variable

Random variables

- two dice experiment

x	2	3	4	5	6	7	8	9	10	11	12
f(x)	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

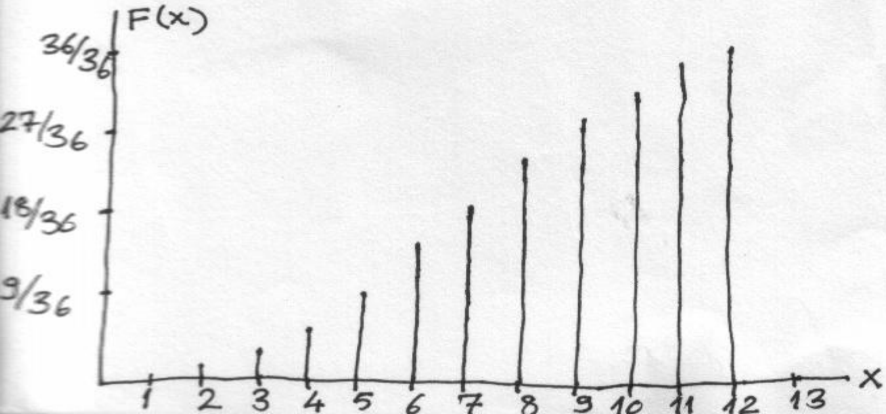
← probability distribution



← probability function (frequency function)

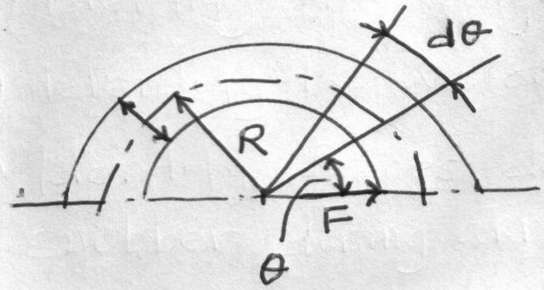
x	2	3	4	5	6	7	8	9	10	11	12
F(x)	1/36	3/36	6/36	10/36	15/36	21/36	26/36	30/36	33/36	35/36	36/36

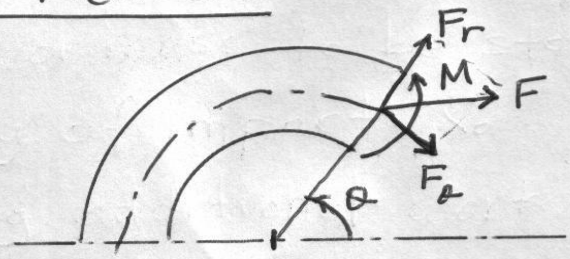
← cumulative probability distribution



$$F(x_i) = \sum_{x_j \leq x_i} f(x_j)$$

Deflection of Curved Members





Bending $U_1 = \int \frac{M^2 d\theta}{2AeE}$ $e = R - r_n$ → radius of the neutral axis

For $\frac{R}{h} > 10 \Rightarrow U_1 \approx \int \frac{M^2 R\, d\theta}{2EI}$

Tension (Normal force) $U_2 = \int \frac{F_\theta^2 R\, d\theta}{2AE}$

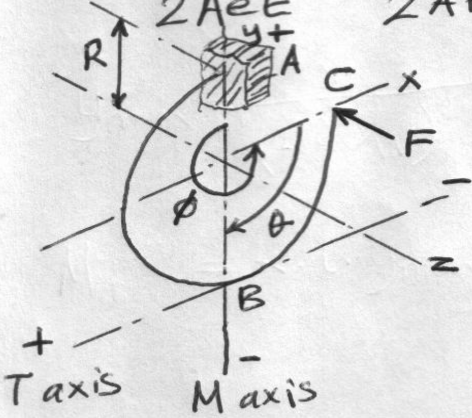
Bending produced by F_θ $U_3 = -\int \frac{M F_\theta\, d\theta}{AE}$

Shear $U_4 = \int \frac{C \cdot F_r^2 R\, d\theta}{2AG}$

$$U = \int \frac{M^2 d\theta}{2AeE} + \int \frac{F_\theta^2 R\, d\theta}{2AE} - \int \frac{M F_\theta\, d\theta}{AE} + \int \frac{C F_r^2 R\, d\theta}{2AG}$$

$\delta = \frac{\partial U}{\partial F}$ $M = FR\sin\theta$ $F_\theta = F\sin\theta$ $F_r = F\cdot\cos\theta$

$$\delta = \frac{\pi F R^2}{2AeE} - \frac{\pi F R}{2AE} + \frac{\pi C F R}{2AG}$$



$$U = \int_0^\phi \frac{M^2 R\, d\theta}{2EI} + \int_0^\phi \frac{T^2 R\, d\theta}{2GJ}$$

$M = FR\sin\theta$ $T = FR(1-\cos\theta)$

$$\delta = \frac{\partial U}{\partial F} = \frac{FR^3}{2}\left(\frac{\alpha}{EI} + \frac{\beta}{GJ}\right)$$

$\alpha = \phi - \sin\phi\cos\phi$

$\beta = 3\phi - 4\sin\phi + \sin\phi\cos\phi$

Deflection of Energy-Dissipative Assemblies

Several types of viscously damped hydraulic cylinders are used to absorb energy in stopping a moving body.

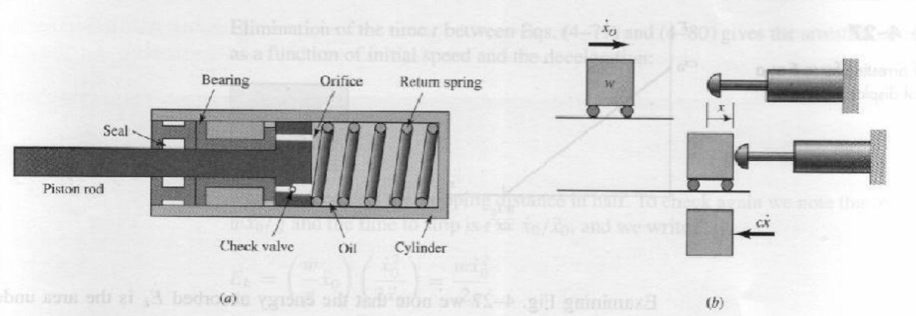


Figure 4–26

Schematic of a single-orifice snubber.

$$\sum F = m\ddot{x} = (W/g)\cdot\ddot{x} = -c\cdot\dot{x}$$

$$\frac{W}{g}\cdot\ddot{x} + c\dot{x} = 0 \qquad \text{with} \qquad \dot{x}\big|_{t=0} = \dot{x}_0 \qquad x\big|_{t=0} = 0$$

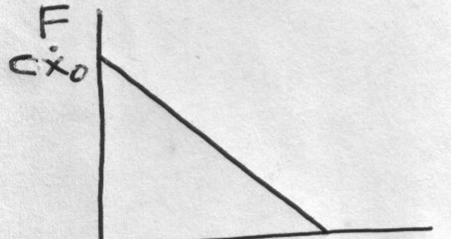
$$x = \frac{W\cdot\dot{x}_0}{c\cdot g}\left[1 - \exp\left(-\frac{cgt}{W}\right)\right]$$

$$\dot{x} = \frac{dx}{dt} = \dot{x}_0 \exp\left(-\frac{cgt}{W}\right)$$

$$F = c\cdot\dot{x} = c\cdot\dot{x}_0 \exp\left(-\frac{cgt}{W}\right)$$

$$F = c\dot{x}_0 - \frac{c^2 g x}{W}$$

The distance to stop $\quad x_s = \dfrac{W\cdot\dot{x}_0}{c\cdot g} \quad (t\to\infty)$



$$E_k = \frac{1}{2}c\dot{x}_0\frac{W\dot{x}_0}{cg} = \frac{1}{2}\frac{W\dot{x}_0^2}{g}$$

absorbed energy (area under the curve and it is equal to the kinetic energy of the body before impact)

Graph of arresting force F as a function of displacement x

$$I = \int_0^\infty F\,dt = c\dot{x}_0\int_0^\infty \exp\left(-\frac{cgt}{W}\right)dt$$

$$I = \frac{W\cdot\dot{x}_0}{g}$$

By arranging orifices of varying size, number, and spacing the force can be made more nearly constant.

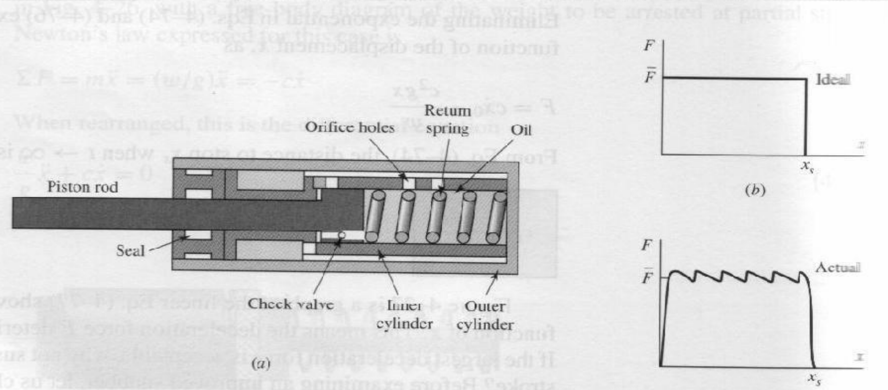


Figure 4–28

Schematic of a multiple-orifice snubber.

$\ddot{x} = -\ddot{x}_0$

$\dot{x} = -\ddot{x}_0 \cdot t + \dot{x}_0$

$x = -\dfrac{\ddot{x}_0 \cdot t^2}{2} + \dot{x}_0 \cdot t$ $\quad > \quad x_s = \dfrac{\dot{x}_0^2}{2 \cdot \ddot{x}_0}$ (cuts the stopping distance in half)

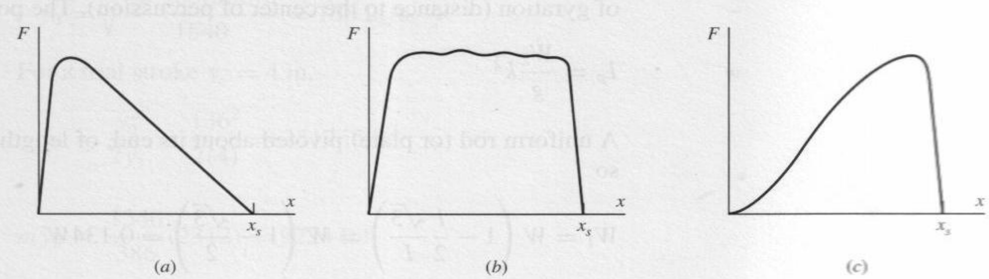
Time to stop $\quad t = \dot{x}_0 / \ddot{x}_0$

$$E_k = \left(\frac{w}{g}\ddot{x}_0\right)\left(\frac{\dot{x}_0^2}{2 \cdot \ddot{x}_0}\right) = \frac{w \cdot \dot{x}_0^2}{2g}$$

$$I = F \cdot t = \frac{w \cdot \ddot{x}_0}{g}\left(\frac{\dot{x}_0}{\ddot{x}_0}\right) = \frac{w}{g}\dot{x}_0$$

Three kinds of snubbers

1) Constant-orifice snubber (force declines with stroke progress)
2) Conventional snubber (constant force)
3) Progressive snubber (increasing force with stroke progress)



ure 4–29

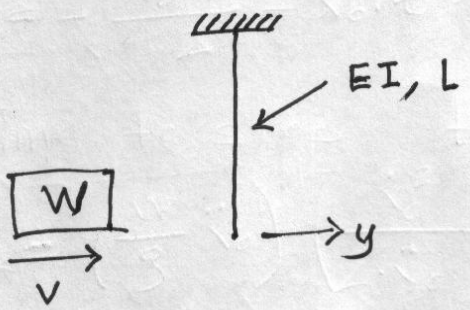
types: (a) declining force; ady force; creasing force.

Selection process

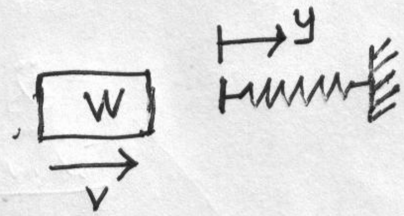
1) Identify the weight or mass to be stopped
2) Identify the velocity of impact $\dot{x}_o$
3) Identify forces doing additional work on the snubber during arrest
4) Identify repetitive cycle frequency

$$F_{max} = F_{ideal}\left(\frac{F_{max}}{F_{ideal}}\right) = \frac{W\cdot \dot{x}_o^2}{2\cdot g\cdot x_s}\left(\frac{1}{0.85}\right) = \frac{(1/2)(W\cdot \dot{x}_o^2/g)}{0.85\, x_s}$$

Suddenly Applied Loading



$\Rightarrow$



$$k = \frac{F}{y} = \frac{3EI}{L^3} \qquad \frac{W}{g}\cdot \ddot{y} = -k\cdot y$$

$$y = A\cos\left(\frac{kg}{W}\right)^{1/2} t + B\cdot \sin\left(\frac{kg}{W}\right)^{1/2}\cdot t$$

$$y = \frac{v}{(kg/W)^{1/2}}\cdot \sin\left(\frac{kg}{W}\right)^{1/2}\cdot t$$

$$y_{max} = \frac{v}{(kg/W)^{1/2}} = v\left(\frac{W\cdot L^3}{3E\cdot I\cdot g}\right)^{1/2}$$

$$M_{max} = k\cdot L\cdot y_{max} = v\left(\frac{3EI\,W}{g\cdot L}\right)^{1/2}$$

The arithmetic mean, the variance, and the standard deviation

- arithmetic mean $\mu = \dfrac{x_1 + x_2 + \cdots + x_N}{N} = \dfrac{1}{N}\sum_{1}^{N} x_j$
- mode is the value that occurs most frequently
- median is the middle value (odd number of cases), the mean of the two middle values (even number)
- variance $s_x^2 = \dfrac{1}{N-1}\sum_{1}^{N}(x_j - \mu)^2$
- standard deviation $s_x = \left[\dfrac{1}{N-1}\sum_{1}^{N}(x_j-\mu)^2\right]^{1/2}$

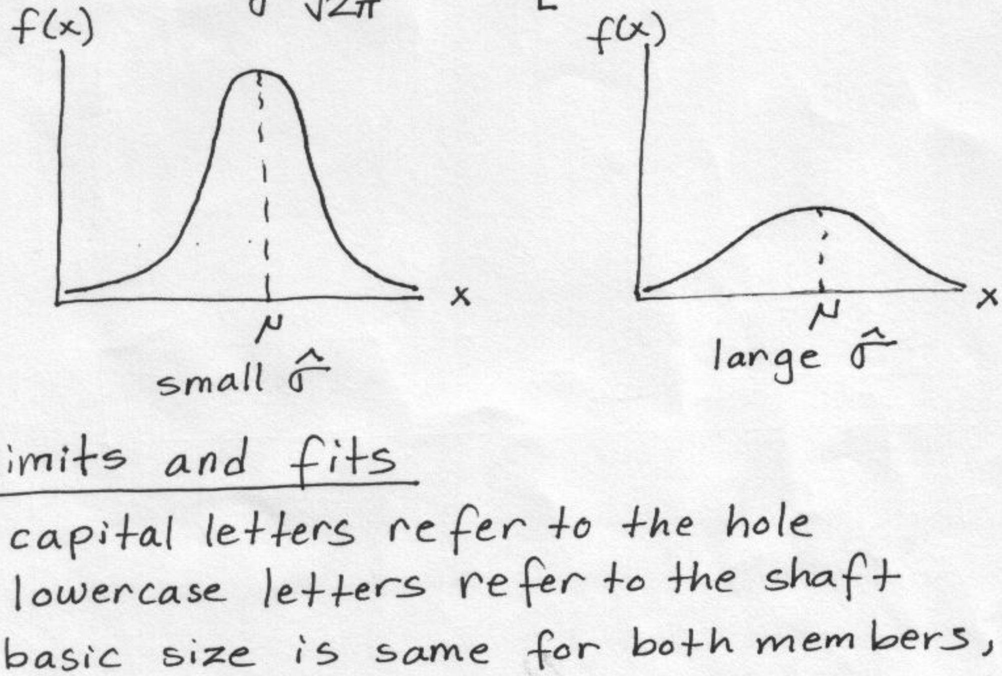
Regression

- fitting the curve a set of data points

$$\underbrace{b}_{\text{y intercept}} \qquad y = b + \underbrace{m}_{\text{slope}}x \qquad m = \frac{\sum xy - \dfrac{\sum x \sum y}{N}}{\sum x^2 - \dfrac{(\sum x)^2}{N}} \qquad b = \frac{\sum y - m\sum x}{N}$$

The normal distribution

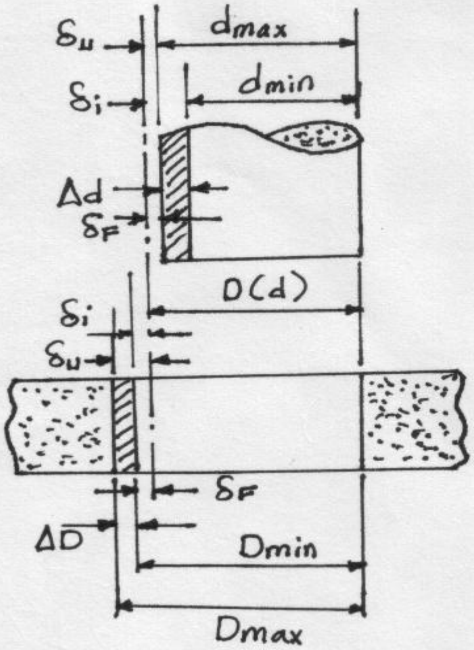
$$f(x) = \frac{1}{\hat{\sigma}\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\hat{\sigma}}\right)^2\right] \Leftarrow \text{probability density function}$$



Limits and fits

- capital letters refer to the hole
  lowercase letters refer to the shaft
- basic size is same for both members, D(d)
- deviation is the algebraic difference between a size and the corresponding basic size

- upper deviation $\Rightarrow d - d_{max} = \delta_u$
- lower deviation $\Rightarrow d - d_{min} = \delta_i$
- tolerance $\Rightarrow d_{max} - d_{min}$
- international tolerance grade numbers, Δd, ΔD
- hole basis $\Rightarrow$ fundamental deviation is H
- shaft basis $\Rightarrow$ fundamental deviation is h



Dimensioning and tolerancing

- nominal size : The size that we use in speaking of an element.
- basic size: The exact theoretical size.
- Limits : maximum and minimum dimensions
- tolerance: the difference between the two limits
- bilateral tolerance: The variation in both directions from the basic dimension $(1.005 \pm 0.002 \text{ in})$
- unilateral tolerance: The basic dimension is taken as one of the limits, and variation is permitted in only one direction $(1.005 \begin{smallmatrix} +0.004 \\ -0.000 \end{smallmatrix} \text{ in})$
- natural tolerance : plus or minus three standard deviations from the mean $(3 \cdot \hat{\sigma})$
- clearance : (internal member is smaller than the external member) $\Rightarrow$ difference in two diameters
- interference: (internal member is larger than the external member) $\Rightarrow$ difference in two diameters

The lognormal distribution

If
- the distribution is asymmetrical about the mean, or
- the variables have only positive values

normal distribution is not convenient

lognormal distribution can be used (particularly for life)

Weibull distribution can be used

Materials

The selection of material is one of the decisions.

Static strength

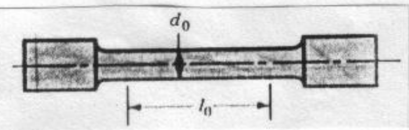


FIGURE 5-1
A typical tension-test specimen. Some of the standard values used for d_0 are 2.5, 6.25, and 12.5 mm and 0.5 in, but other sections and sizes are in use. Common gauge lengths l_0 used are 10, 25, and 50 mm and 1 and 2 in.

P: proportional limit
E: elastic limit
Y: yield point

$$\epsilon = \frac{L_i - L_o}{L_o}$$

L_o → original gage length

L_i → gage length corresponding to any load P_i

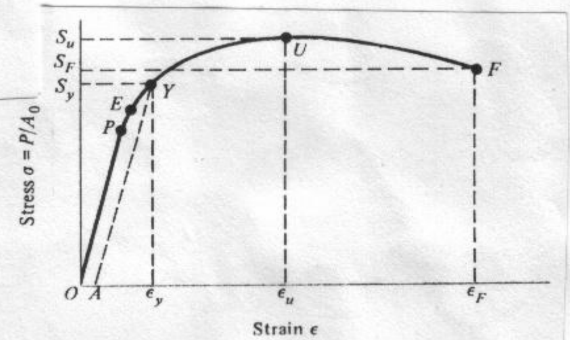


FIGURE 5-2
Stress-strain diagram obtained from the standard tensile test of a ductile material. *P* marks the proportional limit; *E*, the elastic limit; *Y*, the offset yield strength as defined by offset strain *OA*; *U*, the maximum or ultimate strength; and *F*, the fracture strength.

U: ultimate
F: fracture

- S_y is often defined by an offset method (0.2 or 0.5 percent of the original gage length)

$$\varepsilon = \ln \frac{L_i}{L_o}$$

ε → true strain

- When necking occurs, a more satisfactory relation

$$\epsilon = \frac{A_o - A_i}{A_o}$$

A_i → area after deformation

A_o → original area

- There is a buckling problem in compression test
- Torsional strength, $S_{su} = \dfrac{T_u \cdot r}{J}$

r → radius of the bar

J → polar second moment of area

Plastic deformation

- The relationship given by Datsko,

$$\sigma = \sigma_0 \, \varepsilon^m$$

m → strain-strengthening exponent

ε → true plastic strain

σ_0 → strength coefficient

σ → true stress

Strength and cold work

- Cold working is the process of stressing or deforming a material in the plastic region of the stress-strain diagram without the application of heat.

- The total unit strain at point I consists of the plastic (ϵ_p) and elastic (ϵ_e) components. Total strain,

$$\epsilon = \epsilon_p + \epsilon_e$$

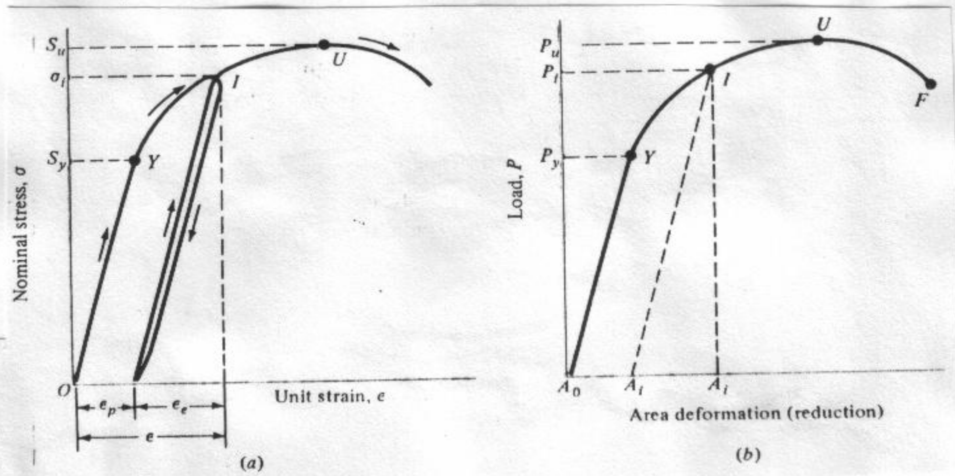


FIGURE 5-8

(a) Stress-strain diagram showing unloading and reloading at point I in the plastic region; (b) analogous load-deformation diagram.

$A_i' = A_0(1-W)$ → cold work factor

↳ area after the load P_i has been relased

$S_y' = \frac{P_i}{A_i'} = \sigma_0 \varepsilon_i^m \qquad P_i \le P_u$

↳ new yield strength

$S_u' = \frac{S_u}{1-W}$

↳ new ultimate strength

Hardness

- The resistance of a material to penetration by a pointed tool is called hardness. The unit of hardness is same as those of stress.
- There are many hardness-measuring systems
- Rockwell hardnes scales are designated as A, B, C, ... (R_A, R_B, R_C, ...)
- Brinell hardness uses a ball (indenting tool). Hardness number

$$H_B = \frac{\text{the applied load}}{\text{spherical surface area}}$$

Impact properties

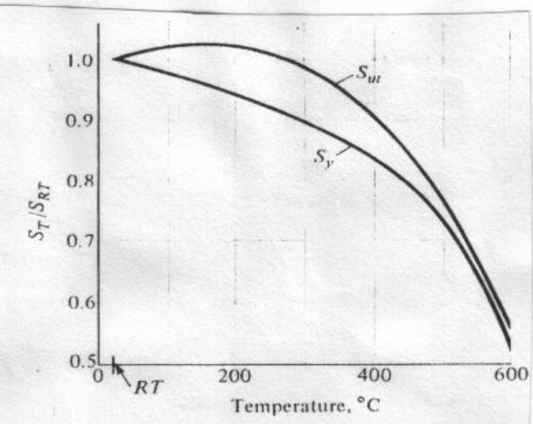
- If the time of application is less than $\frac{1}{3}$ · lowest natural period of vibration, the load is called an impact load.
- Impact value increases as the temperature is increased

Temperature effects

- The tensile strength changes only a small amount until a certain temperature is reached. Then it falls off rapidly.
- The yield strength decreases continuously as the temperature is increased.

FIGURE 5-11

A plot of the results of 145 tests of 21 carbon and alloy steels showing the effect of operating temperature on the yield strength S_y and the ultimate strength S_{ut}. The ordinate is the ratio of the strength at the operating temperature to the strength at room temperature. The standard deviations were $0.0442 \le \hat{\sigma} \le 0.152$ for S_y and $0.099 \le \hat{\sigma} \le 0.110$ for S_{ut}. *[Data source: E. A. Brandes (ed.), Smithells Metals Reference Book, 6th ed., Butterworth, London, 1983, pp. 22-128 to 22-131.]*



Sand casting

- Economical production in large quantities
- No limit to the size, shape, or complexity
- Steel, gray iron, brass, bronze, and aluminum are most often used in castings

Shell molding

- A fine grain, stress free casting
- Smooth surface, close tolerances

Investment casting

- Smooth surfaces, bright surfaces
- High accuracy

Powder-metallurgy process

- Powders from a single, several metals, or a mixture of metals and nonmetals are used for this process.
- Compacting powders at high pressures and heating the compacted part at a temperature less than the melting point
- The elimination of scrap or waste material, the elimination of machining operations, the low unit cost, the exact control of composition are the advantages.

Hot-working processes

- Metal is heated sufficiently and easily worked
- Rolling, forging, hot extrusion, hot pressing
- Steel, aluminum, magnesium, and copper alloys

Cold-working processes

- The forming of metal while at a low temperature
- Bright new finish, more accurate, require less machining

- Cold rolling distorts the grain size

The heat treatment of steel

- The common heat-treating operations are annealing, quenching, tempering, and case hardening
- Annealing :- Heating to a temperature (~100 °F), holding at this temperature, then cooling slowly
  - Soften material and make it more ductile, relieve residual stresses, refine the grain structure
- Quenching
  - A controlled cooling rate (during annealing) is called quenching
  - Carbon steels, medium-carbon, low-alloy steels
- Tempering
  - A modest heating process to relieve the stresses and soften the material
- Case hardening
  - The purpose of case hardening is to produce a hard outer surface on a specimen
  - This is done by increasing the carbon content at the surface

Alloy steels and alloying elements

- Alloy steels are produced by introducing one or more elements other than carbon in sufficient quantities to modify the material properties substantially
- Alloying elements are chromium, nickel, manganese, silicon, molybdenum, vanadium, tungsten

Corrosion-resistant steels

- 12% or more chromium ⇒ stainless steels
- Resistance to many corrosive conditions

Casting materials

Gray cast iron, white cast iron, malleable cast iron, ductile and nodular cast iron, alloy cast irons, cast steels

Nonferrous metals

- Aluminum
  - High strength-weight ratio, high resistance to corrosion, high thermal and electrical conductivity

- Aluminum can be processed by sand casting, die casting, hot or cold working, or extruding
- Aluminum alloys can be machined, press-worked, soldered, brazed or welded

• Magnesium
- The lightest of all commercial metals

• Copper-base alloys
- Brass, bronze

Plastics

- Thermoplastics: Any plastic that flows or is moldable when heat is applied to it.
- Thermosets: A plastic for which the polymerization process is finished in a hot molding press.

Notch sensitivity

• The existence of irregularities or discontinuities in a part increases the theoretical stresses in the immediate vicinity of the discontinuity.

$$\underbrace{K_f}_{\text{reduced value of stress concentration factor}} = \frac{\sigma_{max}}{\sigma_0}$$

$$\underbrace{q}_{\text{notch sensitivity}} = \frac{K_f - 1}{\underbrace{K_t}_{\text{stress concentration factor}} - 1}$$

• In analysis or design work, find K_t first, then find q, and solve for K_f,
(from the geometry) (from the material)

$$K_f = 1 + q(K_t - 1)$$

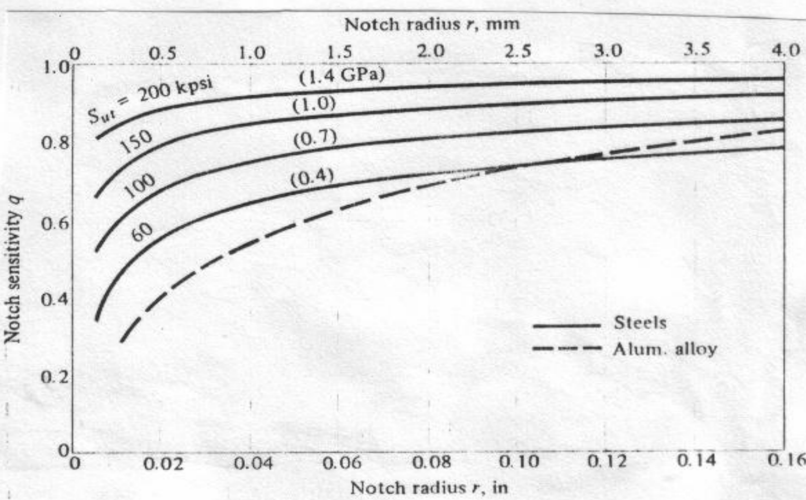
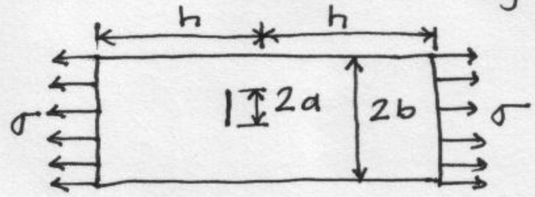


FIGURE 5-16

Notch-sensitivity charts for steels and UNS A92024-T wrought aluminum alloys subjected to reversed bending or reversed axial loads. For larger notch radii, use the values of *q* corresponding to *r* = 0.16 in (4 mm). ***[Reproduced by permission from George Sines and J. L. Waisman (eds.), Metal Fatigue, McGraw-Hill, New York, 1959, pp. 296, 298.]***

Fracture mechanics

- The elastic stress-concentration factors are useful for analysis of loads on a part that will cause fatigue fracture.
- When there exists a crack, flaw, inclusion, or defect of unknown small radius in a part, the elastic stress-concentration factor approaches infinity as the root radius approaches zero.
- Therefore, if there are very sharp cracks, elastic stress-concentration factors are no longer valid.
- $K_0 = \sigma \sqrt{\pi a}$



example

$\frac{h}{b} = 1 \quad \frac{a}{b} = 0.5$

$K_I = 1.32\, \sigma \sqrt{\pi a}$

- Critical stress-intensity factor (fracture toughness)

  $n = \frac{K_c}{K}$ K reaches $K_c \Rightarrow$ the crack will propagate

- There are three possible deformation modes

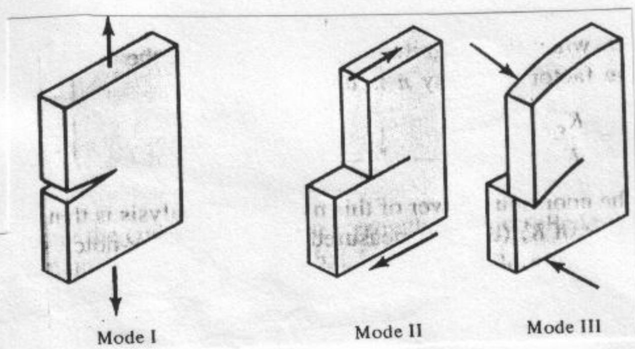


FIGURE 5-20

Deformation modes: mode I is tension; modes II and III are both shear modes.

- Parts subjected to continuous static loads in certain corrosive environments may develop serious cracks. This phenomenon is known as stress-corrosion cracking.

Crack Growth

$$\Delta K_I = \beta\sqrt{\pi a}\,(\sigma_{max} - \sigma_{min}) = \beta\sqrt{\pi a}\,\Delta\sigma$$

Life Prediction

Paris exponent

$$\frac{da}{dN} = A\,(\Delta K_I)^n \Leftarrow \text{Paris equation (in region II)}$$

Paris coefficient

$$N - N_i = \frac{1}{A}\int_{a_i}^{a} \frac{da}{(\Delta K_I)} = \frac{1}{A}\int_{a_i}^{a} \frac{da}{(\beta\sqrt{\pi}\,\Delta\sigma)^n a^{n/2}}$$

$$N - N_i = \frac{a^{1-n/2} - a_i^{1-n/2}}{A(\beta\sqrt{\pi}\,\Delta\sigma)^n (1 - n/2)}$$

$$\frac{da}{dN} = \frac{A\,(\Delta K)^n}{(1-R)K_c - \Delta K}$$

(region II and III) $\Leftarrow$ Forman equation

stress ratio

$$\frac{da}{dN} = \frac{C(1-f)^n\,\Delta K^n\,(1 - \Delta K_{th}/\Delta K)^p}{(1-R)^n\{1 - \Delta K/[(1-R)K_c]\}^q}$$