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Sensitivity-based finite element model updating using constrained optimization with a trust region algorithm

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Abstract

The use of changes in dynamic system characteristics to detect damage has received considerable attention during the last years. Within this context, FE model updating technique, which belongs to the class of inverse problems in classical mechanics, is used to detect, locate and quantify damage. In this study, a sensitivity-based finite element (FE) model updating scheme using a trust region algorithm is developed and implemented in a complex structure. A damage scenario is applied on the structure in which the stiffness values of the beam elements close to the beam–column joints are decreased by stiffness reduction factors. A worst case and complex damage pattern is assumed such that the stiffnesses of adjacent elements are decreased by substantially different stiffness reduction factors. The objective of the model updating is to minimize the differences between the eigenfrequency and eigenmodes residuals. The updating parameters of the structure are the stiffness reduction factors. The changes of these parameters are determined iteratively by solving a nonlinear constrained optimization problem. The FE model updating algorithm is also tested in the presence of two levels of noise in simulated measurements. In all three cases, the updated MAC values are above 99% and the relative eigenfrequency differences improve substantially after model updating. In cases without noise and with moderate levels of noise; detection, localization and quantification of damage are successfully accomplished. In the case with substantially noisy measurements, detection and localization of damage are successfully realized. Damage quantification is also promising in the presence of high noise as the algorithm can still predict 18 out of 24 damage parameters relatively accurately in that case.

1. Introduction

Most of the bridges in USA and Europe are built in the 1960s and the majority of these have reached their critical age. It has been reported that about 125,000 out of the total 585,000 bridges in USA are structurally deficient [1,2]. It is expected that the budget demands for maintenance will peak in 2010 [3]. The earthquakes in Turkey and Greece in 1999 [4,5], the Umbria-Marche earthquake in 1997 in Italy and the recent earthquakes in Portugal and Iceland have damaged many buildings, in which the damage is not visible by

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visual inspections. Vibration monitoring is a useful instrument in evaluating the condition of these bridges and buildings and in making maintenance schedules. Vibration-based damage detection and finite element (FE) model updating have emerged in the 1990s as topics of prominent importance to the design, construction and maintenance of civil engineering structures. An excellent review on all techniques can be found in the book of Friswell and Mottershead [6] and in two reports of the Los Alamos National Laboratory [7,8]. Other comprehensive reviews on the topic can be found in Refs. [9–12].

Structural damage in civil engineering structures results in changes in the modal parameters such as natural frequencies, mode shapes and modal damping values. Modal parameters can be easily obtained from vibration testing. Four levels of damage identification are possible as proposed by Rytter [13]:

(1) Level 1—*detection*: Is the structure damaged or not?
(2) Level 2—*localization*: What is the location of the damage in the structure?
(3) Level 3—*quantification*: What is the extent of damage?
(4) Level 4—*prediction*: What is the remaining service life of the structure?

Many model-based damage detection methods attempt to detect changes in the natural frequencies of a structure. Salawu [14] reviewed different methods of structural damage detection through changes in natural frequencies. The author stresses that the use of natural frequencies as diagnostic parameters in structural assessment procedures using vibration monitoring is an inexpensive structural assessment technique. However, there are limitations that could prevent successful application. The changes in natural frequencies can not or hardly provide the spatial information about structural damage. Therefore, mode shape information is also required to uniquely localize damage. Since this study is on FE model updating by ambient vibration measurements, the identified experimental modes will be unscaled.

This paper describes the sensitivity-analysis-based FE model updating method. Sensitivity-based techniques have been used for FE model updating in Refs. [15–20] but most of these applications are on beam-like structures (e.g. bridges) and plates but not on building type structures where damage is typically caused by an earthquake like action. For building systems designed according to the capacity-based philosophy, the damage resulting from an earthquake is globally distributed throughout the structure, in regions specifically designed to dissipate energy (plastic hinges). The original contribution of the paper is the application of the method on the structure condition assessment with particular reference to buildings. This procedure is applicable to buildings which have been damaged by earthquakes, but that behave in the linear domain during an ambient vibration measurement after the earthquake. Therefore, structures that have been heavily damaged beyond repair are outside the scope of this study. This is an ongoing research and this part of the study is a feasibility investigation of the sensitivity-based FE model updating method using a trust region algorithm on a complex structure with a complicated damage pattern.

2. Theoretical background

Sensitivity-based FE model updating method is the most frequently used updating method [21–23]. This method directly compares eigenfrequencies and mode shapes. The compared quantities can be collected in a vector such as

$$v(\boldsymbol{a}) = [\lambda_1, \ldots, \lambda_{m_f}, \phi_1^{\mathrm{T}}, \ldots, \phi_{m_m}^{\mathrm{T}}]^{\mathrm{T}}, \tag{1}$$

where, $\lambda_j = \omega_j^2$ are the eigenvalues with $\lambda_j = (2\pi v_j)^2$ and eigenfrequency v_j, ϕ_j refers to the eigenvectors of the model, m_f denotes the number of identified eigenfrequencies that are used in the updating process and m_s is equal to the product of the number of identified mode shapes m_m and the number of dofs used N for mode ϕ_j. The residual vector $\mathbf{r}$ is defined as the difference between the measured quantities and quantities calculated from the FE model as shown:

$$\mathbf{r}(\boldsymbol{a}) = \begin{bmatrix} \mathbf{r}_f(\boldsymbol{a}) \\ \mathbf{r}_s(\boldsymbol{a}) \end{bmatrix}, \quad \mathbf{r}_f : \Re^n \to \Re^{m_f}, \quad \mathbf{r}_s : \Re^n \to \Re^{m_s}. \tag{2}$$

As the quantities are of different magnitudes, a weighted residual has to be taken into consideration for the FE model updating:

$$\mathbf{r}(\boldsymbol{a}) = \mathbf{W}(\tilde{\mathbf{v}} - \mathbf{v}(\boldsymbol{a})) = \mathbf{W}\begin{bmatrix} \tilde{\lambda}_1 - \lambda_1 \\ \vdots \\ \tilde{\lambda}_{m_f} - \lambda_{m_f} \\ \tilde{\phi}_1 - \phi_1 \\ \vdots \\ \tilde{\phi}_{m_m} - \phi_{m_m} \end{bmatrix}, \tag{3}$$

where $\tilde{\lambda}$ are the measured eigenvalues, $\tilde{\phi}$ are the measured eigenmodes, λ are the calculated eigenvalues, ϕ are the calculated eigenmodes, $\mathbf{W}$ is a diagonal weighting matrix that scales the eigenvalues and the maximum value of the mode shapes to the order of unity:

$$\mathbf{W} = \mathbf{diag}(1/\tilde{\lambda}_1, \ldots, 1/\tilde{\lambda}_{m_f}, \omega_{\phi_1}, \ldots, \omega_{\phi_{mm}}). \tag{4}$$

Comparison of Eq. (4) with Eq. (3) shows that weighted eigenfrequency residuals can be expressed as

$$r_f(\boldsymbol{a}) = \frac{\lambda_j(\boldsymbol{a}) - \tilde{\lambda}_j}{\tilde{\lambda}_j} \quad \text{with } \lambda_j = (2\pi v_j)^2. \tag{5}$$

In FE calculations, the mode shapes are normalized with respect to the mass matrix. When ambient vibrations are used to extract modal parameters on the other hand, the experimental mode shapes remain unscaled. Therefore, the weighted mode shape residuals $r_s(\boldsymbol{a})$ are defined as

$$r_s(\boldsymbol{a}) = \frac{\phi_j^l(\boldsymbol{a})}{\phi_j^r(\boldsymbol{a})} - \frac{\tilde{\phi}_j^l}{\tilde{\phi}_j^r} = \omega_j^l(\phi_j^l - \tilde{\phi}_j^l), \tag{6}$$

where l and r denote an arbitrary and a reference dof of mode shape ϕ_j (or $\tilde{\phi}_j$), respectively. If Eq. (6) is substituted in Eq. (3), the diagonal entries for the weighting matrix corresponding to mode shapes ω_ϕ are obtained as

$$\omega_j^l = \frac{(\phi_j^l(\boldsymbol{a})\tilde{\phi}_j^r - \tilde{\phi}_j^l\phi_j^r(\boldsymbol{a}))}{\phi_j^r(\boldsymbol{a})\tilde{\phi}_j^r(\phi_j^l(\boldsymbol{a}) - \tilde{\phi}_j^l)}. \tag{7}$$

This factor may be changed to give less importance to the mode shapes with respect to the frequencies. As stated above, the experimental eigenfrequencies are in general the most accurate experimental data that can be obtained and because of their high sensitivity to the physical properties of the structure, they improve the condition of the problem. Experimental mode shapes on the other hand, are much more noisy, can reduce the stability of the minimization problem, but provide more local information. An appropriate weighting is therefore generally necessary. It should be stated that the weight factors influence the result only in the case of over-determined systems when the number of residuals is higher than the number of design variables. The model update is carried out by minimizing the residual:

$$\min \tfrac{1}{2}\mathbf{r}(\boldsymbol{a})^{\mathrm{T}}\mathbf{W}\mathbf{r}(\boldsymbol{a}) = \tfrac{1}{2}\|\mathbf{W}^{\frac{1}{2}}\mathbf{r}(\boldsymbol{a})\|^2. \tag{8}$$

The value of each uncertain variable X^e is determined from the multiplication of the initial value of the variable X_o^e and the dimensionless updating parameter a^e as follows:

$$X^e = X_o^e(1 - a^e). \tag{9}$$

The above-mentioned uncertain physical properties of the numerical model are the stiffness values in the FE model updating problem. Damage results in a stiffness reduction in civil engineering structures. The updated stiffness of an element of the model can be expressed as

$$\mathbf{K}^e = \mathbf{K}_o^e(1 - a^e). \tag{10}$$

The global stiffness matrix will then be:

$$\mathbf{K} = \mathbf{K}^u + \sum_{e=1}^{n} \mathbf{K}_o^e(1 - a^e), \tag{11}$$

where $\mathbf{K}_o^e$ and $\mathbf{K}^e$ are the initial and updated element stiffness matrix, respectively; $\mathbf{K}$ is the global stiffness matrix and $\mathbf{K}^u$ is the stiffness matrix of the elements whose properties remain unchanged, n is the number of elements that are updated.

The optimization algorithm used to minimize the objective function is a trust region Newton method which is a sensitivity-based iterative method [24]. The quadratic model $m(\mathbf{z})$ is defined by a truncated Taylor series of $f(\boldsymbol{a})$:

$$\min_{\mathbf{z}} m(\mathbf{z}) = f_s + [\nabla f_s]^{\mathrm{T}}\mathbf{z} + \tfrac{1}{2}\mathbf{z}^{\mathrm{T}}[\nabla^2 f_s]\mathbf{z}, \tag{12}$$

such that $\|z\| \leqslant \Delta_s$ where $\mathbf{z}$ denotes a step vector from $\boldsymbol{a}$ and where $f_s, \nabla f_s$, and ∇f_s^2 are the values of the function, the gradient and the Hessian of f, respectively. After an iterative process, the minimum $\boldsymbol{a}^*$ of $f(\boldsymbol{a})$ is reached where $\nabla f(\boldsymbol{a}^*) \approx 0$. The gradient and the Hessian of $f(\boldsymbol{a})$ are:

$$\nabla f(\boldsymbol{a}) = \sum_{j=1}^{k} r_j(\boldsymbol{a}) \nabla r_j(\boldsymbol{a}) = \mathbf{J}_{\boldsymbol{a}}(\boldsymbol{a})^{\mathrm{T}} \mathbf{r}(\boldsymbol{a}), \tag{13}$$

$$\nabla^2 f(\boldsymbol{a}) = \mathbf{J}_{\boldsymbol{a}}(\boldsymbol{a})^{\mathrm{T}} \mathbf{J}_{\boldsymbol{a}}(\boldsymbol{a}) + \sum_{j=1}^{k} r_j(\boldsymbol{a}) \nabla^2 r_j(\boldsymbol{a}) \approx \mathbf{J}_{\boldsymbol{a}}(\boldsymbol{a})^{\mathrm{T}} \mathbf{J}_{\boldsymbol{a}}(\boldsymbol{a}), \tag{14}$$

where $\mathbf{J}_{\boldsymbol{a}}$ is the Jacobian matrix (or sensitivity matrix), which contains the partial derivatives of the residuals r_j with respect to $\boldsymbol{a}$:

$$\Delta r_j = \sum_{e=1}^{n} \frac{\delta r_j}{\delta a^e} \Delta a^e \tag{15}$$

or in full form:

$$[\mathbf{J}]_{m \times p} = \begin{bmatrix} \dfrac{\delta r_1}{\delta a_1} & \dfrac{\delta r_1}{\delta a_2} & \dfrac{\delta r_1}{\delta a_3} & \cdots & \dfrac{\delta r_1}{\delta a_p} \\ \dfrac{\delta r_2}{\delta a_1} & \dfrac{\delta r_2}{\delta a_2} & \dfrac{\delta r_2}{\delta a_3} & \cdots & \dfrac{\delta r_2}{\delta a_p} \\ \dfrac{\delta r_3}{\delta a_1} & \dfrac{\delta r_3}{\delta a_2} & \dfrac{\delta r_3}{\delta a_3} & \cdots & \dfrac{\delta r_3}{\delta a_p} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \dfrac{\delta r_m}{\delta a_1} & \dfrac{\delta r_m}{\delta a_2} & \dfrac{\delta r_m}{\delta a_3} & \cdots & \dfrac{\delta r_m}{\delta a_p} \end{bmatrix}, \tag{16}$$

where $m = m_f + m_s$ and p is the number of the updating parameters.

In this paper, Gauss–Newton method is applied with a trust region algorithm to improve the convergence. In the trust region approach, the algorithm constructs a model function m_k, whose behaviour is similar to that of the actual objective function f near the current point $\boldsymbol{a}_k$, and determines a region surrounding $\boldsymbol{a}_k$ where this model can be trusted. A candidate for the new iterate is then computed by approximately minimizing m_k inside the trust region. In case a sufficient decrease is not obtained in f, the subproblem is again solved with a smaller trust region. Otherwise, the candidate is accepted as a new iterate from which the process reiterates [25]. The trust region is a sphere with trust region radius Δ, which restricts the design variables $\|\boldsymbol{a}\| \leqslant \Delta$. The radius Δ_k is adjusted between iterations such that good agreement is obtained between predicted and actual reduction in the function as measured by the ratio a_k:

$$\rho_k = \frac{f(\boldsymbol{a}_k) - f(\boldsymbol{a}_k + \mathbf{z}_k)}{f(\boldsymbol{a}_k) - m_k(\mathbf{z}_k)}. \tag{17}$$

When there is good agreement, ρ_k approaches 1 and Δ_k is increased. If the agreement is poor, ρ_k takes either small or negative values. In that case Δ_k is decreased. Otherwise Δ_k remains unchanged.

The modal sensitivities with respect to the correction factors a^e can be calculated using the finite differences [18,26]. The first partial derivative of each frequency residual r_f and mode shape residual r_s with respect to $\boldsymbol{a}$ are:

$$\frac{\delta r_f}{\delta a^e} = \frac{1}{\tilde{\lambda}_j}\frac{\delta \lambda_j}{\delta a^e}, \tag{18}$$

$$\frac{\delta r_s}{\delta a^e} = \frac{1}{\phi_j^r}\frac{\delta \phi_j^l}{\delta a^e} - \frac{\phi_j^l}{(\phi_j^r)^2}\frac{\delta \phi_j^r}{\delta a^e}. \tag{19}$$

3. FE model updating of a frame type structure

The natural frequencies and mode shapes are directly related to the stiffness of a structure. Therefore, a drop in natural frequencies or mode shapes will indicate a loss of stiffness which is a consequence of damage in certain elements of the structure. As the number of elements to be updated increases, the ill-posedness of the sensitivity matrix increases. In this case, some kind of regularization is necessary. Damage functions have been recommended for decreasing the number of parameters to be updated in the FE model updating of a reinforced concrete beam structure by Teughels et al. [27] which can be interpreted as a kind of regularization method. The principal idea in this methodology is that in order to prevent an ill-conditioned Jacobian matrix due to a high number of updating variables, the distribution of the unknown physical property is approximated by combining a limited set of damage functions. The updating parameters are then selected as the factors by which each of the damage functions has to be multiplied before combining them. By this methodology, not only the number of the updating parameters are reduced, but also the ill posedness of the optimization problem is prevented. However, in more complicated structures, the damage mechanisms depend on engineering judgement and the use of damage functions may not be appropriate. As mentioned above, in this study, the FE model updating problem is applied on a reinforced concrete frame structure in which the use of the damage functions is not appropriate. In the following subsections, the reinforced concrete building type selected is explained first. The damage mechanisms of the reinforced concrete frames are then discussed followed by the description of the damage scenario applied to the building.

3.1. Criteria for the choice of the building type considered and the description of the building

North-west Turkey has been hit by two destructive earthquakes in 1999. A general review of these earthquakes can be found in other papers of the first author [4,5]. In a recent study [28], just over 750,000 buildings in the region to the north of the Sea of Marmara in Turkey are investigated for deriving an earthquake loss model for the particular area due to the high levels of hazard and exposure in this region. The buildings have been separated into ‘good’ and ‘poor’ classes. This classification was based on the considerations of the construction year, evidence of poor construction quality derived from construction type, and the presence of a weak ground floor.

The building type that is going to be studied in the present study is a ‘good’ quality typical existing building in the region to the north of the Marmara Sea with a beam side-sway failure mechanism. The building is chosen such that its geometrical, material and limit state properties are within the confidence intervals of the ‘good’ building classification for the above-mentioned study which is based on 750,000 buildings from the Marmara region.

The building considered is located in Bolu and is a moment resisting reinforced concrete frame system that represents a typical residential building in the Marmara region isolated from other buildings at both sides. The drawings of the building are taken from the curated depository of Turkish building data on the Kocaeli Golcuk and Duzce Bolu earthquakes of 1999 maintained by Purdue University, University of Michigan and University of Texas at Austin [29]. The building is a four storey structure with three bays. The floor system is flat slab with beams. The building has a typical storey height of 2.85 m and considered as regular in elevation. The dimensions of the columns are typically 0.6 m × 0.25 m or 0.25 m × 0.6 m and the dimensions of the beams are 0.25 m × 0.5 m for beams. The concrete design strength is 20 MPa. The building does not have a basement and is fixed at the foundations. The building was under construction when the 1999 $M_w = 7.4$ Kocaeli and $M_w = 7.1$ Duzce earthquakes hit the region. After the 1999 earthquakes, it is reported that the reinforced concrete frame was moderately damaged.

3.2. FE model of the reinforced concrete frame

The structure is modeled by FEs and is idealized as a two-dimensional structure taking into account the weights of the slabs in the transverse direction in the FE model. Eight beam elements per beams and 6 beam elements for columns are generated for the FE model. The measurement locations are shown in Fig. 1.

In the first stage, in order to obtain the natural frequencies and mode shapes of the system, modal analysis is carried out by using the Block Lanczos extraction method with a sparse matrix solver. The mode shapes are normalized to unity. This stage of the analysis gives the undamaged structure's modal properties. The first two mode shapes of the frame are shown in Figs. 2 and 3.

3.3. Damage scenario for the reinforced concrete frame

Typically, in modeling the response of reinforced concrete structures to earthquake loading, it is assumed that the joint regions are rigid and the damage is limited to flexural yielding of beams, columns, slabs and walls. If the beam–column joint regions are adequately designed, and if the reinforced concrete frame is designed appropriately according to the weak beam–strong column philosophy, then the plastic hinges will form in beams close to the face of the columns. On the other hand, the joints are anticipated to fail by bond

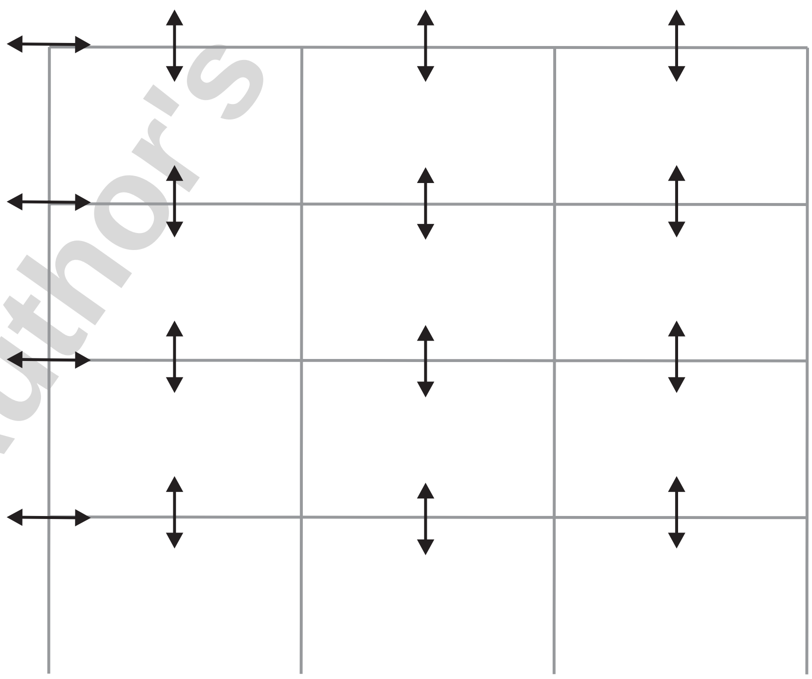

Fig. 1. The location of the response points of the reinforced concrete frame.

Fig. 2. The first vibration mode of the reinforced concrete frame.

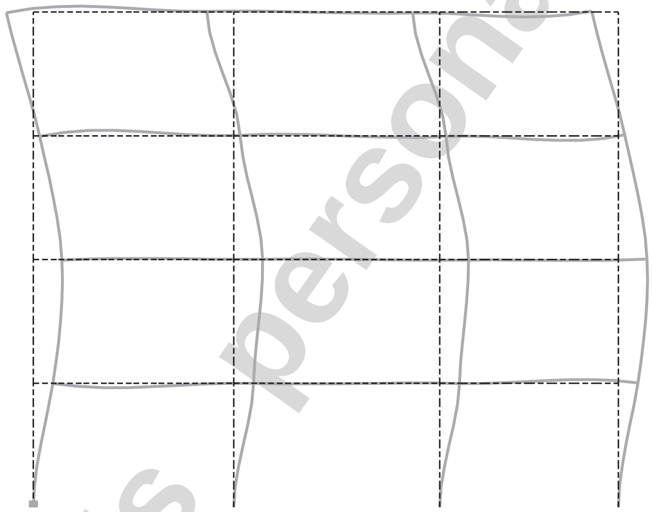

Fig. 3. The second vibration mode of the reinforced concrete frame.

slip or joint shear failure modes if the beam–column joint region does not have transverse reinforcement, the development lengths of beam bars or column depth to beam bar diameter ratio are less than the recommended values in codes and smooth bars are used for the beam longitudinal reinforcement. In this study, a damage scenario is considered such that the joint region is assumed to be adequately detailed with closely spaced stirrups and the frame is assumed to be designed according to the weak beam strong column philosophy. Consequently, the joints are assumed to remain rigid during seismic excitations and the plastic hinges are expected to occur in the beams close to the beam column joint regions spreading towards the point of contra flexure in the beams. Research [30] has shown that the damaged region in this case can not be represented by two springs because cracking spreads over a finite region at the ends of the reinforced concrete girders. For this type of scenario, the stiffnesses of the beam elements in the FE model close to the beam–column joint regions are decreased. It is reasonable to assume that the stiffness at this cracked region is constant. This is primarily due to the fact that the reinforcement layout will not change along the cracked zone length provided this zone does not extend beyond the quarter span point. Fig. 4 shows the elements of the reinforced concrete frame for which damage is simulated by reducing their stiffnesses. The stiffness reduction factors of these elements are given in Table 1.

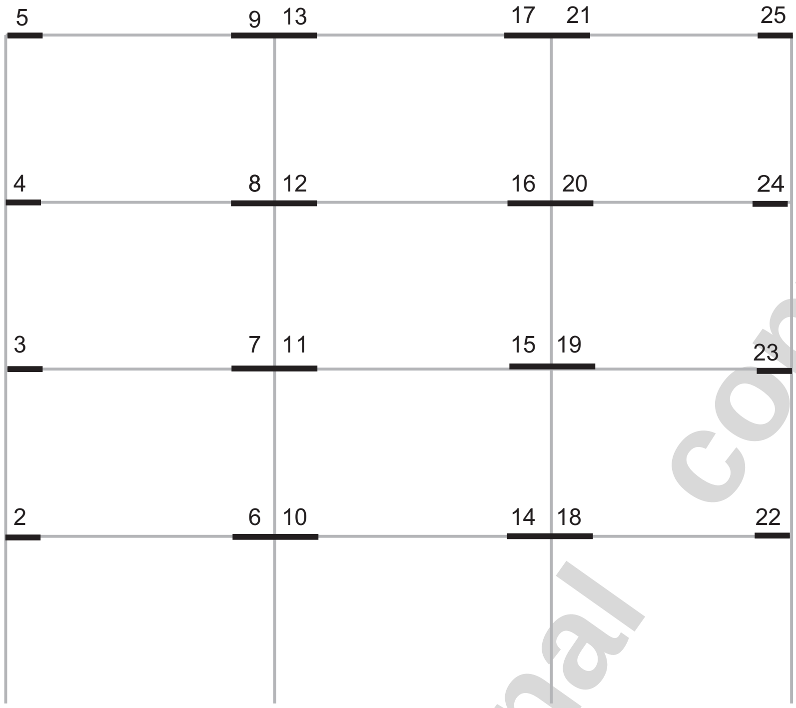

Fig. 4. The material numbers for the damaged elements in the FE model of the reinforced concrete frame. Materials numbers 2 to 25 represent the damaged elements whose stiffnesses will be updated. Material number 1 corresponds to the undamaged elements.

Table 1
Stiffness reduction factors for the 24 damaged elements

Material no.	Damage scenario *a*	FEM update *a*	Detected damage (yes/no)	Actual damage (yes/no)
1	0	NA	NA	NA
2	0.65	0.65	Yes	Yes
3	0.50	0.50	Yes	Yes
4	0.40	0.40	Yes	Yes
5	0.30	0.30	Yes	Yes
6	0.80	0.80	Yes	Yes
7	0.90	0.90	Yes	Yes
8	0.20	0.20	Yes	Yes
9	0	≈ 0	No	No
10	0.50	0.50	Yes	Yes
11	0.55	0.55	Yes	Yes
12	0	≈ 0	No	No
13	0.10	0.10	Yes	Yes
14	0.85	0.85	Yes	Yes
15	0.70	0.70	Yes	Yes
16	0	≈ 0	No	No
17	0	≈ 0	No	No
18	0.60	0.60	Yes	Yes
19	0.40	0.40	Yes	Yes
20	0.10	0.10	Yes	Yes
21	0.15	0.15	Yes	Yes
22	0.90	0.90	Yes	Yes
23	0.60	0.60	Yes	Yes
24	0.40	0.40	Yes	Yes
25	0.30	0.30	Yes	Yes

In the rest of the paper, the model of the damaged frame will be referred to as the model of the simulated experiment.

3.4. Justification for the selection of the number of modes and the trust region algorithm

In updating the FE model of the reinforced concrete frame, several parametric studies are carried out for selecting the optimum number of modes to include in the model updating. Within this context, model quality indices are introduced in which percentage average error in natural frequencies (IAENF) and percentage average error in mode shapes (IAEMS) are calculated using the following expressions:

$$\mathrm{IAENF} = \frac{100}{m_f} \sum_{i=1}^{m_f} \mathrm{abs}\left(\frac{v_u - \tilde{v}}{\tilde{v}} \right), \tag{20}$$

$$\mathrm{IAEMS} = \frac{100}{m_m N} \sum_{i=1}^{m_m} \sum_{j=1}^{N} \mathrm{abs}\left(\frac{\phi_{i_u}^{j} - \tilde{\phi}_i^{j}}{\tilde{\phi}_i^{j}} \right), \tag{21}$$

where v_u and ϕ_u are the updated natural frequencies and mode shapes, respectively. The results of the parametric studies are summarized in Table 2 in which the error indices are given and in Table 3 in which the condition numbers, number of iterations, CPU time for a computer with an Intel-Core 2 Duo processor are compared.

It is apparent from the tables that the best condition number for the Jacobian is obtained for the case of four modes. Four modes also give the lowest IAEMS error index after updating. For this reason, four modes are selected for inclusion in to the updating process.

For the case when a single mode is included in the updating process, no convergence was achieved. In this case, the trust region method could not be implemented, as it requires at least as many equations as design

Table 2
Error indices before and after model updating for different number of modes

Mode no.	IAENF (BU) (%)	IAENF (AU) (%)	IAEMS (BU) (%)	IAEMS (AU) (%)
1[a]	16.57	6.28	77.16	39.87
2	11.37	0.02	139.07	0.34
3	8.96	0.0015	109.52	0.23
4	7.25	0.0034	102.86	0.16
5	7.08	0.013	120.28	0.77
6	7.08	3.51	154.23	82.98
7	6.77	3.48	171.24	186.56

[a]The case of no convergence.

Table 3
Number of iterations, CPU time and the condition number of the Jacobian for the model updating algorithm for different number of modes

Mode no.	No. of iterations	Condition number of the Jacobian	CPU time (s)
1[a]	4	2.43E7	350.594
2	9	1986	172.63
3	9	339.65	166.59
4	22	191.17	397.81
5	20	162.30	356.61
6	32	1029.8	605.39
7	23	6446	405.84

[a]The case of no convergence.

variables. Consequently, a line-search algorithm using Levenberg–Marquardt method is implemented for which bound-constraints are not permitted. In model updating problems, due to the ill-conditioned nature of the Jacobian matrix, the calculated search directions are very sensitive to approximations and errors. Consequently, if the design variables are not bounded in the line searches along the incorrect directions, extreme step lengths are obtained that go beyond unrealistic limits. As a result, the optimization process did not converge for the case of one mode as shown in Tables 2 and 3.

In this study, a large-scale trust region method is selected for solving the nonlinear optimization problem in FE model updating. There are two main reasons behind this selection. First, other eigensensitivity techniques based on first-order approximations of FE model properties that use line search methods such as the Gauss–Newton and the Levenberg–Marquardt do not allow bound-constrained problems. As stated above, even very small errors in the line searches result in extreme step lengths, hence divergence of the algorithm. Secondly, the trust region strategy improves the condition number of the Jacobian and acts as a regularization technique. There are also two reasons for this. First, the radius of the trust region is updated in each iteration according to the accuracy of the approximating model function. This strategy, reduces oscillations in the design variables and results in a more robust optimization method. Second, in addition to the application of a trust region, explicit bound constraints are introduced into the optimization procedure which prevents overshooting and improves stability. It is true that the convergence speed of the large-scale trust method, where the subproblems are solved with an iterative conjugate gradient algorithm, will be lower than the other algorithms. However, since robustness is the major issue to be considered in ill-conditioned FE model updating problems, the convergence speed may be considered as rather secondary.

A last remark to be noted is that, in this study, the modes are mainly presented in increasing number. However, throughout the updating algorithm, possible eigenvalue crossing troubles, which may occasionally (though not necessarily) occur are also taken into account. This phenomenon occurs when the order of modes change in the damaged structure. In order to solve it, it is important to look to the modeshapes between two successive damage steps (or iterations in the updating process) to be sure that the same vibration modes are compared. This is achieved via the MAC matrix as described in the following subsection.

3.5. The MAC matrix and the analysis of results

The index MAC (modal assurance criterion) indicates the correlation between two sets of mode shapes [31]. MAC produces a matrix of inner products between the mode shape vectors as

$$\mathrm{MAC}(\phi_i, \tilde{\phi}_j) = \frac{|\phi_i \tilde{\phi}_j|^2}{(\phi_i^{\mathrm{T}} \phi_i)(\tilde{\phi}_j^{\mathrm{T}} \tilde{\phi}_j)}. \tag{22}$$

MAC matrix values change between 0 and 1. A MAC value close to 1 indicates a good correlation, and a MAC value close to 0 indicates a poor correlation. All the analytical modes are correlated with all the measured modes and the results are placed in a matrix. After the updating algorithm is completed, the MAC matrix is calculated in order to see the improvements in the eigenfrequencies and the eigenmodes of the building. In the first step, the correlation between the initial FE model and the damaged model is investigated. In the second step, the reference FE model is updated and the correlation between the updated FE model and the damaged model is again calculated. As a total four eigenfrequencies and four mode shapes are used in the FE model updating problem. Sixteen measurement points are used in the numerical simulation. Table 1 shows the actual and detected damage states by the model as well as the actual and predicted stiffness reduction factors. The table shows that all the damage states and the stiffness reduction factors are predicted accurately by the model. The FE model updating scheme used can accomplish the first three levels of damage identification, namely; detection, localization and quantification, successfully. Tables 4 and 5 also show that the relative eigenfrequency differences and the MAC values are considerably improved after FE model updating and the updated model gives accurate prediction of these modal parameters.

Next, the FE model updating scheme is tested in the presence of noise. Two noise levels are considered to simulate measurement errors. The first is moderate noise level in which random noise with normal distribution and 0.5% standard deviation is applied to the eigenvalues. For the mode shapes, noise with 1% standard

Table 4
Relative eigenfrequency differences and MAC values between modes obtained from the damaged and the initial reference undamaged FEM

Mode no.	Damaged $\tilde{v}$ (Hz)	Undamaged v (Hz)	$\frac{(v-\tilde{v})}{\tilde{v}}$ (%)	MAC (%)
1	1.45	1.70	16.57	99.82
2	5.18	5.50	6.17	99.28
3	9.82	10.23	4.13	98.18
4	13.11	13.39	2.13	87.50

Table 5
Relative eigenfrequency differences and MAC values between modes obtained from the damaged and the updated FEM

Mode no.	Damaged $\tilde{v}$ (Hz)	Updated v (Hz)	$\frac{(v-\tilde{v})}{\tilde{v}}$ (%)	MAC (%)
1	1.45	1.45	−1e−3	100
2	5.18	5.18	−1.88e−4	100
3	9.82	9.82	−3.76e−5	100
4	13.11	13.11	−3.95e−5	100

Table 6
Stiffness reduction factors predicted for the 24 damaged elements in the presence of random noise with normal distribution and 0.5% standard deviation applied to the eigenvalues; 1% standard deviation relative to the maximum amplitude applied to the mode shapes

Material no.	Damage + noise a	FEM update a	Detected damage (yes/no)	Actual damage (yes/no)
1	0	NA	NA	NA
2	0.65	0.65	Yes	Yes
3	0.50	0.51	Yes	Yes
4	0.40	0.42	Yes	Yes
5	0.30	0.29	Yes	Yes
6	0.80	0.76	Yes	Yes
7	0.90	0.91	Yes	Yes
8	0.20	0.23	Yes	Yes
9	0	≈ 0	No	No
10	0.50	0.63	Yes	Yes
11	0.55	0.55	Yes	Yes
12	0	≈ 0	No	No
13	0.10	0.14	Yes	Yes
14	0.85	0.83	Yes	Yes
15	0.70	0.70	Yes	Yes
16	0	≈ 0	No	No
17	0	0.06	Yes	No
18	0.60	0.13	Yes	Yes
19	0.40	0.44	Yes	Yes
20	0.10	0.13	Yes	Yes
21	0.15	0.098	Yes	Yes
22	0.90	0.92	Yes	Yes
23	0.60	0.60	Yes	Yes
24	0.40	0.42	Yes	Yes
25	0.30	0.25	Yes	Yes

deviation relative to the maximum amplitude is applied. The second noise level simulates substantially noisy measurements. Random noise with normal distribution and 1.5% standard deviation is applied to the eigenvalues in this case. For the mode shapes, noise with 3% standard deviation relative to the maximum amplitude is applied.

Table 6 shows the detected and actual damage as well as the predicted stiffness reduction factors in the presence of noise. The stiffness reduction factors are predicted very closely except for the damaged element 18. This may be most probably due to the fact that a relatively complicated damage pattern is assumed for the

Table 7
Relative eigenfrequency differences and MAC values between modes obtained from the initial FEM and the damaged in the presence of moderate noise

Mode no.	Damaged + noise $\tilde{v}$ (Hz)	Undamaged v (Hz)	$\frac{(v-\tilde{v})}{\tilde{v}}$ (%)	MAC (%)
1	1.457	1.696	16.42	99.75
2	5.171	5.496	6.28	99.16
3	9.825	10.230	4.13	98.02
4	13.108	13.387	2.13	87.19

Table 8
Relative eigenfrequency differences and MAC values between modes obtained from the updated FEM and the damaged in the presence of moderate noise

Mode no.	Damaged + noise $\tilde{v}$ (Hz)	Updated v (Hz)	$\frac{(v-\tilde{v})}{\tilde{v}}$ (%)	MAC (%)
1	1.457	1.456	0.05	99.97
2	5.171	5.177	0.10	99.97
3	9.825	9.812	0.13	99.99
4	13.108	13.094	0.10	99.97

Table 9
Stiffness reduction factors predicted for the 24 damaged elements in the presence of substantial noise

Material no.	Damage + noise a	FEM update a	Detected damage (yes/no)	Actual damage (yes/no)
1	0	NA	NA	NA
2	0.65	0.22	Yes	Yes
3	0.50	0.49	Yes	Yes
4	0.40	0.47	Yes	Yes
5	0.30	0.31	Yes	Yes
6	0.80	0.70	Yes	Yes
7	0.90	0.93	Yes	Yes
8	0.20	0.33	Yes	Yes
9	0	0.07	Yes	No
10	0.50	0.70	Yes	Yes
11	0.55	0.66	Yes	Yes
12	0	≈ 0	No	No
13	0.10	0.21	Yes	Yes
14	0.85	0.82	Yes	Yes
15	0.70	0.67	Yes	Yes
16	0	≈ 0	No	No
17	0	0.25	Yes	No
18	0.60	0.75	Yes	Yes
19	0.40	0.49	Yes	Yes
20	0.10	0.42	Yes	Yes
21	0.15	0.37	Yes	Yes
22	0.90	0.94	Yes	Yes
23	0.60	0.52	Yes	Yes
24	0.40	0.57	Yes	Yes
25	0.30	0.064	Yes	Yes

Table 10
Relative eigenfrequency differences and MAC values between modes obtained from the initial FEM and the simulated experiment in the presence of substantial noise

Mode no.	Damaged + noise $\tilde{v}$ (Hz)	Undamaged v (Hz)	$\frac{(v-\tilde{v})}{\tilde{v}}$ (%)	MAC (%)
1	1.437	1.696	17.99	99.30
2	5.17	5.496	6.31	99.01
3	9.82	10.230	4.17	97.71
4	13.09	13.387	2.27	85.36

Table 11
Relative eigenfrequency differences and MAC values between modes obtained from the updated FEM and the simulated experiment in the presence of substantial noise

Mode no.	Damaged + noise $\tilde{v}$ (Hz)	Updated v (Hz)	$\frac{(v-\tilde{v})}{\tilde{v}}$ (%)	MAC (%)
1	1.437	1.451	0.94	99.51
2	5.170	5.138	−0.62	99.60
3	9.82	9.772	−0.50	99.84
4	13.09	12.95	−1.07	99.78

damage scenario. Most damaged elements are adjacent to each other and the adjacent elements' stiffness reduction factors are assumed to be substantially different. However, overall results are again quite accurate. Tables 7 and 8 show the relative eigenfrequency differences and the MAC values between the numerical and the simulated experimental modes in the presence of moderate noise. The results show that the relative differences in the eigenfrequencies and the MAC values are considerably improved after the FE model updating. The results show that the first three levels of damaged identification are again possible with the FE model updating scheme in the presence of moderate noise.

The second case considered is the substantial noise in measurements. Table 9 shows the stiffness reduction factors predicted. The results show that in the presence of substantial amount of noise, the model can under or overestimate the stiffness reduction factors, but 18 out of 24 damage parameters are predicted relatively accurately again even for the complex damage pattern adopted. Tables 10 and 11 also show that the MAC values as well as the relative eigenfrequency differences are improved substantially after FE model updating.

4. Conclusions

This study investigates the feasibility of the FE model updating method on a multistorey complex structure with a complex damage pattern in the absence and presence of noise. A sensitivity-based FE model updating scheme using constrained optimization with a trust region algorithm is applied on a numerical model of an actual residential building from Turkey that had been subjected to the 1999 Kocaeli and Duzce earthquakes. A damage scenario is applied on the reinforced concrete frame such that the stiffnesses of the elements close to the beam–column joints are deliberately decreased to simulate damage. A complex worst case damage scenario in which adjacent elements have substantially different stiffness levels is considered. The initial FE model is then updated to tune the initial modal parameters with the modal parameters from the numerical model in which the damage is simulated. The results of the updating showed that the first three levels of damage identification as proposed by Rytter; namely, damage detection, localization and quantification are successfully accomplished by the FE model updating algorithm used. The relative eigenfrequency differences and the MAC values are substantially improved after updating. The FE model updating algorithm is also tested in the presence of two noise levels which simulate moderate and substantial noise in measurements, respectively. In the presence of moderate noise levels, damage is successfully detected and located in all elements. With the exception of one element, all the 23 stiffness reduction factors are predicted quite accurately. The relative eigenfrequency differences and the MAC values are considerably improved after model updating. In the presence of high amount of noise, damage is detected and located correctly in all the damaged elements except for one element. The majority of the stiffness reduction factors (18 out of 24) are predicted quite accurately. The MAC values and the relative eigenfrequency differences are substantially improved after model updating. The results demonstrate that a sensitivity-based FE model updating using a trust region algorithm is promising for the detection of damaged elements in actual multistorey buildings.

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References

[1] D.L. Brown, A.J. Dalal, A.J. Helmicki, V.J. Hunt, A.E. Aktan, D.F. Farhey, S.J. Shelley, Condition assessment for bridge assessment, *Journal of Infrastructure Systems, ASCE* 2 (3) (1996) 108–117.

[2] H. Hao, J.M.W. Brownjohn, P.Q. Xia, Y. Xia, Civil structure condition assessment by FE model updating: methodology and case studies, *Journal of Sound and Vibration* 37 (2001) 761–775.

[3] B. Peeters, System Identification and Damage Detection in Civil Engineering, PhD Thesis, K.U. Leuven, Belgium, 2000.

[4] P.G. Bakir, H.M. Boduroglu, Earthquake risk and hazard mitigation in Turkey, *Earthquake Spectra* 18 (3) (2002) 427–447.

[5] P.G. Bakir, Proposal for a national mitigation strategy against earthquakes in Turkey, *Natural Hazards* 33 (3) (2004) 405–425.

[6] M.I. Friswell, J.E. Mottershead, *Finite Element Model Updating in Structural Dynamics*, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1995.

[7] S.W. Doebling, C.R. Farrar, M.B. Prime, D.W. Shevitz, Damage identification and health monitoring of structural and mechanical systems from changes in their vibration characteristics, Technical Report LA-13070-MS, Los Alamos National Laboratory, May 1996.

[8] H. Sohn, C.R. Farrar, F.M. Hemez, D.D. Shunk, D.W. Stinemates, B.R. Nadler, A review of structural health monitoring literature: 1996–2001, Technical Report Annex to SAMCO Summer Academy, Los Alamos National Laboratory, Cambridge, 2003.

[9] A. Alvandi, C. Cremona, Assessment of vibration-based damage identification techniques, *Journal of Sound and Vibration* 292 (2006) 179–202.

[10] C.R. Farrar, N.A.J. Lieven, Damage prognosis: the future of structural health monitoring, *Philosophical Transactions of the Royal Society A* 365 (2007) 623–632.

[11] S.V. Modak, T.K. Kundra, B.C. Nakra, Comparative study of model updating methods using simulated experimental data, *Computers and Structures* 80 (2002) 437–447.

[12] S. Kanev, F. Weber, M. Verhaegen, Experimental validation of a finite-element model updating procedure, *Journal of Sound and Vibration* 300 (2007) 394–413.

[13] A. Rytter, Vibration Based Inspection of Civil Engineering Structures, PhD Thesis, Department of Building Technology and Structural Engineering, Aalborg University, Denmark, 1993.

[14] O.S. Salawu, Detection of structural damage through changes in frequency: a review, *Engineering Structures* 19 (9) (1997) 718–723.

[15] J.M.W. Brownjohn, P.Q. Xia, H. Hao, Y. Xia, Civil structure condition assessment by FE model updating: methodology and case studies, *Finite Elements in Analysis and Design* 37 (2001) 761–775.

[16] J.R. Wu, Q.S. Li, Finite element model updating for a high-rise structure based on ambient vibration measurements, *Engineering Structures* 26 (2004) 979–990.

[17] D.A. Rade, G. Lallement, A strategy for the enrichment of the experimental data as applied to an inverse eigensensitivity-based fe model updating method, *Mechanical Systems and Signal Processing* 12 (2) (1998) 293–307.

[18] B. Jaishi, W.X. Ren, Structural finite element model updating using ambient vibration test results, *Journal of Structural Engineering, ASCE* 1231 (4) (2005) 617–628.

[19] B. Jaishi, W.X. Ren, Damage detection by finite element model updating using modal flexibility residual, *Journal of Sound and Vibration* 290 (2006) 369–387.

[20] J.F. Unger, A. Teughels, G. De Roeck, System identification and damage detection of a prestressed concrete beam, *Journal of Structural Engineering, ASCE* 132 (11) (2006) 1691–1698.

[21] A. Teughels, G. De Roeck, Damage detection and parameter identification by FE model updating, *Archives of Computational Methods in Engineering* 12 (2) (2005) 123–164.

[22] A. Teughels, G. De Roeck, Damage assessment of the Z24 bridge by FE model updating, *Key Engineering Materials* 245 (2) (2003) 19–26.

[23] A. Teughels, G. De Roeck, Structural damage identification of the highway bridge Z24 by FE model updating, *Journal of Sound and Vibration* 278 (3) (2004) 589–610.
[24] J. Nocedal, J. Wright, *Numerical Optimization*, Springer, New York, USA, 1999.
[25] A. Teughels, Inverse Modeling of Civil Engineering Structures Based on Operational Modal Data, PhD Thesis, K.U. Leuven, Belgium, 2003.
[26] E. Reynders, G. De Roeck, P. Gundes Bakir, C. Sauvage, Damage identification on the Tilff Bridge by vibration monitoring using optical fiber strain sensors, *Journal of Engineering Mechanics, ASCE* 133 (2) (2007) 185–193.
[27] J. Maeck, A. Teughels, G. De Roeck, Damage assessment by FE model updating using damage functions, *Computers and Structures* 80 (2002) 1869–1879.
[28] H. Crowley, J.J. Bommer, R. Pinho, J. Bird, The impact of epistemic uncertainty on an earthquake loss model, *Earthquake Engineering and Structural Dynamics* 14 (2005) 1653–1685.
[29] AnatolianQuake.Org. http://bridge.ecn.purdue.edu/ anatolia/.
[30] A. D'Ambrisi, F.C. Filippou, A. Issa, Nonlinear static and dynamic analysis of reinforced concrete subassemblages, Report No. UCB/EERC-92/08, University of California, Berkeley, 1992.
[31] R.J. Allemang, D.L. Brown, A correlation coefficient for modal vector analysis, *Proceedings of IMAC I: 1st International Modal Analysis Conference*, Orlando, Florida, 1982, pp. 110–116.