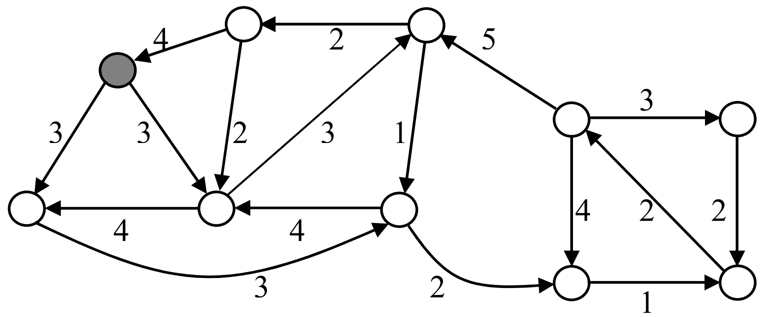
GRAPH THEORY AND APPLICATIONS **November 16, 2007**

1. Find a directed minimum spanning tree for the following graph. Choose gray colored vertex as the root.



2. Give an example to show that if P is a (u,v) path in a 2 connected graph G, then G does not necessarily contain a (u,v) path Q internally disjoint from P.

3.a. How would you create different graphs of the same partition?
b. Draw 2 non-isomorphic graphs for the following partition.
4, 4, 3, 3, 2, 2

4.Use the following theorem (h.6 in slides) to show that the graph on right is Hamiltonian.

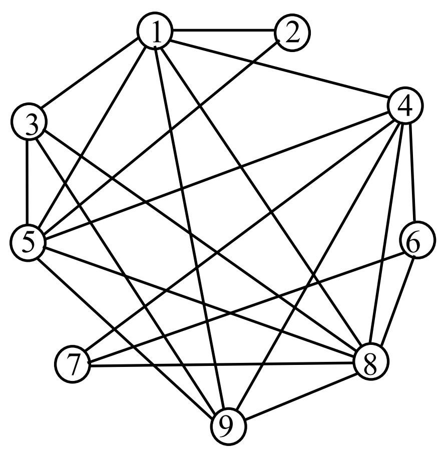
Theorem: Let G be a simple graph with degree sequence ($d_1, d_2, \ldots, d_n$), where:

$d_1 \leq d_2 \leq \ldots \leq d_n$
$n \geq 3$

Suppose that there is no value of m less than $n/2$ for which:

$d_m \leq m$ and
$d_{n-m} < n - m$

Then, G is Hamiltonian.



5. Short questions:
a. What is the thickness of $K_{3,5}$? Draw the planar subgraphs, whose union is $K_{3,5}$.
b. Can you find an Euler trail on the graph of question 4? Why?
c. What are the radius and diameter of the graph of question 1, assuming it is undirected? How do you find them?
d. What is the minimum number of edges of a 3-connected graph with 9 vertices. Draw this graph.
e. Is it possible to embed K_9 on a surface of genus 2? Explain.

Duration: 90 min.
Points: 1: 18pts, 2: 12pts, 3: 18pts, 4: 18pts, 5: (8+6+6+7+7)pts.