Chapter 3

Local Energy (Head) Losses

3.1. Introduction

Local head losses occur in the pipes when there is a change in the area of the cross-section of the pipe (enlargement, contraction), a change of the direction of the flow (bends), and application of some devices on the pipe (vanes). Local head losses are also named as **minor losses**. Minor losses can be neglected for long pipe systems.

Minor losses happen when the magnitude or the direction of the velocity of the flow changes. In some cases the magnitude and the direction of the velocity of the flow may change simultaneously. Minor losses are proportional with the velocity head of the flow and is defined by,

$$h_L = \xi \frac{V^2}{2g} \qquad (3.1)$$

Where ξ is the **loss coefficient**.

3.2. Abrupt Enlargement

There is certain amount of head loss when the fluid moves through a **sudden enlargement** in a pipe system such as that shown in Fig. (3.1)

Impuls-momentum equation is applied to the 1221 control volume along the horizontal flow direction. The forces acting on to the control volume are,

a) Pressure force on the 1-1 cross-section, $\vec{F}_1 = p_1 A_1$

b) Pressure force on the 2-2 cross-section, $\overleftarrow{F}_2 = p_2 A_2$

c) Pressure force on the 3-3 cross-section, $\vec{F}_3 = p_3 \left(A_2 - A_1\right)$

Laboratory experiments have verified that $p_3 = p_1$, $\vec{F}_3 = p_1 \left(A_2 - A_1\right)$

d) Momentum on the 1-1 cross-section, $\vec{M}_1 = \rho Q V_1$

e) Momentum on the 2-2 cross-section, $\vec{M}_2 = \rho Q V_2$

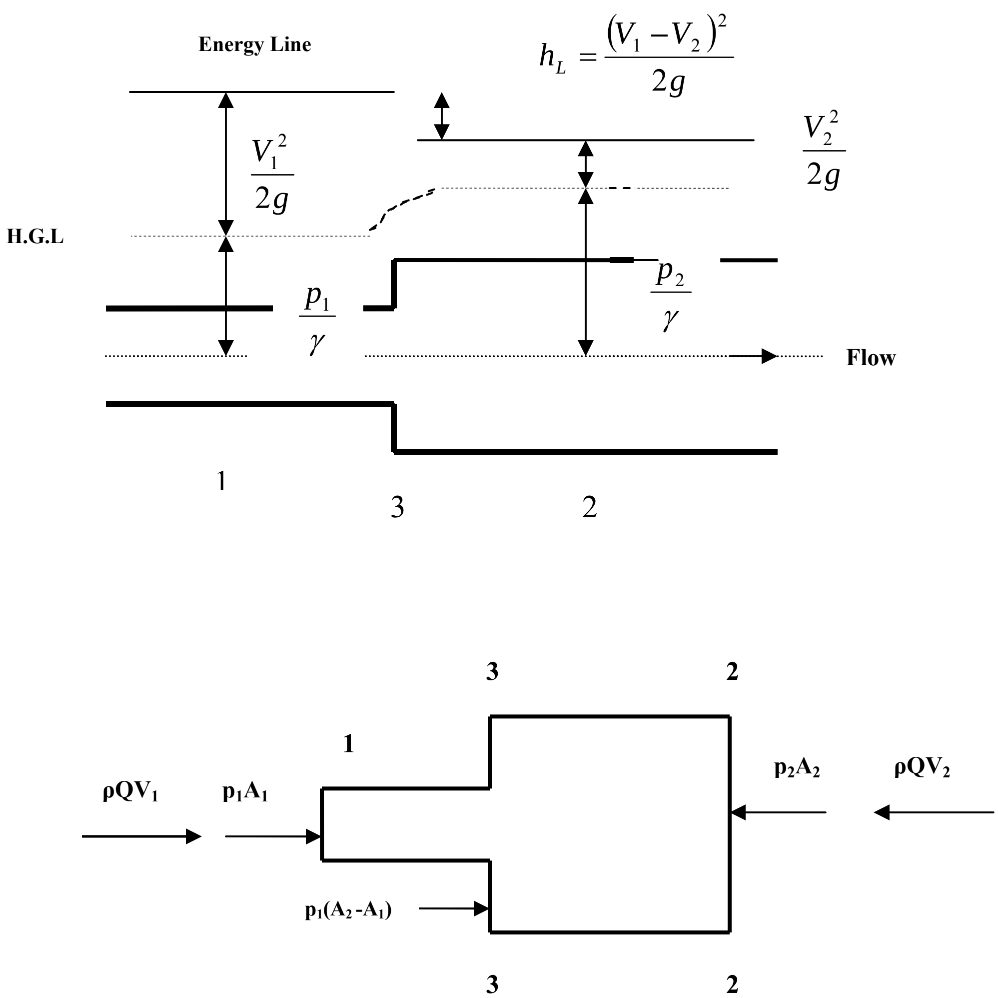


Fig. 3.1.

Neglecting the shearing force on the surface of the control volume,

$$\rho Q V_1 + p_1 A_1 + p_1 (A_2 - A_1) - p_2 A_2 - \rho Q V_2 = 0$$

$$(p_1 - p_2) A_2 = \rho Q (V_2 - V_1)$$

$$\rho = \frac{\gamma}{g}$$

$$\frac{p_1 - p_2}{\gamma} = \frac{Q}{g A_2} (V_2 - V_1)$$

Applying the continuity equation,

$$Q = V_1 A_1 = V_2 A_2$$

$$V_2 = V_1 \frac{A_1}{A_2}$$

Substituting this,

$$\frac{p_1 - p_2}{\gamma} = \frac{1}{gA_2}\left(V_1 A_1 V_1 \frac{A_1}{A_2} - V_1 A_1 V_1 \right)$$

$$\frac{p_1 - p_2}{\gamma} = \frac{V_1^2}{g}\left[\left(\frac{A_1}{A_2} \right)^2 - \frac{A_1}{A_2} \right] \qquad (3.2)$$

Applying Bernoulli equation between cross-sections 1-2 for the horizontal pipe flow,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + h_L$$

$$\frac{p_1 - p_2}{\gamma} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} + h_L$$

Substituting Equ. (3.2),

$$\frac{V_1^2}{2g}\left[\left(\frac{A_1}{A_2} \right)^2 - \frac{A_1}{A_2} \right] = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} + h_L$$

$$V_1 A_1 = V_2 A_2 \rightarrow \frac{A_1}{A_2} = \frac{V_2}{V_1}$$

$$h_L = \frac{V_1^2}{2g} - \frac{V_2^2}{2g} + \frac{V_1^2}{g}\left[\left(\frac{V_2}{V_1} \right)^2 - \frac{V_2}{V_1} \right]$$

$$h_L = \frac{V_1^2}{2g} - \frac{V_2^2}{2g} + \frac{V_2^2}{g} - \frac{V_1 V_2}{g}$$

$$h_L = \frac{V_1^2}{2g} - \frac{V_1 V_2}{g} + \frac{V_2^2}{2g}$$

$$h_L = \frac{(V_1 - V_2)^2}{2g} \qquad (3.3)$$

This equation is known as **Borda-Carnot** equation. The **loss coefficient** can be derived as,

$$h_L = \frac{V_1^2}{2g}\left(1 - 2\frac{V_2}{V_1} + \frac{V_2^2}{V_1^2}\right)$$

$$h_L = \frac{V_1^2}{2g}\left(1 - 2\frac{A_1}{A_2} + \frac{A_1^2}{A_2^2}\right)$$

$$h_L = \frac{V_1^2}{2g}\left(1 - \frac{A_1}{A_2}\right)^2 \qquad (3.4)$$

The loss coefficient of Equ. (3.1) is then,

$$\xi = \left(1 - \frac{A_1}{A_2}\right)^2 \qquad (3.5)$$

The loss coefficient ξ is a function of the geometry of the pipe and is not dependent on to the flow characteristics.

Special case: Connecting to a reservoir,

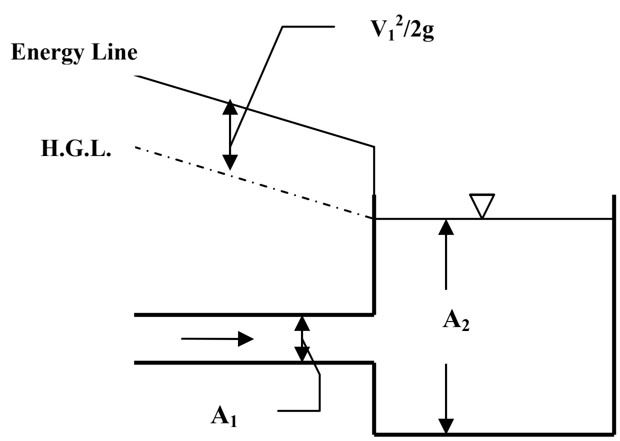


Fig. 3.2.

Since the reservoir cross-sectional area is too big comparing to the area pipe cross-section, $A_2 >> A_1$, $\frac{A_1}{A_2} \cong 0$, the loss coefficient will be equal to zero, ζ=0. The head loss is then,

$$h_L = \frac{V_1^2}{2g} \qquad (3.6)$$

3.3. Abrupt Contraction

Flow through an **abrupt contraction** is shown in Fig. 3.3.

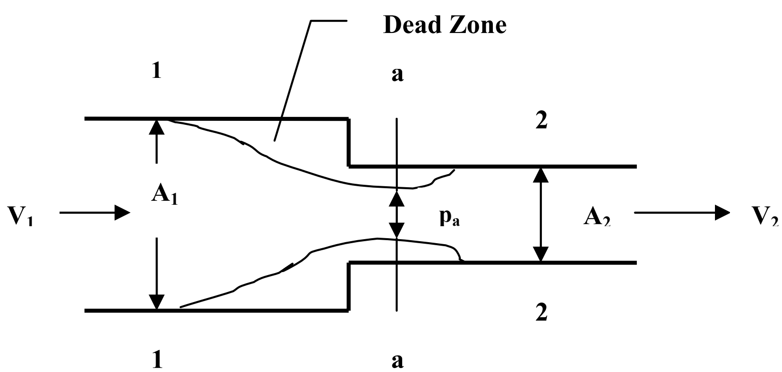


Fig. 3.3

Derivation of the ζ loss coefficient may be done by following these steps.

a) Bernoulli equation between 1, C, and 2 cross-sections by neglecting the head loss between 1 and C cross-sections is for the horizontal pipe system,

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_C}{\gamma} + \frac{V_C^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + h_L$$

b) Continuity equation of the system,

$$Q = V_1 A_1 = V_C A_C = V_2 A_2$$

c) Impuls-Momentum equation between C and 2 cross-sections,

$$\left(p_C - p_2\right)A_2 = \rho Q\left(V_2 - V_C\right)$$

Solving these 3 equations gives us head loss equation as,

$$h_L = \frac{\left(V_C - V_2\right)^2}{2g}$$
$$h_L = \left(\frac{A_2}{A_C} - 1\right)^2 \frac{V_2^2}{2g} \qquad (3.7)$$

Where,

$$C_C = \frac{A_C}{A_2} = \textbf{Coefficient of contraction} \qquad (3.8)$$

The loss coefficient for abrupt contraction is then,

$$\xi = \left(\frac{1}{C_C} - 1\right)^2 \qquad (3.9)$$

$$h_L = \xi \frac{V_2^2}{2g}$$

C_C contraction coefficients, and ζ loss coefficients have determined by laboratory experiments depending on the A_2/A_1 ratios and given in Table 3.1.

Table 3.1.

A_2/A_1	0	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
C_C	0.62	0.62	0.63	0.64	0.66	0.68	0.71	0.76	0.81	0.89	1.00
ζ	0.50	0.46	0.41	0.36	0.30	0.24	0.18	0.12	0.06	0.02	0

Example: At an abrupt contraction in a pressured pipe flow, Q = 1 m^3/sec, D_1 = 0.80 m, D_2 = 0.40 m. Calculate the local head when contraction coefficient is C_C = 0.60.

Solution: Since discharge has been given,

$$V_2 = \frac{4Q}{\pi D_2^2} = \frac{4\times 1}{\pi\times 0.40^2} = 7.96\, m/\sec$$
$$A_2 = \frac{\pi D_2^2}{4} = \frac{\pi\times 0.40^2}{4} = 0.1257 m^2$$
$$A_C = C_c A_2 = 0.60\times 0.1257 = 0.0754 m^2$$
$$V_C = \frac{Q}{A_C} = \frac{1}{0.0754} = 13.26\, m/\sec$$

$$h_L = \frac{(V_C - V_2)^2}{2g} = \frac{(13.26-7.96)^2}{19.62} = 1.43 m$$

Or,

$$h_L = \left(\frac{1}{C_C} - 1\right)^2 \times \frac{V_2^2}{19.62}$$
$$h_L = \left(\frac{1}{0.60} - 1\right)^2 \times \frac{7.96^2}{19.62} = 1.44 m$$

Special case: Pipe entrance from a reservoir,

Loss coefficient ζ depends on the geometry of the pipe connection to the reservoir. The loss coefficient ζ is 0.50 for square-edge entrance. Fig. (3.4)

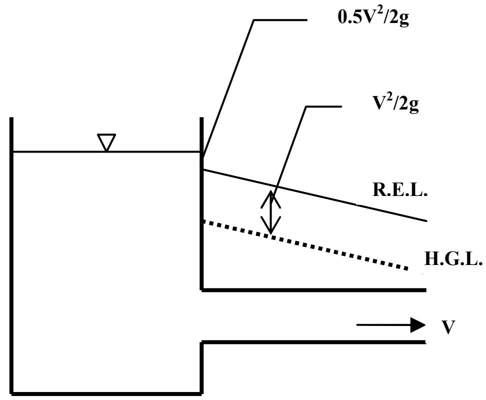


Fig. 3.4

$$h_L = 0.5\frac{V^2}{2g} \qquad (3.10)$$

The head loss at the pipe fittings, bends and vanes is calculated with the general local loss equation (Equ. 3.1). The loss coefficients can be taken from the tables prepared for the bends and vanes.

Example 3.1. Two reservoirs is connected with a pipe length L. What will the elevation difference between the water surfaces of the reservoir? Draw the energy and hydraulic grade line of the system.

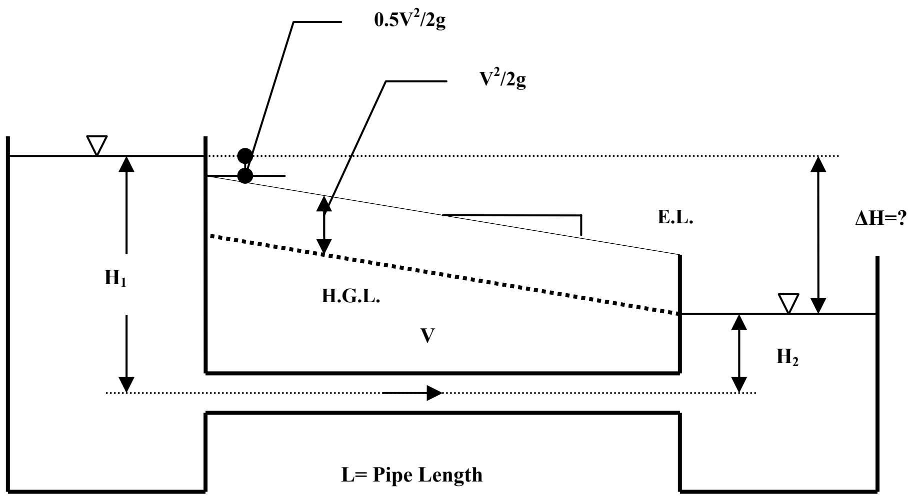


Fig. 3.5

Solution: Bernoulli equation between reservoir (1) and (2),

$$z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + h_L$$

Since the velocities in the reservoirs are taken as zero, $V_1 = V_2 = 0$, and the pressure on the surface of reservoirs is equal to the atmospheric pressure,

$$z_1 - z_2 = h_L$$

The head losses along the pipe system are,

1) Local loss to the entrance of the pipe, $h_{L_1} = 0.5\frac{V^2}{2g}$
2) Head loss along the pipe, $h_{L_2} = JL = \frac{f}{D}\frac{V^2}{2g}L$
3) Local loss at the entrance to the reservoir, $h_{L_3} = \frac{V^2}{2g}$

The total loss is then will be equal to the elevation difference between the reservoir levels,

$$z_1 - z_2 = h_L$$

$$z_1 - z_2 = 0.5\frac{V^2}{2g} + \frac{f}{D}\frac{V^2}{2g}L + \frac{V^2}{2g}$$

$$z_1 - z_2 = 1.5\frac{V^2}{2g} + \frac{f}{D}\frac{V^2}{2g}L$$

Numerical Application: If the physical characteristics of the system are given as below, what will be difference between the reservoir levels? Calculate the ratio of local losses to the total head loss and give a physical explanation of this ratio.

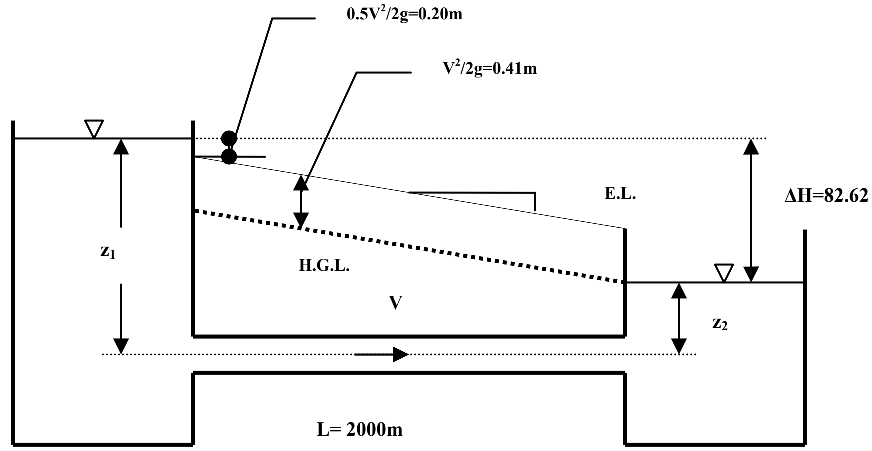


Figure 3.6

Q = 0.2 m^3/sec, f = 0.02, L = 2000 m, D = 0.30 m

The velocity in the pipe,

$$V = \frac{4Q}{\pi D^2} = \frac{4 \times 0.2}{\pi \times 0.3^2}$$
$$V = 2.83\, m/\sec$$

The velocity head,

$$\frac{V^2}{2g} = \frac{2.83^2}{19.62} = 0.41m$$

The elevation difference is then,

$$z_1 - z_2 = 1.5\frac{V^2}{2g} + \frac{f}{D}\frac{V^2}{2g}L$$
$$z_1 - z_2 = 1.5 \times 0.41 + 0.41 \times \frac{0.02}{0.2} \times 2000$$
$$z_1 - z_2 = 0.62 + 82.0$$
$$z_1 - z_2 = 82.62m$$

The ratio of local losses to the total head loss is,

$$\frac{0.62}{82.62} = 0.0075$$

As can be seen from the result, the ratio is less than 1%. Therefore, local losses are generally neglected for long pipes.

3.4 Conduit Systems

The other applications which are commonly seen in practical applications for conduit systems are,

a) **The Three-Reservoir Problems,**
b) **Parallel Pipes,**
c) **Branching Pipes,**
d) **Pump Systems.**

3.4.1 The Three-Reservoir Problems

Consider the case where three reservoirs are connected by a branched-pipe system. The problem is to determine the discharge in each pipe and the head at the junction point D. There are four unknowns (V_{AD}, V_{BC}, V_{DC} and p_D/γ), and the solution is obtained by solving the energy equations for the pipes (neglecting velocity heads and including only pipe losses and not the minor losses) and the continuity equation. The physical characteristics of the system such as lengths, diameters and f friction factors of the pipes, the geometric elevations of the water surfaces at the reservoirs and the piezometric head at the junction D are given. The reservoirs are located as, $z_A > z_B > z_C$. There are three possible solutions of this problem,

a) $\left(z + \frac{p}{\gamma}\right)_D = z_A$

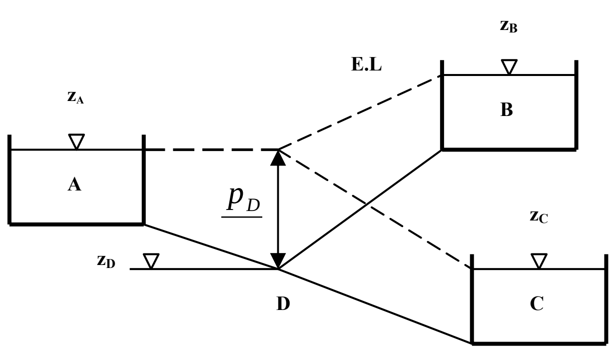


Figure 3.7

Since $\left(z+\frac{p}{\gamma}\right)_D = z_A$, there will be no flow from or to reservoir A, and therefore,

$$Q_{AD} = 0, V_{AD} = 0, J_{AD} = 0$$

$$Q_{BD} = Q_{DC}$$

The relations between the reservoir levels B, and D and the piezometric head at the junction D are,

$$z_B - h_{L_{BD}} = z_D + \frac{p_D}{\gamma} = z_C + h_{L_{CD}}$$

Energy line slope (J_{BD}) of the BD pipe can be found as,

$$J_{BD} = \frac{z_B - z_D - \frac{p_D}{\gamma}}{L_{BD}} = \frac{z_B - z_A}{L_{BD}} = \frac{h_{L_{BD}}}{L_{BD}} \qquad (3.11)$$

Using the Darcy-Weisbach equation,

$$J = \frac{fV^2}{D2g}$$

The flow velocity in the BD pipe can be calculated by,

$$V_{BD} = \left(\frac{2gD_{BD}J_{BD}}{f_{BD}}\right)^{1/2} \qquad (3.12)$$

And the discharge is,

$$Q_{BD} = A_{BD}V_{BD}$$

The energy line slope, the velocity and the discharge of the CD pipe are then,

$$J_{CD} = \frac{z_A - z_C}{L_{CD}} = \frac{h_{L_{CD}}}{L_{CD}}$$

$$V_{CD} = \left(\frac{2gD_{CD}J_{CD}}{f_{CD}}\right)^{1/2}$$

$$Q_{CD} = A_{CD}V_{Cd} = Q_{BD}$$

b) $z_C < z_A < \left(z + \frac{p}{\gamma}\right)_D < z_B$

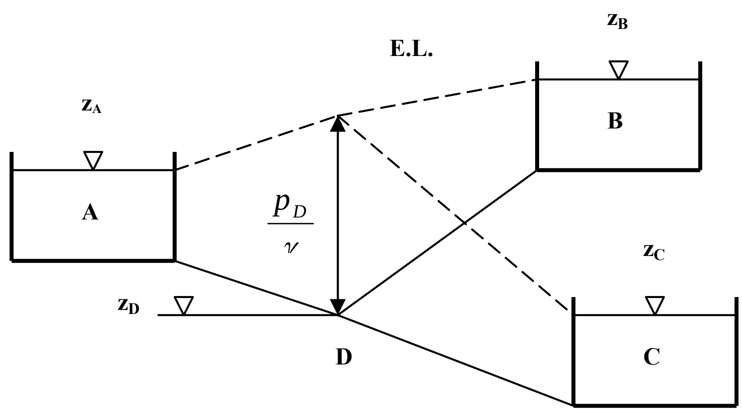


Figure 3.7

The continuity equation for this case is,

$$Q_{Bd} = Q_{AD} + Q_{DC}$$

The relations between the reservoir levels B, A, and C and the piezometric head at the junction D are,

$$z_B - h_{L_{BD}} = \left(z + \frac{p}{\gamma}\right)_D = z_A + h_{L_{AD}} = z_C + h_{L_{CD}}$$

The energy line slopes of the pipes are,

$$J_{BD} = \frac{z_B - \left(z + \frac{p}{\gamma}\right)_D}{L_{BD}} = \frac{h_{L_{BD}}}{L_{BD}}$$

$$J_{AD} = \frac{\left(z + \frac{p}{\gamma}\right)_D - z_A}{L_{AD}} = \frac{h_{L_{AD}}}{L_{AD}}$$

$$J_{CD} = \frac{\left(z + \frac{p}{\gamma}\right)_D - z_C}{L_{CD}} = \frac{h_{L_{CD}}}{L_{CD}}$$

c) $z_C < \left(z + \frac{p}{\gamma}\right)_D < z_A < z_B$

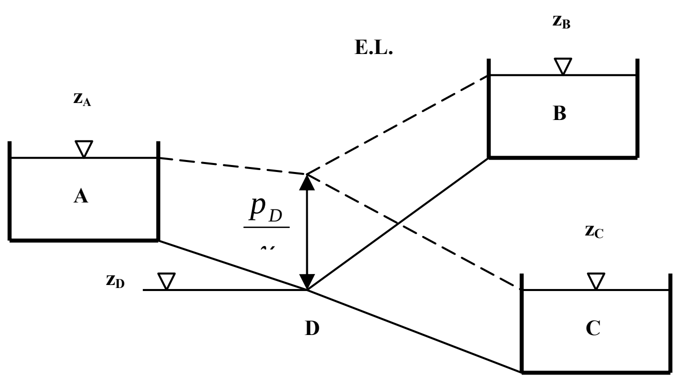


Figure 3.8

The continuity equation is,

$$Q_{BD} + Q_{AD} = Q_{CD}$$

The relations between the reservoir levels B, A, and C and the piezometric head at the junction D are,

$$z_B - h_{L_{BD}} = \left(z + \frac{p}{\gamma}\right)_D = z_A - h_{L_{AD}} = z_C + h_{L_{CD}}$$

The energy line slopes of the pipes are,

$$J_{BD} = \frac{z_B - \left(z + \frac{p}{\gamma}\right)_D}{L_{BD}} = \frac{h_{L_{BD}}}{L_{BD}}$$

$$J_{AD} = \frac{z_A - \left(z + \frac{p}{\gamma}\right)_D}{L_{AD}} = \frac{h_{L_{AD}}}{L_{AD}}$$

$$J_{CD} = \frac{\left(z + \frac{p}{\gamma}\right)_D - z_C}{L_{CD}} = \frac{h_{L_{CD}}}{L_{CD}}$$

Example: The water surface levels at the A and C reservoirs are respectively z_A=100 m and z_C= 70 for the given three-reservoir system. Reservoirs A and B are feeding reservoir C and $Q_A = Q_B$. Calculate the water surface level of the reservoir B. Draw the energy line of the system. The physical characteristics of the pipes are,

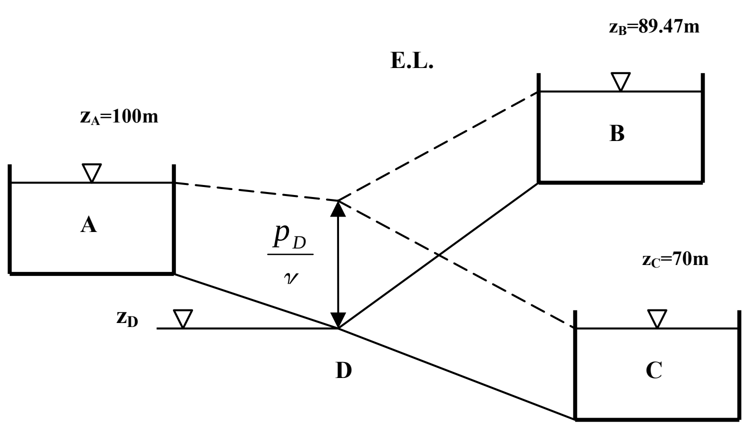


Figure 3.9

Pipe	Length (m)	Diameter (mm)	f
AD	500	150	0.03
BD	1000	200	0.02
DC	1500	250	0.03

Solution: The head loss along the pipes 1 and 3 is,

$$h_{L_1} + h_{L_3} = z_A - z_C = 100 - 70 = 30m \quad (1)$$

Since,

$$Q_1 = Q_2 = Q$$
$$Q_3 = 2Q$$

Using the Darcy-Weisbach equation for the head loss,

$$h_L = \frac{fV^2 L}{2gD} = \frac{fL}{2g} \times \left(\frac{4Q}{\pi D^2} \right)^2$$
$$h_L = \frac{8f}{g\pi^2} \times \frac{LQ^2}{D^5}$$

Using Equ. (1),

$$h_L = \frac{8}{g\pi^2} \left(\frac{f_1 L_1 Q_1^2}{D_1^5} + \frac{f_2 L_2 Q_2^2}{D_2^5} \right)$$
$$30 = \frac{8 \times 0.03}{9.81 \times \pi^2} \left[\frac{500 \times Q^2}{0.15^5} + \frac{1500 \times (2Q)^2}{0.25^5} \right]$$
$$30 = 31566 Q^2$$
$$Q = 0.0308\, m^3/\sec = 30.8\, lt/\sec$$

$$Q_1 = 0.0308\, m^3/\sec$$
$$V_1 = \frac{4Q_1}{\pi D_1^2} = \frac{4 \times 0.0308}{\pi \times 0.15^2} = 1.74\, m/\sec$$

$$Q_3 = 2Q_1 = 2 \times 0.0308 = 0.0616\, m^3/\sec$$
$$V_3 = \frac{4Q_3}{\pi D_3^2} = \frac{4 \times 0.0616}{\pi \times 0.25^2} = 1.26\, m/\sec$$

Head losses along the pipes are,

$$h_{L_1} = \frac{f_1 V_1^2 L_1}{2gD_1} = \frac{0.03 \times 1.74^2 \times 500}{19.62 \times 0.15} = 15.43m$$

$$h_{L_3} = \frac{f_3 V_3^2 L_3}{2gD_3} = \frac{0.03 \times 1.26^2 \times 1500}{19.62 \times 0.25} = 14.57m$$

$$h_{l_1} + h_{L_3} = 15.43 + 14.57 = 30.00m$$

For the pipe 2,

$$Q_1 = Q_2 = 0.0308\, m^3/\sec$$

$$V_2 = \frac{4Q_2}{\pi D_2^2} = \frac{4 \times 0.0308}{\pi \times 0.20^2} = 0.98\, m/\sec$$

$$h_{L_2} = \frac{f_2 V_2^2 L_2}{2gD_2} = \frac{0.02 \times 0.98^2 \times 1000}{19.62 \times 0.20} = 4.90m$$

Water surface level of the reservoir B is,

$$\left(z + \frac{p}{\gamma}\right)_D = z_A - h_{L_1} = 100.00 - 15.43 = 84.57m$$

$$\left(z + \frac{p}{\gamma}\right)_D = z_C + h_{L_3} = 70.00 + 14.57 = 84.57m$$

$$z_B = \left(z + \frac{p}{\gamma}\right)_D + h_{L_2} = 84.57 + 4.90 = 89.47m$$

3.4.2 Parallel Pipes

Consider a pipe that branches into two parallel and then rejoins as in Fig….. . In pipeline practice, looping or laying a pipeline parallel to an existing pipeline is a standard method of increasing the capacity of the line. A problem involving this configuration might be to determine the division of flow in each pipe given the total flow rate. In such problems, velocity heads, and minor losses are usually neglected, and calculations are made on the basis of coincident energy and hydraulic grade lines.

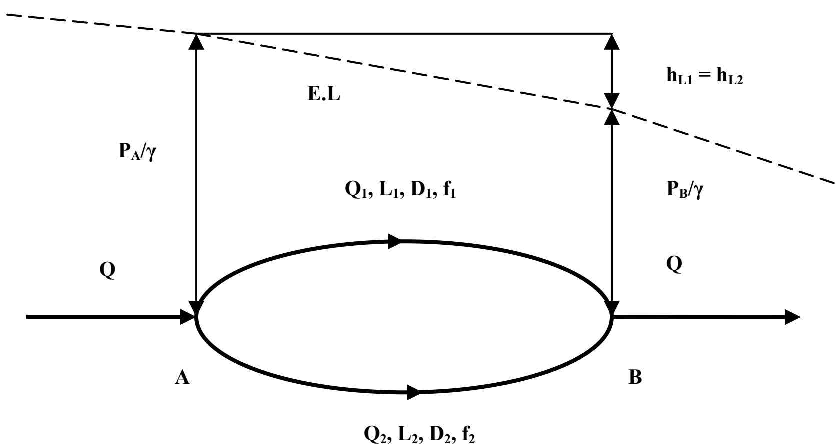


Figure 3.10

Evidently, the distribution of flow in the branches must be such that the same head loss occurs in each branch; if this were not so there would be more than one energy line for the pipe upstream and downstream from the junctions.

Application of the continuity principle shows that the discharge in the mainline is equal to the sum of the discharges in the branches. Thus the following simultaneous equations may be written.

$$h_{L_1} = h_{L_2}$$
$$Q = Q_1 + Q_2 \qquad (3.13)$$

Using the equality of head losses,

$$\frac{f_1 L_1 V_1^2}{D_1 2g} = \frac{f_2 L_2 V_2^2}{D_2 2g}$$

$$\frac{V_1}{V_2} = \left(\frac{f_2 L_2 D_1}{f_1 L_1 D_2} \right)^{1/2} \qquad (3.14)$$

If f_1 and f_2 are known, the division of flow can be easily determined.

Expressing the head losses in terms of discharge through the Darcy-Weisbach equation,

$$h_L \frac{fV^2 L}{D2g} = \frac{fL}{2gD} \times \frac{16Q^2}{\pi^2 D^4}$$

$$h_L = \left(\frac{16 fL}{2\pi^2 gD^5} \right) \times Q^2 \qquad (3.14)$$

$$h_L = KQ^2$$

Substituting this to the simultaneous Equs (3.12),

$$K_1 Q_1^2 = K_2 Q_2^2 \qquad (3.15)$$

$$Q = Q_1 + Q_2$$

Solution of these simultaneous equations allows prediction of the division of a discharge Q into discharges Q_1 and Q_2 when the pipe characteristics are known. Application of these principles allows prediction of the increased discharge obtainable by looping an existing pipeline.

Example: A 300 mm pipeline 1500 m long is laid between two reservoirs having a difference of surface elevation of 24. The maximum discharge obtainable through this line is $0.15 m^3$/sec. When this pipe is looped with a 600 m pipe of the same size and material laid parallel and connected to it, what increase of discharge may be expected?

Solution: For the single pipe, using Equ. (3.14),

$$h_L = KQ^2$$

$$24 = K \times 0.15^2$$

$$K = 1067$$

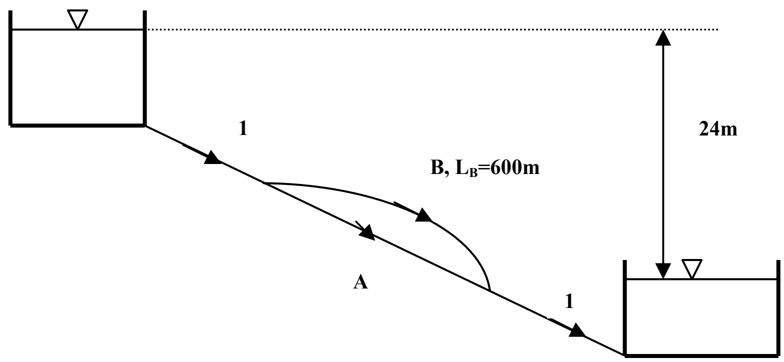


Figure 3.11

K for the looped section is,

$$K_A = \frac{L_2}{L} K = \frac{600}{1500} \times 1067$$
$$K_A = 427$$

For the unlooped section,

$$K' = \frac{L - L_2}{L} K$$
$$K' = \frac{1500 - 600}{1500} \times 1067$$
$$K' = 640$$

For the looped pipeline, the headloss is computed first in the common section plus branch A as,

$$h_L = K'Q^2 + K_A Q_A^2$$
$$24 = 640 \times Q^2 + 427 \times Q_A^2 \qquad (1)$$

And then in the common section plus branch B as,

$$24 = 640 \times Q^2 + 427 \times Q_B^2 \qquad (2)$$

In which Q_A and Q_B are the discharges in the parallel branches. Solving these by eliminating Q shows that $Q_A=Q_B$ (which is expected to be from symmetry in this problem). Since, from continuity,

$$Q = Q_A + Q_B$$
$$Q_A = \frac{Q}{2}$$

Substituting this in the Equ (1) yields,

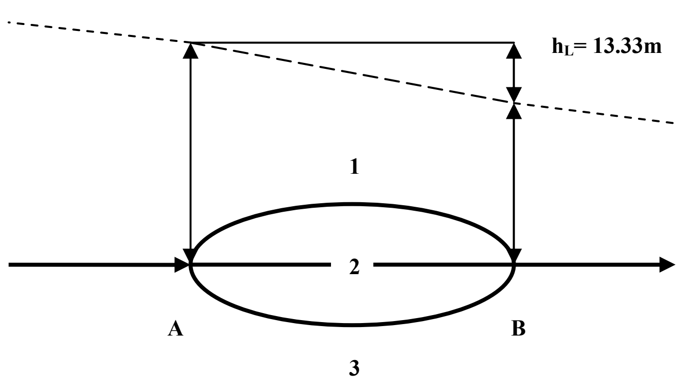
$$24 = 640 \times (2Q_A)^2 + 427 \times Q_A^2$$
$$24 = 2987 Q_A^2$$
$$Q_A = 0.09\, m^3/\sec$$
$$Q = 0.18\, m^3/\sec$$

Thus the gain of discharge capacity by looping the pipe is 0.03 m^3/sec or 20%.

Example: Three pipes have been connected between the points A and B. Total discharge between A and B is 200 lt/sec. Physical characteristics of the pipes are,

$$f_1 = f_2 = f_3 = 0.02$$
$$L_1 = L_2 = L_3 = 1000m$$
$$D_1 = 0.10m, D_2 = 0.20m, D_3 = 0.30m$$

Calculate the discharges of each pipe and calculate the head loss between the points A and B. Draw the energy line of the system.



Figure

Solution: Head losses through the three pipes should be the same.

$$h_{L_1} = h_{L_2} = h_{L_3}$$

$$J_1 L_1 = J_2 L_2 = J_3 L_3$$

Since the pipe lengths are the same for the three pipes,

$$J_1 = J_2 = J_3$$

$$J = \frac{f}{D} \times \frac{V^2}{2g} = \frac{f}{D} \times \frac{1}{2g} \times \frac{16Q^2}{\pi^2 D^4} = \frac{8f}{g\pi^2} \times \frac{Q^2}{D^5}$$

And also roughness coefficients f are the same for three pipe, the above equation takes the form of,

$$\frac{Q_1^2}{D_1^5} = \frac{Q_2^2}{D_2^5} = \frac{Q_3^2}{D_3^5}$$

Using this equation,

$$\frac{Q_1}{Q_2} = \left(\frac{D_1}{D_2}\right)^{5/2} = \left(\frac{0.10}{0.20}\right)^{2.5} = 0.177$$

$$Q_1 = 0.177 Q_2$$

$$\frac{Q_3}{Q_2} = \left(\frac{D_3}{D_2}\right)^{5/2} = \left(\frac{0.30}{0.20}\right)^{2.5} = 2.76$$

$$Q_3 = 2.76 Q_2$$

Substituting these values to the continuity equation,

$$Q_1 + Q_2 + Q_3 = 0.200$$

$$0.177 Q_2 + Q_2 + 2.76 Q_2 = 0.200$$

$$3.937 Q_2 = 0.200$$

$$Q_2 = 0.051\, m^3/\text{sec}$$

$$Q_1 = 0.177 \times 0.051 = 0.009\, m^3/\text{sec}$$

$$Q_3 = 2.76 \times 0.051 = 0.140\, m^3/\text{sec}$$

$$Q_1 + Q_2 + Q_3 = 0.009 + 0.051 + 0.140 = 0.200\, m^3/\text{sec}$$

Calculated discharges for the parallel pipes satisfy the continuity equation.

The head loss for the parallel pipes is,

$$h_{L_{AB}} = \frac{8f}{g\pi^2} \times \frac{Q_3^2}{D_3^5} \times L_3$$

$$h_{L_{AB}} = \frac{8 \times 0.02}{9.81 \times \pi^2} \times \frac{0.14^2}{0.30^5} \times 1000 = 13.33m$$

3.4.2 Branching Pipes

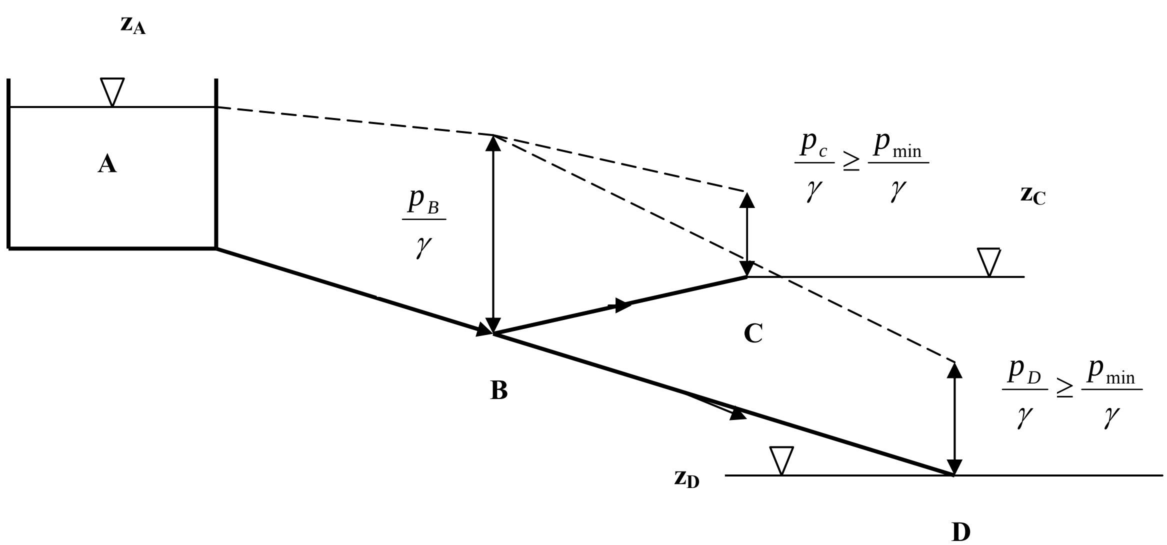


Figure 3.12

Consider the reservoir A supplying water with branching pipes BC and BD to a city. Pressure heads (p/γ) are required to be over 25 m to supply water to the top floors in multistory buildings.

$$\frac{p_C}{\gamma} \geq 25m, \frac{p_D}{\gamma} \geq 25m$$

Using the continuity equation,

$$Q_{AB} = Q_{BC} + Q_{BD}$$

Two possible problems may arise for these kinds of pipeline systems;

a) The physical characteristics of the system such as the lengths, diameters and friction factors, and also the discharges of each pipe are given. Water surface level in the reservoir is searched to supply the required $(p/\gamma)_{min}$ pressure head at points C and D. The problem can be solved by following these steps,

1) Velocities in the pipes are calculated using the given discharges and diameters.

$$V = \frac{Q}{A} = \frac{4Q}{\pi D^2}$$

2) Head losses for the BC and BD pipes are calculated.

$$h_L = \frac{f}{D} \times \frac{V^2}{2g} \times L$$

Piezometric head at junction B is calculated by using the given geometric elevations of the points C and D.

$$\left(z + \frac{p}{\gamma}\right)_B = \left(\frac{p}{\gamma}\right)_{min} + z_C - h_{L_{BC}} \qquad (1)$$

$$\left(z + \frac{p}{\gamma}\right)_B = \left(\frac{p}{\gamma}\right)_{min} + z_D - h_{L_{BD}} \qquad (2)$$

Since there should be only one piezometric head $(z+p/\gamma)$ at any junction, the largest value obtained from the equations (1) and (2) is taken as the piezometric head for the junction B. Therefore the minimum pressure head requirement for the points C and D has been achieved.

3) The minimum water surface level at the reservoir to supply the required pressure head at points C and D is calculated by taking the chosen piezometric head for the junction,

$$z_A = \left(z + \frac{p}{\gamma}\right)_B + h_{L_{AB}}$$

b) The physical characteristics and the discharges of the pipes are given. The geometric elevations of the water surface of the reservoir and at points C and D are also known. The minimum pressure head requirement will be checked,

1) V_{AB} , V_{BC} and V_{BD} velocities in the pipes are calculated.

2) Head losses along the pipes AB, BC and BD are calculated.

3) Piezometric heads at points B, C and D are calculated.

$$\left(z+\frac{p}{\gamma}\right)_B = z_A - h_{L_{AB}}$$

$$\left(z+\frac{p}{\gamma}\right)_C = \left(z+\frac{p}{\gamma}\right)_B - h_{L_{BC}} \geq \left(\frac{p}{\gamma}\right)_{\min}$$

$$\left(z+\frac{p}{\gamma}\right)_D = \left(z+\frac{p}{\gamma}\right)_B - h_{L_{BD}} \geq \left(\frac{p}{\gamma}\right)_{\min}$$

4) If the minimum pressure head requirement is supplied at points C and D, the pipeline system has been designed according to the project requirements. If the minimum pressure head is not supplied either at one of the points or at both points,

a) The reservoir water level is increased up to supply the minimum pressure head at points C and D by following the steps given above.
b) Head losses are reduced by increasing the pipe diameters.

Example: Geometric elevation of the point E is 50 m for the reservoir system given. The required minimum pressure at point E is 300 kPa. The discharge in the pipe DE is 200 lt/sec. The physical characteristics of the reservoir-pipe system are given as,

Pipe	Length(m)	Diameter(mm)	f
ABD	1000	200	0.03
ACD	1000	300	0.03
DE	1500	400	0.03

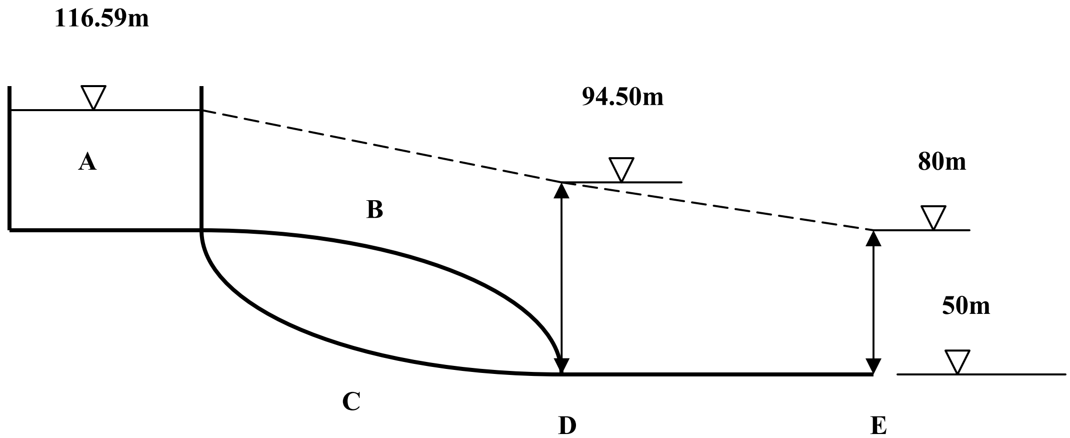


Figure 3.13

Calculate the minimum water surface level to supply the required pressure at the outlet E. Draw the energy line of the system. γ_{water}= 10 kN/m^3.

Solution: The velocity and the head loss along the pipe DE are,

$$V_{De} = \frac{4Q}{\pi D^2} = \frac{4\times 0.2}{\pi \times 0.4^2} = 1.59\, m/\sec$$

$$h_{L_{DE}} = \frac{fV_{DE}^2 L_{DE}}{2gD} = \frac{0.03\times 1.59^2 \times 1500}{19.62\times 0.4} = 14.50m$$

Since,

$$h_{L_{ABD}} = h_{L_{ACD}}$$

$$\frac{fV_{ABD}^2 L_{ABD}}{2gD_{ABD}} = \frac{fV_{ACD}^2 L_{ACD}}{2gD_{ACD}}$$

$$\frac{0.03\times V_{ABD}^2 \times 1000}{2g\times 0.2} = \frac{0.03\times V_{ACD}^2 \times 1000}{2g\times 0.3}$$

$$0.3\times V_{ABD}^2 = 0.2\times V_{ACD}^2$$

$$V_{ABD} = 0.816 V_{ACD}$$

Applying the continuity equation,

$$Q = Q_{ABD} + Q_{ACD}$$

$$\frac{\pi}{4}\times\left(0.2^2 \times V_{ABD} + 0.3^2 \times V_{ACD}^2\right) = 0.200$$

$$0.04\times V_{ABD} + 0.09\times V_{ACD} = 0.255$$

$$0.04\times 0.816\times V_{ACD} + 0.09\times V_{ACD} = 0.255$$

$$V_{ACD} = 2.08\, m/\sec$$

$$V_{ABD} = 0.816\times 2.08 = 1.70\, m/\sec$$

The discharges of the looped pipes are,

$$Q_{ABD} = \frac{\pi D_{ABD}^2}{4}\times V_{ABD}$$

$$Q_{ABD} = \frac{\pi \times 0.2^2}{4}\times 1.70$$

$$Q_{ABD} = 0.053\, m^3/\sec$$

$$Q_{ACD} = \frac{\pi \times D_{ACD}^2}{4}\times V_{ACD}$$

$$Q_{ACD} = \frac{\pi \times 0.3^2}{4}\times 2.08$$

$$Q_{ACD} = 0.147\, m^3/\sec$$

The head loss along the looped pipes is,

$$h_{L_{ACD}} = h_{L_{ABD}} = \frac{fV_{ABD}^2 L_{ABD}}{2gD_{ABD}}$$

$$h_L = \frac{0.03\times 1.70^2 \times 1000}{19.62\times 0.2}$$

$$h_L = 22.09m$$

Piezometric head at the outlet E is,

$$\left(z+\frac{p}{\gamma_w}\right)_E = 50+\frac{300}{10} = 80m$$

The required minimum water surface level at the reservoir is,

$$z_A = \left(z+\frac{p}{\gamma}\right)_E + h_{L_{De}} + h_{L_{ACD}}$$

$$z_A = 80+14.50+22.09$$
$$z_A = 116.59m$$

Example: The discharges in the AB and AC pipes are respectively Q_1=50 lt/sec and Q_2=80 lt/sec for the pipe system given. The required pressure at the B and C outlets is 200 kPa and the geometric elevations for these points are z_B= 50 m and z_c= 45 m. The physical characteristics of the pipe system are,

Pipe	Length (m)	Diameter (mm)	f
RA	2000	300	0.02
AB	1000	350	0.02
AC	1500	400	0.02

Calculate the minimum water surface level of the reservoir R to supply the required pressure at the outlets. Draw the energy line of the system. γ_{water}= 10 kN/m^3.

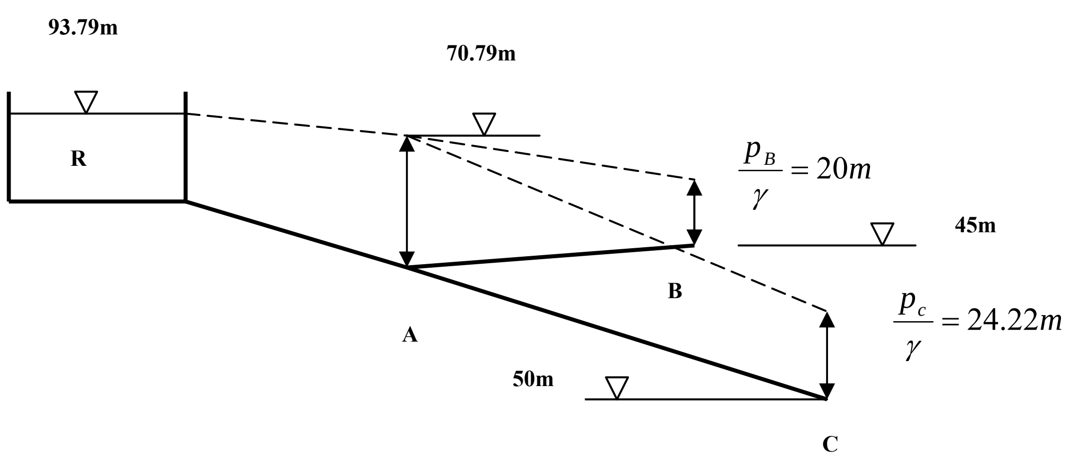


Figure 3.14

Solution: Since the discharges in the pipes are given,

$$V_{AB} = \frac{4Q_{AB}}{\pi D_{AB}^2} = \frac{4\times 0.050}{\pi\times 0.35^2} = 0.52\, m/\sec$$

$$h_{L_{AB}} = \frac{fV_{AB}^2 L_{AB}}{2gD_{AB}} = \frac{0.02\times 0.52^2\times 1000}{19.62\times 0.35} = 0.79m$$

$$\left(z+\frac{p}{\gamma}\right)_A = \left(z+\frac{p}{\gamma}\right)_B + h_{L_{AB}}$$

$$\left(z+\frac{p}{\gamma}\right)_A = \left(50+\frac{200}{10}\right) + 0.79 = 70.79m$$

$$V_{AC} = \frac{4Q_{AC}}{\pi D_{AC}^2} = \frac{4\times 0.080}{\pi\times 0.40^2} = 0.64\, m/\sec$$

$$h_{L_{AC}} = \frac{fV_{AC}^2 L_{AC}}{2gD_{AC}} = \frac{0.02\times 0.64^2\times 1500}{19.62\times 0.40} = 1.57m$$

$$\left(z+\frac{p}{\gamma}\right)_A = \left(z+\frac{p}{\gamma}\right)_C + h_{L_{AC}}$$

$$\left(z+\frac{p}{\gamma}\right)_A = 45+\frac{200}{10}+1.57 = 66.57m$$

If we take the largest piezometric head of the outlets 70.79 m, the pressure will be 200kPa at the outlet B, and the pressure at outlet C is,

$$\left(\frac{p}{\gamma}\right)_C = \left(z+\frac{p}{\gamma}\right)_A - h_{L_{AC}} - z_C$$

$$\left(\frac{p}{\gamma}\right)_C = 70.79-1.57-45.00 = 24.22m$$

$$p = \gamma_w \times 24.22 = 10\times 24.22 = 242.2kPa > 200kPa$$

Water surface level at the reservoir R is,

$$Q_{RA} = Q_{AB} + Q_{AC} = 0.05 + 0.08 = 0.13\, m^3/\text{sec}$$

$$V_{RA} = \frac{4Q_{RA}}{\pi D_{RA}^2} = \frac{4 \times 0.13}{\pi \times 0.30^2} = 1.84\, m/\text{sec}$$

$$h_{L_{RA}} = \frac{fV_{RA}^2 L_{RA}}{2gD_{RA}} = \frac{0.02 \times 1.84^2 \times 2000}{19.62 \times 0.30} = 23.00m$$

$$z_R = \left(z + \frac{p}{\gamma}\right)_A + h_{L_{RA}} = 70.79 + 23.00 = 93.79m$$

3.4.4. Pump Systems

When the energy (head) in a pipe system is not sufficient enough to overcome the head losses to convey the liquid to the desired location, energy has to be added to the system. This is accomplished by a pump. The power of the pump is calculated by,

$$N = \gamma QH \qquad (3.16)$$

Where γ = Specific weight of the liquid (N/m^3), Q = Discharge (m^3/sec), H = Head to be supplied by pump (m), N = Power of the pump (Watt). Calculated pump power should be divided by η = Efficiency factor of the pump.

Example: Reservoir D is fed by a pump and pipe system. Pressure heads in the pipe flow at the entrance and at the outlet from the pump are respectively 5 and 105 m. The geometric of the pipe before the pump is 0.50 m. The efficient power of the pump is 75 kW. The friction factor at all the pipe system is f = 0.03. (Minor losses will be neglected).

- **a)** Calculate the discharge in pipe BC,
- **b)** What will be the discharges in the parallel C1D and C2D pipes?
- **c)** Calculate the water surface level of the reservoir D.
- **d)** Draw the energy line of the system.

Pipe	Length (m)	Diameter (mm)	f
BC	1000	300	0.03
C1D	3000	400	0.03
C2D	1500	200	0.03

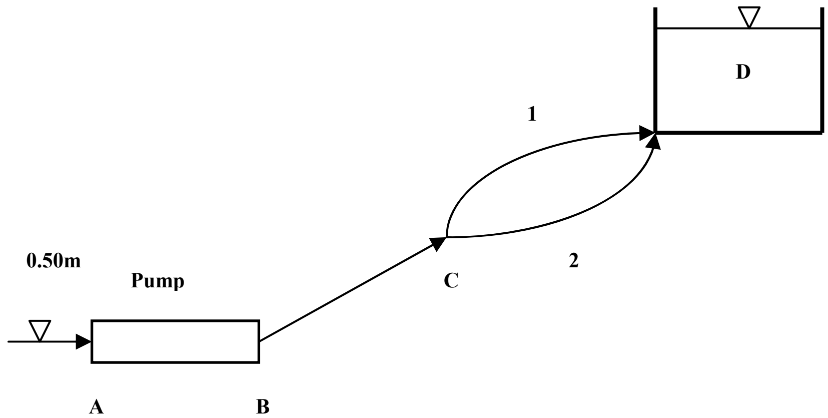


Figure 3.

Solution:

a) The head to be supplied by the pump is,

$$\Delta H_{pump} = 105 - 5 = 100m$$

The efficient power of the pump is,

$$N = \frac{\gamma Q H}{\eta}$$

$$Q = \frac{N\eta}{\gamma H} = \frac{75000}{1000 \times 9.81 \times 100} = 0.076\, m^3/\sec = 76\, lt/\sec$$

b) The velocity in the pipe BC,

$$V_{BC} = \frac{4Q}{\pi D_{BC}^2} = \frac{4 \times 0.076}{\pi \times 0.30^2} = 1.08\, m/\sec$$

The head loss is,

$$h_{L_{BC}} = \frac{f}{D_{BC}} \times \frac{V_{BC}^2}{2g} \times L_{BC}$$

$$h_{L_{BC}} = \frac{0.03}{0.30} \times \frac{1.08^2}{19.62} \times 1000 = 5.94m$$

Since,

$$h_{L_1} = h_{L_2}$$
$$\frac{f}{D_1} \times \frac{V_1^2}{2g} \times L_1 = \frac{f}{D_2} \times \frac{V_2^2}{2g} \times L_2$$
$$\frac{0.03}{0.40} \times \frac{V_1^2}{2g} \times 3000 = \frac{0.03}{0.20} \times \frac{V_2^2}{2g} \times 1500$$
$$V_1^2 = V_2^2$$
$$V_1 = V_2 = V$$

By equation of continuity,

$$Q = Q_1 + Q_2$$
$$Q_1 = \frac{\pi D_1^2}{4} \times V_1$$
$$Q_2 = \frac{\pi D_2^2}{4} \times V_2$$

$$Q_1 = \frac{\pi \times 0.4^2}{4} \times V_1 = \frac{0.16\pi}{4} \times V$$
$$Q_2 = \frac{\pi \times 0.2^2}{4} \times V_2 = \frac{0.40\pi}{4} \times V$$
$$Q_1 = 4Q_2$$

$$Q_1 + Q_2 = 76\, m^3/\sec$$
$$5Q_2 = 76\, lt/\sec$$

$$Q_1 = 15.2\, lt/\sec \qquad Q_2 = 60.8\, lt/\sec$$

c) The velocity and the head loss in the parallel pipes are,

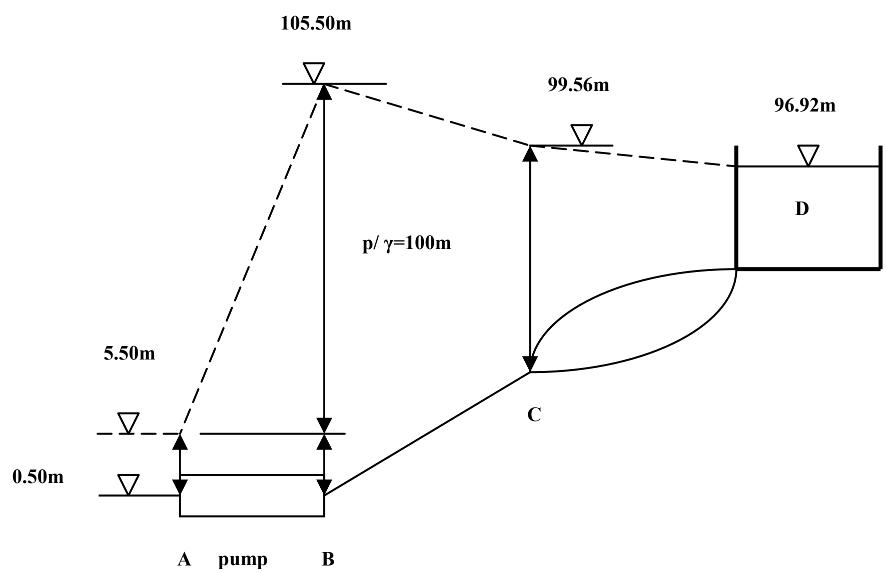
$$V_1 = V_2 = V = \frac{4 \times 0.0152}{\pi \times 0.20^2} = 0.48\, m/\sec$$
$$h_{L_1} = h_{L_2} = \frac{f}{D_1} \times \frac{V^2}{2g} \times L_1 = \frac{0.03}{0.40} \times \frac{0.48^2}{19.62} \times 3000 = 2.64m$$

Total head loss is,

$$\sum h_L = h_{L_{BC}} + h_{L_1}$$
$$\sum h_L = 5.94 + 2.64 = 8.58m$$

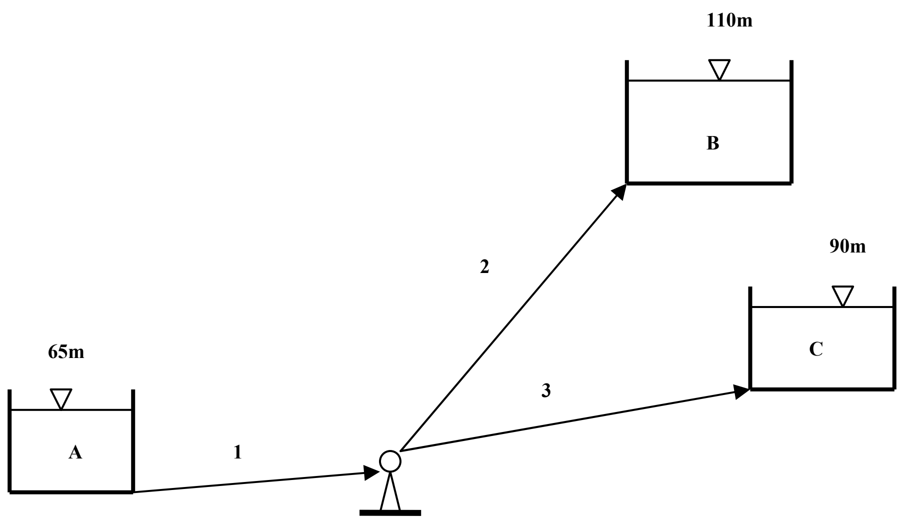
The water surface level of the reservoir D is,

$$z_D = \left(z + \frac{p}{\gamma} \right)_B - \sum h_L$$
$$z_D = 0.50 + 105 - 8.58 = 96.92m$$



Example: Reservoir A is feeding reservoirs B and C by a pump and pipe system. The discharge to reservoir C is Q_2 = 0.10 m^3/sec. If the efficient coefficient of the pump is η = 0.70 n, what will be required power of pump? Draw the energy line of the system . The physical characteristics of the pipe system are,

Pipe	Diameter (mm)	Length (m)	f
1	300	400	0.020
2	150	300	0.015
3	200	1000	0.025



Figure…..

Solution: Beginning with pipe 2,

$$V_2 = \frac{Q_2}{A_2} = \frac{4Q_2}{\pi D_2^2}$$
$$V_2 = \frac{4 \times 0.10}{\pi \times 0.15^2} = 5.66\, m/\sec$$

$$h_{L_2} = \frac{f_2}{D_2} \times \frac{V_2^2}{2g} \times L_2$$
$$h_{L_2} = \frac{0.015}{0.15} \times \frac{5.66^2}{19.62} \times 300 \cong 49m$$

Energy level at the outlet of the pump is,

$$H_{P_{outlet}} = \left(z + \frac{p}{\gamma} \right)_P = z_C + h_{L_2} = 90 + 49 = 139m$$

Pipe 3:

Head loss along the pipe 3 is,

$$h_{L_3} = H_{P_{outlet}} - z_B = 139 - 110 = 29m$$

$$h_{L_3} = \frac{f_3}{D_3} \times \frac{V_3^2}{2g} \times L_3$$

$$29 = \frac{0.025}{0.200} \times \frac{V_3^2}{19.62} \times 1000$$

$$V_3 = 2.13\, m/\sec$$

$$Q_3 = \frac{\pi D_3^2}{4} \times V_3 = \frac{\pi \times 0.20^2}{4} \times 2.13 = 0.067\, m^3/\sec$$

Pipe 1:

$$Q_1 = Q_2 + Q_3$$

$$Q_1 = 0.100 + 0.067 = 0.167\, m^3/\sec$$

$$V_1 = \frac{4Q_1}{\pi D_1^2} = \frac{4 \times 0.167}{\pi \times 0.30^2} = 2.36\, m/\sec$$

$$h_{L_1} = \frac{f_1}{D_1} \times \frac{V_1^2}{2g} \times L_1 = \frac{0.020}{0.30} \times \frac{2.36^2}{19.62} \times 400 = 7.57m$$

Energy level at the entrance to the pump is,

$$H_{P_{ent}} = z_A - h_{L_1} = 65.00 - 7.57 = 57.43m$$

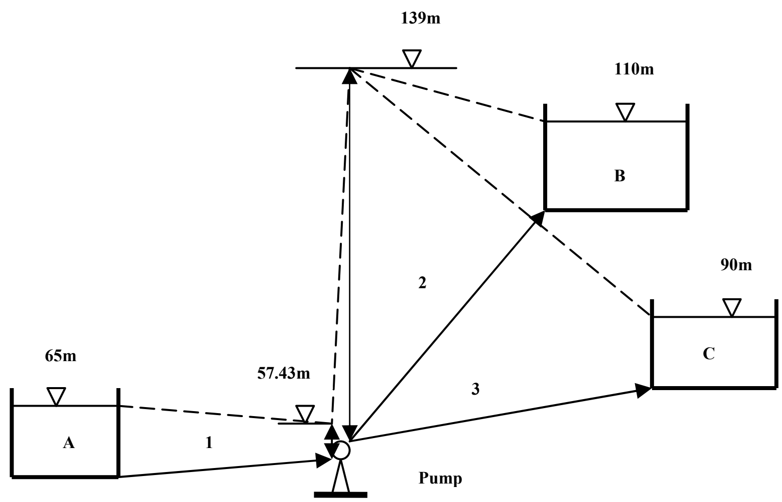
The energy (head) to be supplied by pump is,

$$\Delta H_p = H_{P_{outlet}} - H_{P_{ent}} = 139.00 - 57.43 = 81.57m$$

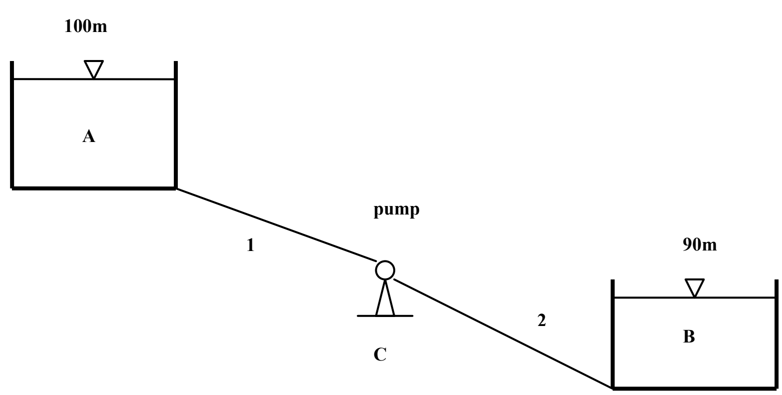
The required power of the pump is,

$$N_P = \frac{\gamma Q \Delta H_P}{\eta}$$

$$N_P = \frac{9.81 \times 1000 \times 0.167 \times 81.59}{0.70} = 190952W \cong 191kW$$



Example: Reservoir B is fed by reservoir A with 600 lt/sec discharge. The water surface levels in the reservoirs are z_A = 100 m and z_B = 90 m. If the pipe diameter is D = 0.50m, the friction factor is f = 0.02, and the efficiency factor of the pump is η = 0.70, calculate the required pump power. The lengths of the pipes are respectively L_1 = 500 m and L_2 = 1000 m. Draw the energy line of the system.



Figure

Solution: Since the discharge of the system is known, the head loss from reservoir A to the pump can be calculated as,

$$V = \frac{4Q}{\pi D^2} = \frac{4 \times 0.60}{\pi \times 0.50^2} = 3.06\, m/\sec$$

$$h_{L_{AC}} = \frac{f}{D} \times \frac{V^2}{2g} \times L_{AC}$$

$$h_{L_{AC}} = \frac{0.02}{0.50} \times \frac{3.06^2}{19.62} \times 500 = 9.54m$$

The energy level at the entrance to the pump is,

$$H_{C_{ent}} = 100 - 9.54 = 90.46m$$

Since the pipe diameter is same along the pipe, and $L_2 = 2L_1$, the head loss along the pipe 2 is,

$$h_{L_2} = 2h_{L_1} = 2 \times 9.54 = 19.08m$$

The energy level at the outlet of the pump should be,

$$H_{C_{out}} = 90.00 + 19.08 = 109.80m$$

The pumping is then,

$$\Delta H_{pump} = 109.80 - 90.46 = 18.62m$$

The required power of the pump is,

$$N = \frac{\gamma Q H}{\eta} = \frac{9810 \times 0.600 \times 18.62}{0.70} = 156568W \cong 157kW$$