Advanced Spatial Modulation Techniques for MIMO Systems

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Outline

V-BLAST vs Spatial Modulation (SM)

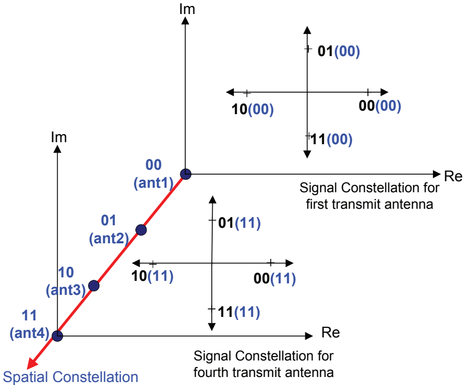
- The use of multiple antennas at both transmitter and receiver sides has been shown to be an effective way to improve the capacity and reliability of single antenna wireless systems.
- Two general MIMO (multiple-input multiple-output) transmission strategies, space-time block coding (STBC) and spatial multiplexing, have been proposed in the past decade.
- A novel concept known as spatial modulation (SM) has been introduced in by Mesleh et al. as an alternative to these two MIMO transmission techniques[1].
- The basic idea of SM is an extension of two dimensional signal constellations (such as M-PSK or M-QAM) to a third dimension, which is the spatial (antenna) dimension.

[1] **Mesleh, R.**, **Haas, H.**, **Sinanovic, S.**, **Ahn, C.W. and Yun, S.**, 2008. Spatial Modulation, *IEEE Trans. Veh. Technol.*, **57(4)**, 2228–2241.

SM Concept

There are two information carrying units in SM scheme

1. antenna indices
2. constellation symbols



Advances in SM

- It has been shown by Jeganathan, et al. that the error performance of the SM scheme can be greatly improved by the use of an optimal detector and that SM provides better error performance than V-BLAST.
- A different form of SM, called space-shift keying (SSK) is proposed by eliminating amplitude/phase modulation. SSK modulation uses only antenna indices to convey information and therefore, has a simpler structure than SM.
- The inventors of SM have proposed a trellis coded spatial modulation scheme, where the key idea of trellis coded modulation (TCM) is partially applied to SM to improve its performance in correlated channels.
- It has been shown that this scheme does not provide any error performance advantage compared to uncoded SM in uncorrelated channel conditions.

Motivation

- Despite the fact that the SM scheme has been concerned with exploiting the multiplexing gain of multiple transmit antennas, the potential of the transmit diversity of MIMO systems is not explored.
- This motivates the introduction *Space-Time Block Coded Spatial Modulation (STBC-SM)*, designed for taking advantage of both SM and STBC.
- In addition to the transmit diversity advantage of the STBC-SM, to obtain additional coding gains, a novel coded MIMO transmission scheme, called *Trellis Coded Spatial Modulation (TC-SM)*, which directly combines trellis coding and SM, is proposed.

Space-Time Block Coded Spatial Modulation (STBC-SM)[2]

- A new MIMO transmission scheme, called STBC-SM, is proposed, in which information is conveyed with an STBC matrix that is transmitted from combinations of the transmit antennas of the corresponding MIMO system.
- The Alamouti code is chosen as the target STBC to exploit.
- As a source of information, we consider not only the two complex information symbols embedded in Alamouti's STBC, but also the indices (positions) of the two transmit antennas employed.
- A general framework is presented to construct the STBC-SM scheme for any number of transmit antennas.

[2] **Başar, E.**, **Aygölü, Ü.**, **Panayırcı, E. and Poor, H.V.**, 2011. Space-Time Block Coded Spatial Modulation, *IEEE Trans. Commun.*, **59(3)**, 823–832.

Space-Time Block Coded Spatial Modulation (STBC-SM) con'td.

- Diversity and coding gain analyses are performed.
- A low complexity maximum likelihood (ML) decoder is derived for the proposed STBC-SM system.
- It is shown via computer simulations that the proposed STBC-SM scheme has significant performance advantages over the SM with optimal decoding and over V-BLAST, due to its diversity advantage.
- Furthermore, it is shown that the new scheme achieves significantly better error performance than Alamouti's STBC and rate-$3/4$ orthogonal STBC (OSTBC).

Trellis Coded Spatial Modulation (TC-SM)[3]

- In TC-SM scheme, the trellis encoder and the SM mapper are jointly designed and a soft decision Viterbi decoder which is fed with the soft information supplied by the optimal SM decoder, is used at the receiver.
- The general conditional pairwise error probability (CPEP) for TC-SM is derived, and then for quasi-static Rayleigh fading channels, by averaging over channel coefficients, the unconditional PEP (UPEP) of TC-SM is obtained for error events with path lengths two and three.
- Code design criteria are given for the TC-SM scheme, which are then used to obtain the best codes with optimized distance spectra.

[3] **Başar, E.**, **Aygölü, Ü.**, **Panayırcı, E. and Poor, H.V.**, 2010. New Trellis Code Design for Spatial Modulation, *to appear in IEEE Trans. on Wireless Commun.*

Trellis Coded Spatial Modulation (TC-SM) con'td.

- New TC-SM schemes with 4, 8 and 16 states are proposed for $2, 3$ and 4 bits/s/Hz spectral efficiencies.
- It is shown via computer simulations that the proposed TC-SM schemes for uncorrelated and correlated Rayleigh fading channels provide significant error performance improvements over space-time trellis codes (STTCs), coded V-BLAST systems and the trellis coded SM scheme proposed in the literature in terms of bit error rate (BER) and frame error rate (FER) yet with a lower decoding complexity.

Performance Evaluation of SM

- We consider a MIMO system operating over quasi-static Rayleigh flat fading and having n_T transmit and n_R receive antennas. The channel fading coefficient between tth transmit and rth receive antenna, denoted by $\alpha_{t,r}$, is distributed as $\mathcal{CN}(0,1)$.
- The spatially modulated symbol is denoted by $x = (i, s)$, where s is transmitted over ith transmit antenna.
- The received signal at the rth receive antenna $(r = 1, \cdots, n_R)$ is given by

$$y_r = \alpha_{i,r} s + w_r$$

where w_r is the additive white Gaussian noise sample with distribution $\mathcal{CN}(0, N_0)$.

Conditional Pairwise Error Probability of SM

- Assuming the SM symbol $x = (i, s)$ is transmitted and it is erroneously detected as $\hat{x} = (j, \hat{s})$, when CSI is perfectly known at the receiver, the conditional pairwise error probability (CPEP) is given by

$$P(x \to \hat{x} \mid \mathbf{H}) = Q\left(\sqrt{\frac{\gamma}{2}\sum\nolimits_{r=1}^{n_R}\left|\alpha_{i,r}s - \alpha_{j,r}\hat{s}\right|^2}\right)$$

where $\mathbf{H} = [\alpha_{t,r}]_{n_T \times n_R}$ is the channel matrix with independent and identically distributed entries and $\gamma = E\{|s|^2\}/N_0$ is the average SNR at each receiver antenna.

Unconditional Pairwise Error Probability of SM

- Defining $d_r \triangleq \left|\beta_{i,r} s - \beta_{j,r} \hat{s}\right|^2$, we derive its MGF as

$$M_{d_r}(t) = \frac{1}{1 - \lambda t}$$

where

$$\lambda = \begin{cases} |s|^2 + |\hat{s}|^2, & \text{if } i \neq j \\ |s - \hat{s}|^2, & \text{if } i = j. \end{cases}$$

After simple manipulation, the unconditional PEP (UPEP) of the SM scheme is derived as follows:

$$P(x \to \hat{x}) = \frac{1}{\pi} \int_0^{\pi/2} \left(\frac{\sin^2 \theta}{\sin^2 \theta + \frac{\lambda \gamma}{4}} \right)^{n_R} d\theta$$

Performance of SM with Imperfect Channel Knowledge

- In a practical system, the channel estimator at the receiver provides fading coefficient estimates $\beta_{t,r}$, which can be assumed to be of the form

$$\beta_{t,r} = \alpha_{t,r} + \epsilon_{t,r}$$

  where $\epsilon_{t,r}$ represents the channel estimation error that is independent of $\alpha_{t,r}$, and is distributed according to $\mathcal{CN}\left(0, \sigma_\epsilon^2\right)$.
- Consequently, the distribution of $\beta_{t,r}$ becomes $\mathcal{CN}\left(0, 1+\sigma_\epsilon^2\right)$, and $\beta_{t,r}$ is dependent on $\alpha_{t,r}$ with the correlation coefficient

$$\rho = 1/\sqrt{1+\sigma_\epsilon^2}$$

  i.e, when $\sigma_\epsilon^2 \rightarrow 0$, then $\rho \rightarrow 1$.
- We assume that ρ is known at the receiver.

- In the presence of channel estimation errors, assuming the SM symbol $x = (i, s)$ is transmitted, the mean and variance of the received signal $y_r, r = 1, \cdots, n_R$ conditioned on $\beta_{i,r}$ is given as

$$E\{y_r \mid \beta_{i,r}\} = \rho^2 \beta_{i,r} s$$
$$Var\{y_r \mid \beta_{i,r}\} = N_0 + \left(1 - \rho^2\right) |s|^2 .$$

- Then, the optimal receiver of SM decides in favor of the symbol $\hat{s}$ and transmit antenna index j that minimizes the following metric for M-ary phase-shift keying (M-PSK) $\left(|s|^2 = 1, \forall s\right)$

$$(j, \hat{s}) = \arg\min_{i,s} \sum_{r=1}^{n_R} \left| y_r - \rho^2 \beta_{i,r} s \right|^2$$

to maximize the a posteriori probability of $y_r, r = 1, \cdots, n_R$, which are complex Gaussian r.v.'s.

Performance Analysis for M-PSK

- Assuming $x = (i, s)$ is transmitted, the probability of deciding in favor of $\hat{x} = (j, \hat{s})$ is given as

$$P(x \to \hat{x} \mid \hat{\mathbf{H}}) = P\left(\sum_{r=1}^{n_R} \left|y_r - \rho^2 \beta_{j,r}\hat{s}\right|^2 < \sum_{r=1}^{n_R} \left|y_r - \rho^2 \beta_{i,r} s\right|^2\right)$$

CPEP

$$P(x \to \hat{x} \mid \hat{\mathbf{H}}) = Q\left(\rho^2 \sqrt{\frac{\sum_{r=1}^{n_R} \left|\beta_{i,r} s - \beta_{j,r}\hat{s}\right|^2}{2\left(N_0 + (1-\rho^2)\right)}}\right)$$

UPEP

$$P(x \to \hat{x}) = \frac{1}{\pi}\int_0^{\pi/2} \left(\frac{\sin^2\theta}{\sin^2\theta + \frac{\lambda\rho^2}{4(N_0 + (1-\rho^2))}}\right)^{n_R} d\theta^{a}$$

[a] $\lambda = 2$ for $i \neq j$, $\lambda = |s - \hat{s}|^2$ for $i = j$

Performance Analysis for M-QAM (Mismatched Receiver)

- We consider the mismatched ML receiver which uses the ML decision metric of the P-CSI case by replacing $\alpha_{t,r}$ by $\beta_{t,r}$,

$$P(x \to \hat{x} \mid \hat{\mathbf{H}}) = P\left(\sum_{r=1}^{n_R} \left|y_r - \beta_{j,r}\hat{s}\right|^2 < \sum_{r=1}^{n_R} \left|y_r - \beta_{i,r}s\right|^2\right)$$

CPEP, $\left(1+\sigma_\epsilon^2\right)^2 \approx \left(1+\sigma_\epsilon^2\right)$

$$P(x \to \hat{x} \mid \hat{\mathbf{H}}) \approx Q\left(\sqrt{\frac{\sum_{r=1}^{n_R} \left|\beta_{i,r}s - \beta_{j,r}\hat{s}\right|^2}{2\left(N_0 + (1-\rho^2)\left|s\right|^2\right)}}\right)$$

UPEP

$$P(x \to \hat{x}) \approx \frac{1}{\pi}\int_0^{\pi/2} \left(\frac{\sin^2\theta}{\sin^2\theta + \frac{\lambda}{4\left(N_0+(1-\rho^2)|s|^2\right)}}\right)^{n_R} d\theta^{a}$$

[a] $\lambda = |s|^2 + |\hat{s}|^2$ for $i \neq j$, $\lambda = |s - \hat{s}|^2$ for $i = j$

Evaluation of Average Bit Error Probability (ABEP)

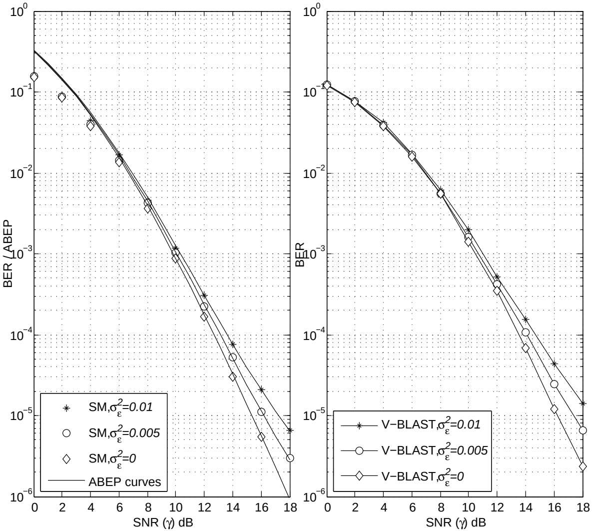
- After the evaluation of the UPEP, the ABEP of the SM scheme can be upper bounded by the following asymptotically tight union bound:

$$P_b \leq \frac{1}{2^k} \sum_{n=1}^{2^k} \sum_{m=1}^{2^k} \frac{P\left(x_n \rightarrow x_m\right) e_{n,m}}{k}$$

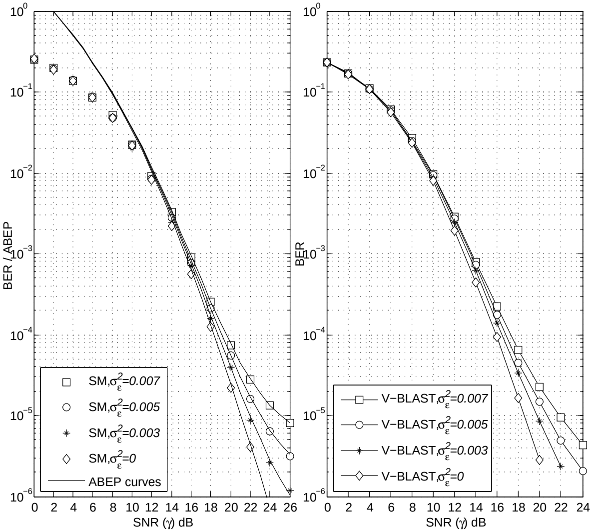
where

- $\{x_n\}_{n=1}^{2^k}$ is the set of all possible SM symbols,
- $k = \log_2\left(Mn_T\right)$ is the number of information bits per SM symbol, and
- $e_{n,m}$ is the number of bit errors associated with the corresponding PEP event.

BER performance of SM with $n_T = 4$, QPSK and V-BLAST with $n_T = 4$, BPSK (4 bits/s/Hz) with optimal receivers



BER performance of SM with $n_T = 4$, 16-QAM and V-BLAST with $n_T = 3$, QPSK (6 bits/s/Hz) with mismatched receivers



From STBC to STBC-SM

Alamouti's STBC

$$\mathbf{X} = \begin{pmatrix} \mathbf{x}_1 & \mathbf{x}_2 \end{pmatrix} = \begin{pmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{pmatrix} \begin{matrix} \rightarrow \textit{space} \\ \downarrow \\ \textit{time} \end{matrix}$$

- In the STBC-SM scheme, both STBC symbols and the indices of the transmit antennas from which these symbols are transmitted, carry information:

Example (STBC-SM, Four Transmit Antennas ($n_T = 4$))

$$\chi_1 = \{\mathbf{X}_{11}, \mathbf{X}_{12}\} = \left\{ \begin{pmatrix} x_1 & x_2 & 0 & 0 \\ -x_2^* & x_1^* & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & x_1 & x_2 \\ 0 & 0 & -x_2^* & x_1^* \end{pmatrix} \right\}$$

$$\chi_2 = \{\mathbf{X}_{21}, \mathbf{X}_{22}\} = \left\{ \begin{pmatrix} 0 & x_1 & x_2 & 0 \\ 0 & -x_2^* & x_1^* & 0 \end{pmatrix}, \begin{pmatrix} x_2 & 0 & 0 & x_1 \\ x_1^* & 0 & 0 & -x_2^* \end{pmatrix} \right\} e^{j\theta}$$

Here,

- $\chi_i, i = 1, 2$ are called the STBC-SM codebooks each containing two STBC-SM codewords $\mathbf{X}_{ij}, j = 1, 2$ which do not interfere to each other.
- θ is a rotation angle to be optimized for a given modulation format to ensure maximum diversity and coding gain at the expense of expansion of the signal constellation.
- However, if θ is not considered, overlapping columns of codeword pairs from different codebooks would reduce the transmit diversity order to one.

STBC-SM Mapping Rule for 2 bits/s/Hz (BPSK, 4 Transmit Antennas)

		Input Bits	Transmission Matrices			Input Bits	Transmission Matrices
χ_1	$\ell = 0$	0000	$\begin{pmatrix} 1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{pmatrix}$	χ_2	$\ell = 2$	1000	$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 \end{pmatrix} e^{j\theta}$
		0001	$\begin{pmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}$			1001	$\begin{pmatrix} 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix} e^{j\theta}$
		0010	$\begin{pmatrix} -1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 \end{pmatrix}$			1010	$\begin{pmatrix} 0 & -1 & 1 & 0 \\ 0 & -1 & -1 & 0 \end{pmatrix} e^{j\theta}$
		0011	$\begin{pmatrix} -1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix}$			1011	$\begin{pmatrix} 0 & -1 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{pmatrix} e^{j\theta}$
	$\ell = 1$	0100	$\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$		$\ell = 3$	1100	$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \end{pmatrix} e^{j\theta}$
		0101	$\begin{pmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$			1101	$\begin{pmatrix} -1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix} e^{j\theta}$
		0110	$\begin{pmatrix} 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & -1 \end{pmatrix}$			1110	$\begin{pmatrix} 1 & 0 & 0 & -1 \\ -1 & 0 & 0 & -1 \end{pmatrix} e^{j\theta}$
		0111	$\begin{pmatrix} 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & -1 \end{pmatrix}$			1111	$\begin{pmatrix} -1 & 0 & 0 & -1 \\ -1 & 0 & 0 & 1 \end{pmatrix} e^{j\theta}$

STBC-SM System Design and Optimization

- An important design parameter for quasi-static Rayleigh fading channels is the minimum coding gain distance (CGD) between two STBC-SM codewords $\mathbf{X}_{ij}$ and $\hat{\mathbf{X}}_{ij}$:

$$\delta_{\min}(\mathbf{X}_{ij}, \hat{\mathbf{X}}_{ij}) = \min_{\mathbf{X}_{ij}, \hat{\mathbf{X}}_{ij}} \det(\mathbf{X}_{ij} - \hat{\mathbf{X}}_{ij})(\mathbf{X}_{ij} - \hat{\mathbf{X}}_{ij})^H$$

- The minimum CGD between two codebooks:

$$\delta_{\min}\left(\chi_i, \chi_j\right) = \min_{k,l} \delta_{\min}\left(\mathbf{X}_{ik}, \mathbf{X}_{jl}\right)$$

- The minimum CGD of an STBC-SM code:

$$\delta_{\min}\left(\chi\right) = \min_{i,j,i \neq j} \delta_{\min}\left(\chi_i, \chi_j\right)$$

STBC-SM Design Algorithm

Unlike in the SM scheme, the number of transmit antennas in the STBC-SM scheme need not be an integer power of 2, since the pairwise combinations are chosen from n_T available transmit antennas for STBC transmission.

Step 1

Given the total number of transmit antennas n_T, calculate the number of possible antenna combinations for the transmission of Alamouti's STBC from (this must be an integer power of 2!)

$$c = \left\lfloor \binom{n_T}{2} \right\rfloor_{2^p}.$$

($\lfloor x \rfloor$: floor function, $\lceil x \rceil$: ceiling function)

Step 2

Calculate the number of codewords in each codebook $\chi_i, i = 1, 2, \ldots, n-1$ from $a = \lfloor n_T/2 \rfloor$ and the total number of codebooks from $n = \lceil c/a \rceil$.

Step 3

Start with the construction of χ_1 which contains a non-interfering codewords as

$$\chi_1 = \Big\{ \begin{pmatrix} \mathbf{X} & \mathbf{0}_{2\times(n_T-2)} \end{pmatrix} \\ \begin{pmatrix} \mathbf{0}_{2\times 2} & \mathbf{X} & \mathbf{0}_{2\times(n_T-4)} \end{pmatrix} \\ \begin{pmatrix} \mathbf{0}_{2\times 4} & \mathbf{X} & \mathbf{0}_{2\times(n_T-6)} \end{pmatrix} \\ \vdots \\ \begin{pmatrix} \mathbf{0}_{2\times 2(a-1)} & \mathbf{X} & \mathbf{0}_{2\times(n_T-2a)} \end{pmatrix} \Big\}$$

where $\mathbf{X}$ is the Alamouti's STBC.

Step 4

Using a similar approach, construct χ_i for $2 \leq i \leq n$ by considering the following two important facts:

- Every codebook must contain non-interfering codewords chosen from pairwise combinations of n_T available transmit antennas.
- Each codebook must be composed of codewords with antenna combinations that were never used in the construction of a previous codebook.

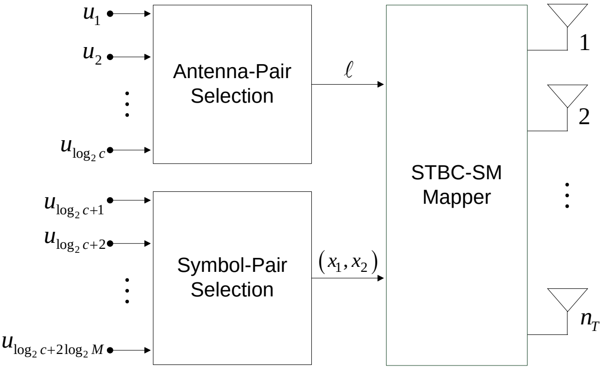
Step 5

Determine the rotation angles θ_i for each χ_i, $2 \leq i \leq n$, that maximize $\delta_{\min}(\chi)$ for a given signal constellation and antenna configuration; that is

$$\boldsymbol{\theta}_{opt} = \arg\max_{\boldsymbol{\theta}} \delta_{\min}(\chi)$$

where $\boldsymbol{\theta} = (\theta_2, \theta_3, \ldots, \theta_n)$.

Block Diagram of the STBC-SM Transmitter



- Since we have c antenna combinations, the spectral efficiency of the STBC-SM scheme is calculated as

$$m = \frac{1}{2}\log_2 cM^2 = \frac{1}{2}\log_2 c + \log_2 M \ \text{[bits/s/Hz]}.$$

A Design Example for $n_T = 6$

- Number of possible antenna combinations: $c = \left\lfloor \binom{6}{2} \right\rfloor_{2^p} = 8$
- Number of codewords in each codebook: $a = \lfloor n_T/2 \rfloor = \lfloor 6/2 \rfloor = 3$
- Number of codebooks: $n = \lceil c/a \rceil = \lceil 8/3 \rceil = 3$
- According to the design algorithm, a possible construction of the STBC-SM codebooks should be

$$\begin{aligned}
\chi_1 &= \left\{ \left(\mathbf{x}_1 \ \mathbf{x}_2 \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \right), \left(\mathbf{0} \ \mathbf{0} \ \mathbf{x}_1 \ \mathbf{x}_2 \ \mathbf{0} \ \mathbf{0} \right), \left(\mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{x}_1 \ \mathbf{x}_2 \right) \right\} \\
\chi_2 &= \left\{ \left(\mathbf{0} \ \mathbf{x}_1 \ \mathbf{x}_2 \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \right), \left(\mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{x}_1 \ \mathbf{x}_2 \ \mathbf{0} \right), \left(\mathbf{x}_2 \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \ \mathbf{x}_1 \right) \right\} e^{j\theta_2} \\
\chi_3 &= \left\{ \left(\mathbf{x}_1 \ \mathbf{0} \ \mathbf{x}_2 \ \mathbf{0} \ \mathbf{0} \ \mathbf{0} \right), \left(\mathbf{0} \ \mathbf{x}_1 \ \mathbf{0} \ \mathbf{x}_2 \ \mathbf{0} \ \mathbf{0} \right) \right\} e^{j\theta_3}
\end{aligned}$$

where $\mathbf{X} = (\mathbf{x}_1 \ \ \mathbf{x}_2) = \begin{pmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{pmatrix}$ and $\mathbf{0} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

- But how we can determine θ_2 and θ_3 ? $\Rightarrow$ Optimization Problem!

STBC-SM System Optimization

- *Case 1* $(n_T \leq 4)$: We have, in this case, two codebooks χ_1 and χ_2 and only one non-zero angle, say θ, to be optimized. It can be seen that $\delta_{\min}(\chi_1, \chi_2)$ is equal to the minimum CGD between any two interfering codewords from χ_1 and χ_2 such as

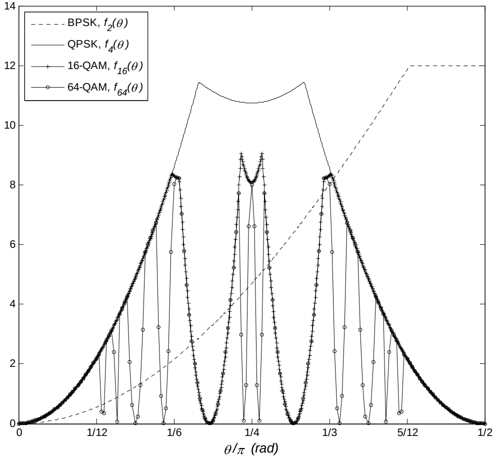
$$\begin{aligned} \mathbf{X}_{1k} &= \begin{pmatrix} \mathbf{x}_1 & \mathbf{x}_2 & \mathbf{0}_{2\times(n_T-2)} \end{pmatrix} \\ \mathbf{X}_{2l} &= \begin{pmatrix} \mathbf{0}_{2\times 1} & \hat{\mathbf{x}}_1 & \hat{\mathbf{x}}_2 & \mathbf{0}_{2\times(n_T-3)} \end{pmatrix} e^{j\theta} \end{aligned} \quad (1)$$

where $\mathbf{X}_{1k} \in \chi_1$ is transmitted and $\hat{\mathbf{X}}_{1k} = \mathbf{X}_{2l} \in \chi_2$ is erroneously detected. We calculate the minimum CGD between $\mathbf{X}_{1k}$ and $\hat{\mathbf{X}}_{1k}$ as

$$\begin{aligned} \delta_{\min}\left(\mathbf{X}_{1k}, \hat{\mathbf{X}}_{1k}\right) = \min_{\mathbf{X}_{1k}, \hat{\mathbf{X}}_{1k}} \Big\{ & \left(\kappa - 2\,\mathrm{Re}\left\{\hat{x}_1^* x_2 e^{-j\theta}\right\}\right)\left(\kappa + 2\,\mathrm{Re}\left\{x_1 \hat{x}_2^* e^{j\theta}\right\}\right) \\ & - |x_1|^2 |\hat{x}_1|^2 - |x_2|^2 |\hat{x}_2|^2 + 2\,\mathrm{Re}\left\{x_1 \hat{x}_1 x_2^* \hat{x}_2^* e^{j2\theta}\right\} \Big\} \end{aligned}$$

where $\kappa = \sum_{i=1}^{2}\left(|x_i|^2 + |\hat{x}_i|^2\right)$.

- We compute $\delta_{\min}\left(\mathbf{X}_{1k}, \hat{\mathbf{X}}_{1k}\right)$ as a function of $\theta \in [0, \pi/2]$ for BPSK, QPSK, 16-QAM and 64-QAM signal constellations via computer search.



The value of the single optimization parameter is determined as follows:

$$\max_{\theta} \delta_{\min}(\chi) = \begin{cases} \max_{\theta} f_2(\theta) = 12, & \text{if } \theta = 1.57 \text{ rad} \\ \max_{\theta} f_4(\theta) = 11.45, & \text{if } \theta = 0.61 \text{ rad} \\ \max_{\theta} f_{16}(\theta) = 9.05, & \text{if } \theta = 0.75 \text{ rad} \\ \max_{\theta} f_{64}(\theta) = 8.23, & \text{if } \theta = 0.54 \text{ rad} \end{cases}$$

- *Case 2* $(n_T > 4)$: In this case, the number of codebooks is greater than 2. Let the corresponding rotation angles to be optimized be denoted in ascending order by

$$\theta_1 = 0 < \theta_2 < \theta_3 < \cdots < \theta_n < p\pi/2$$

where $p = 2$ for BPSK and $p = 1$ for QPSK. For BPSK and QPSK, choosing

$$\theta_k = \begin{cases} \frac{(k-1)\pi}{n}, & \text{for BPSK} \\ \frac{(k-1)\pi}{2n}, & \text{for QPSK} \end{cases}$$

for $1 \leq k \leq n$ guarantees the maximization of the minimum CGD for the STBC-SM scheme! This is accomplished due to the fact that the minimum CGD between two codebooks is given as

$$\max \delta_{\min}(\chi) = \max \min_{i,j,i \neq j} \delta_{\min}(\chi_i, \chi_j) = \max \min_{i,j,i \neq j} f_M(\theta_j - \theta_i)$$

- In order to maximize $\delta_{\min}(\chi)$, it is sufficient to maximize the minimum CGD between the consecutive codebooks χ_i and $\chi_{i+1}, i = 1, 2, \ldots, n-1$. For QPSK signaling, this is accomplished by dividing the interval $[0, \pi/2]$ into n equal sub-intervals and choosing, for $i = 1, 2, \ldots, n-1$,

$$\theta_{i+1} - \theta_i = \frac{\pi}{2n}.$$

which results in

$$\begin{aligned} \max \delta_{\min}(\chi) &= \min \{f_4(\theta_2), f_4(\theta_3), \ldots, f_4(\theta_n)\} \\ &= f_4(\theta_2) = f_4\left(\frac{\pi}{2n}\right). \end{aligned}$$

- Similar results are obtained for BPSK signaling except that $\pi/2n$ is replaced by π/n.

- For 16-QAM and 64-QAM signaling, the selection of $\{\theta_k\}$'s in integer multiples of $\pi/2n$ would not guarantee to maximize the minimum CGD for the STBC-SM scheme since the behavior of the functions $f_{16}(\theta)$ and $f_{64}(\theta)$ is very non-linear, having several zeros in $[0, \pi/2]$.
- However, our extensive computer search has indicated that for 16-QAM with $n \leq 6$, the rotation angles chosen as $\theta_k = (k-1)\pi/2n$ for $1 \leq k \leq n$ are still optimum.
- But for 16-QAM signaling with $n > 6$ as well as for 64-QAM signaling with $n > 2$, the optimal $\{\theta_k\}$'s must be determined by an exhaustive computer search.

Basic Parameters of the STBC-SM System for Different Number of Transmit Antennas

n_T	c	a	n	$\delta_{\min}(\chi)$			m [bits/s/Hz]
				$M=2$	$M=4$	$M=16$	
3	2	1	2	12	11.45	9.05	$0.5+\log_2 M$
4	4	2	2	12	11.45	9.05	$1+\log_2 M$
5	8	2	4	4.69	4.87	4.87	$1.5+\log_2 M$
6	8	3	3	8.00	8.57	8.31	$1.5+\log_2 M$
7	16	3	6	2.14	2.18	2.18	$2+\log_2 M$
8	16	4	4	4.69	4.87	4.87	$2+\log_2 M$

- Increasing the number of transmit antennas results in an increasing number of antenna combinations and, consequently, increasing spectral efficiency achieved by the STBC-SM scheme.
- However, this requires a larger number of angles to be optimized and causes some reduction in the minimum CGD.

Optimal ML Decoder for the STBC-SM System

- n_T transmit and n_R receive antennas
- quasi-static Rayleigh flat fading MIMO channel

$$\mathbf{Y} = \sqrt{\frac{\rho}{\mu}}\mathbf{X}_\chi \mathbf{H} + \mathbf{N}$$

- $\mathbf{Y}$: $2 \times n_R$ received signal matrix
- $\mathbf{X}_\chi$: $2 \times n_T$ STBC-SM transmission matrix
- μ: normalization factor which ensures ρ is the received SNR
- $\mathbf{H}$: $n_T \times n_R$ channel matrix $\sim \mathcal{CN}(0,1)$
- $\mathbf{N}$: $2 \times n_R$ AWGN matrix $\sim \mathcal{CN}(0,1)$

- The STBC-SM code has c codewords, from which cM^2 different transmission matrices can be constructed.

ML Detection Problem with Exponential (cM^2) Complexity

$$\hat{\mathbf{X}}_\chi = \arg\min_{\mathbf{X}_\chi \in \chi} \left\| \mathbf{Y} - \sqrt{\frac{\rho}{\mu}}\mathbf{X}_\chi \mathbf{H} \right\|^2 .$$

Simplified ML Decoder for STBC-SM

Equivalent Channel Model

$$\mathbf{y} = \sqrt{\frac{\rho}{\mu}}\mathcal{H}_{\chi}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \mathbf{n}$$

- $\mathcal{H}_{\chi}$: $2n_R \times 2$ equivalent channel matrix of the Alamouti coded SM scheme, which has c different realizations $\mathcal{H}_{\ell},\ 0 \leq \ell \leq c-1$ according to the STBC-SM codewords.
- Due to the orthogonality of Alamouti's STBC, the columns of $\mathcal{H}_{\ell} = \begin{bmatrix} \mathbf{h}_{\ell,1} & \mathbf{h}_{\ell,2} \end{bmatrix}$ are orthogonal to each other for all cases. For the ℓth combination, the receiver determines the ML estimates of x_1 and x_2 using the decomposition as follows

$$\hat{x}_{1,\ell} = \arg\min_{x_1 \in \gamma} \left\| \mathbf{y} - \sqrt{\frac{\rho}{\mu}}\mathbf{h}_{\ell,1}\ x_1 \right\|^2$$
$$\hat{x}_{2,\ell} = \arg\min_{x_2 \in \gamma} \left\| \mathbf{y} - \sqrt{\frac{\rho}{\mu}}\mathbf{h}_{\ell,2}\ x_2 \right\|^2$$

with the associated minimum ML metrics $m_{1,\ell}$ and $m_{2,\ell}$ for x_1 and x_2 are

$$(m_{1,\ell}, m_{2,\ell}) = \left(\min_{x_1 \in \gamma} \left\| \mathbf{y} - \sqrt{\rho/\mu}\mathbf{h}_{\ell,1}\ x_1 \right\|^2, \min_{x_2 \in \gamma} \left\| \mathbf{y} - \sqrt{\rho/\mu}\mathbf{h}_{\ell,2}\ x_2 \right\|^2 \right)$$

- Since $m_{1,\ell}$ and $m_{2,\ell}$ are calculated by the ML decoder for the ℓth combination, their summation

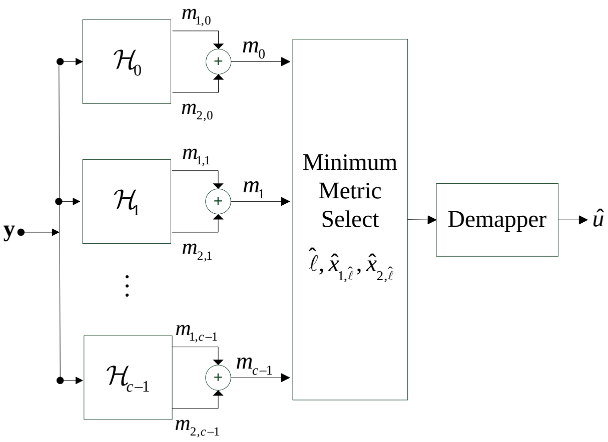
$$m_\ell = m_{1,\ell} + m_{2,\ell}, 0 \leq \ell \leq c-1$$

gives the total ML metric for the ℓth combination.
- Finally, the receiver makes a decision by choosing the minimum antenna combination metric as

$$\hat{\ell} = \arg\min_{\ell} m_\ell$$

for which $(\hat{x}_1, \hat{x}_2) = (\hat{x}_{1,\hat{\ell}}, \hat{x}_{2,\hat{\ell}})$.
- As a result, the total number of ML metric calculations, which was cM^2, is reduced to $2cM$, yielding a linear decoding complexity as is also true for the SM scheme.
- The last step of the decoding process is the demapping operation based on the look-up table used at the transmitter, to recover the input bits.

Block Diagram of the STBC-SM ML Receiver

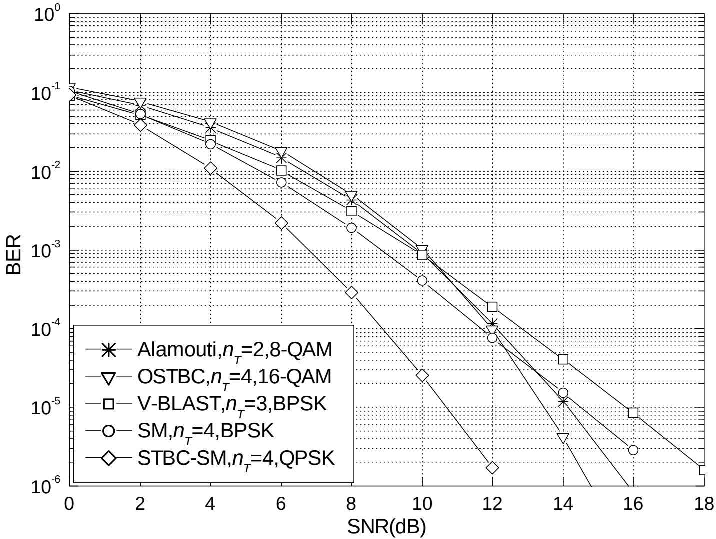


- c different equivalent channel matrices, corresponding different pairwise antenna combinations for STBC-SM, operates on $\mathbf{y}$.

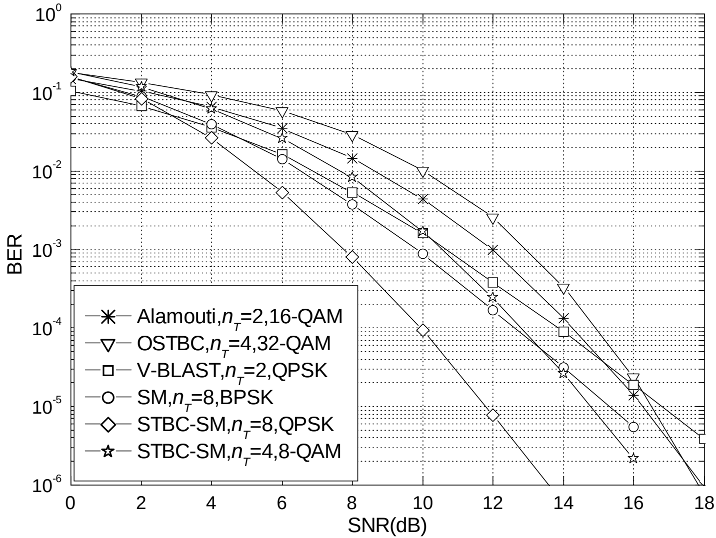
Simulation Results and Comparisons

- STBC-SM systems with different numbers of transmit antennas are considered.
- Comparisons with the SM, V-BLAST, rate-$3/4$ orthogonal STBC for four transmit antennas and Alamouti's STBC are given.
- BER performance of these systems was evaluated via Monte Carlo simulations as a function of the average SNR per receive antenna.
- In all cases we assumed four receive antennas.
- SM system uses the optimal decoder.
- V-BLAST system uses minimum mean square error (MMSE) detection.
- Spatial correlation channel model: $\mathbf{H}_{corr} = \mathbf{R}_t^{1/2}\mathbf{H}\mathbf{R}_r^{1/2}$, $\mathbf{R}_t = [r_{ij}]_{n_T \times n_T}$, $\mathbf{R}_r = [r_{ij}]_{n_R \times n_R}$.
- Exponential correlation matrix model: $r_{ij} = r_{ji}^* = r^{|j-i|}$ and $|r| < 1$.

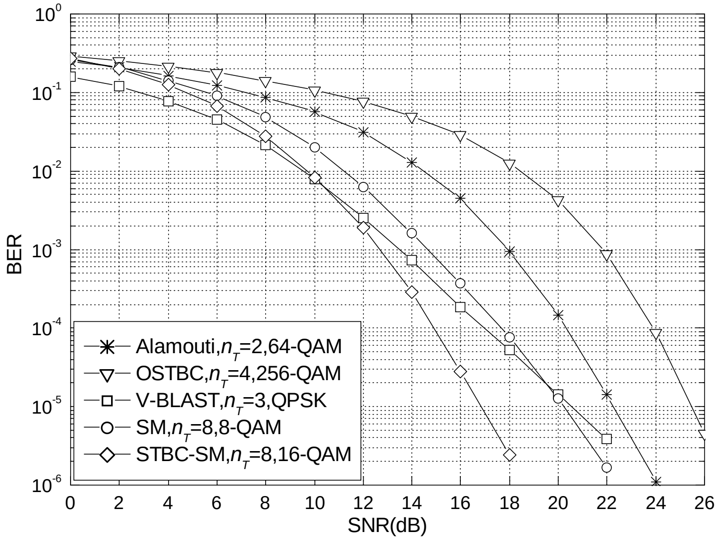
BER performance at 3 bits/s/Hz for STBC-SM, SM, V-BLAST, OSTBC and Alamouti's STBC schemes



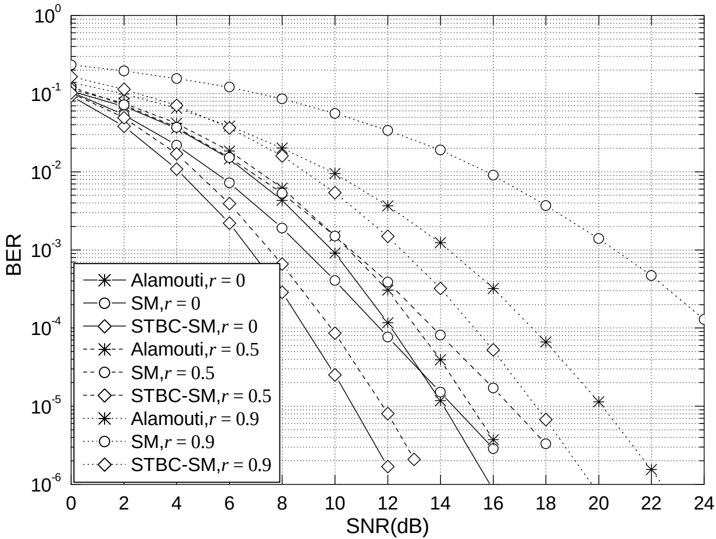
BER performance at 4 bits/s/Hz for STBC-SM, SM, V-BLAST, OSTBC and Alamouti's STBC schemes



BER performance at 6 bits/s/Hz for STBC-SM, SM, V-BLAST, OSTBC and Alamouti's STBC schemes



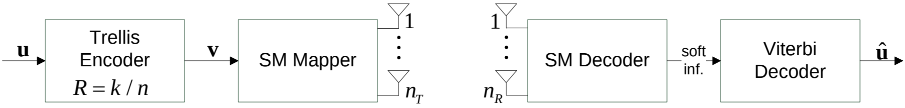
BER performance at 3 bits/s/Hz for STBC-SM, SM, and Alamouti's STBC schemes for Spatially Correlated channel ($\mathbf{r} = 0, 0.5$ and 0.9)



Conclusions

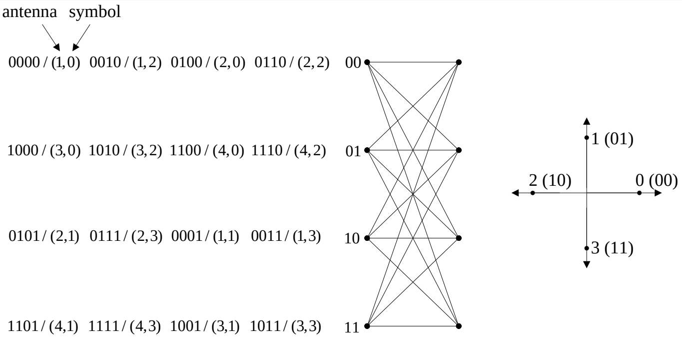
- A novel high-rate, low complexity MIMO transmission scheme, called STBC-SM, has been introduced as an alternative to existing techniques such as SM and V-BLAST.
- A general algorithm has been presented for the construction of the STBC-SM scheme for any number of transmit antennas in which the STBC-SM scheme was optimized by deriving its diversity and coding gains to reach optimal performance.
- It was shown via computer simulations that the STBC-SM offers significant improvements in BER performance compared to SM and V-BLAST systems (approximately 3-5 dB depending on the spectral efficiency) with an acceptable linear increase in decoding complexity.
- However, to obtain additional coding gains, trellis coding is incorporated to SM.

System Model of Trellis Coded Spatial Modulation (TC-SM)



- The spatial modulator is designed in conjunction with the trellis encoder to transmit $n = \log_2 (M n_T)$ coded bits in a transmission interval.
- The SM mapper first specifies the identity of the transmit antenna determined by the first $\log_2 n_T$ bits of the coded sequence $\mathbf{v}$. It than maps the remaining $\log_2 M$ bits of the coded sequence onto the signal constellation employed for transmisson of the data symbols.
- Due to trellis coding, the overall spectral efficiency of the TC-SM would be k bits/s/Hz.

Trellis diagram of the TC-SM scheme with $R = 2/4$ trellis encoder, four transmit antennas and QPSK ($k = 2$ bits/s/Hz)



- At each coding step, the first two coded bits determine the active transmit antenna over which the QPSK symbol determined by the last two coded bits is transmitted.
- The new signal generated by the SM is denoted by $x = (i, s)$ where $s \in \chi$ is the data symbol transmitted over the antenna labeled by $i \in \{1, 2, \cdots, n_T\}$.

- That is, the spatial modulator generates an $1 \times n_T$ signal vector $\left[0 \;\; 0 \;\; \cdots \;\; s \;\; 0 \;\; \cdots \;\; 0\right]$ whose ith entry is s at the output of the n_T transmit antennas for transmission.
- The MIMO channel over which the spatially modulated symbols are transmitted, is characterized by an $n_T \times n_R$ matrix $\mathbf{H}$, whose entries are i.i.d. r.v.'s having the $\mathcal{CN}\left(0,1\right)$ distribution.
- We assume that $\mathbf{H}$ remains constant during the transmission of a frame and takes independent values from one frame to another.
- The transmitted signal is corrupted by an n_R-dimensional AWGN vector with i.i.d. entries distributed as $\mathcal{CN}\left(0,N_0\right)$.
- At the receiver, a soft decision Viterbi decoder, which is fed with the soft information supplied by the optimal SM decoder, is employed to provide an estimate $\hat{\mathbf{u}}$ of the input bit sequence.

Pairwise-Error Probability (PEP) Derivation of the TC-SM Scheme

- The conditional PEP (CPEP) of the TC-SM scheme is derived, and then for quasi-static Rayleigh fading channels, by averaging over channel fading coefficients, the unconditional PEP (UPEP) of the TC-SM scheme is obtained for error events with path lengths two and three.
- For the sake of simplicity, one receive antenna is assumed.
- A pairwise error event of length N occurs when the Viterbi decoder decides in favor of the spatially modulated symbol sequence

$$\hat{\mathbf{x}} = (\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_N)$$

when

$$\mathbf{x} = (x_1, x_2, \ldots, x_N)$$

is transmitted.

- Let the received signal is given as

$$y_n = \alpha_n s_n + w_n$$

for $1 \leq n \leq N$, where α_n is the complex fading coefficient from i_nth transmit antenna to the receiver at the nth transmission inverval, and w_n is the noise sample with $\mathcal{CN}\left(0, N_0\right)$.

- Let $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \ldots, \alpha_N)$ and $\boldsymbol{\beta} = (\beta_1, \beta_2, \ldots, \beta_N)$ denote the sequences of fading coefficients corresponding to transmitted and erroneously detected SM symbol sequences, $\mathbf{x}$ and $\hat{\mathbf{x}}$, respectively.
- The CPEP for this case is given by

$$\Pr\left(\mathbf{x} \rightarrow \hat{\mathbf{x}} \middle| \boldsymbol{\alpha}, \boldsymbol{\beta}\right) = \Pr\left\{ m\left(\mathbf{y}, \hat{\mathbf{x}}; \boldsymbol{\beta}\right) \geq m\left(\mathbf{y}, \mathbf{x}; \boldsymbol{\alpha}\right) \middle| \mathbf{x}\right\}$$

where $m\left(\mathbf{y}, \mathbf{x}; \boldsymbol{\alpha}\right) = \sum_{n=1}^{N} m\left(y_n, s_n; \alpha_n\right) = -\sum_{n=1}^{N} \left|y_n - \alpha_n s_n\right|^2$ is the decision metric for $\mathbf{x}$.

- After some algebraic manipulations,

$$\Pr\left(\mathbf{x} \to \hat{\mathbf{x}} \middle| \boldsymbol{\alpha}, \boldsymbol{\beta}\right) \leq \frac{1}{2} \exp\left(-\frac{\gamma}{4} \sum_{n=1}^{N} \left|\alpha_n s_n - \beta_n \hat{s}_n\right|^2\right)$$

where $\gamma = E_s/N_0 = 1/N_0$ is the average received SNR.

- Note that, if $\alpha_n = \beta_n$ for all n, $1 \leq n \leq N$, this expression reduces the CPEP of the conventional TCM scheme

$$\Pr\left(\mathbf{x} \to \hat{\mathbf{x}} \middle| \boldsymbol{\alpha}, \boldsymbol{\beta}\right) \leq \frac{1}{2} \exp\left(-\frac{\gamma}{4} \sum_{n=1}^{N} \left|\alpha_n\right|^2 \left|s_n - \hat{s}_n\right|^2\right)$$

for which the UPEP can be evaluated easily.

- On the other hand, the derivation of the UPEP for the considered TC-SM scheme in which an interleaver is not included, is quite complicated because of the varying statistical dependence between α and β through error events of path length N.

- The CPEP upper bound of the TC-SM scheme can be alternatively rewritten in matrix form as

$$\Pr\left(\mathbf{x} \rightarrow \hat{\mathbf{x}} | \boldsymbol{\alpha}, \boldsymbol{\beta}\right) \leq \frac{1}{2} \exp\left(-\frac{\gamma}{4} \mathbf{h}^{H} \mathbf{S} \mathbf{h}\right)$$

where $\mathbf{h} = \left[\begin{array}{cccc} h_1 & h_2 & \cdots & h_{n_T} \end{array}\right]^T$ is the $n_T \times 1$ channel vector with $h_i, i = 1, 2, \cdots, n_T$ representing the channel fading coefficient from ith transmit antenna to the receiver.

- $\mathbf{S} = \sum_{n=1}^{N} \mathbf{S}_n$ where $\mathbf{S}_n$ is an $n_T \times n_T$ Hermitian matrix representing a realization of α_n and β_n which are related to the channel coefficients as $\alpha_n = h_{i_n}$, $\beta_n = h_{j_n}$, i_n and $j_n \in \{1, 2, \cdots, n_T\}$ being the transmitted and detected antenna indices, respectively.

Example ($n_T = 4$, $\alpha_n = h_1$, $\beta_n = h_3$, i.e., $i_n = 1$, $j_n = 3$)

$$\mathbf{S}_n = \begin{bmatrix} |s_n|^2 & 0 & -s_n^* \hat{s}_n & 0 \\ 0 & 0 & 0 & 0 \\ -s_n \hat{s}_n^* & 0 & |\hat{s}_n|^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

- In order to obtain the UPEP, the CPEP should be averaged over the multivariate complex Gaussian p.d.f. of $\mathbf{h}$ which is given as

$$f(\mathbf{h}) = (1/\pi^{n_T})\, e^{-\mathbf{h}^H\mathbf{h}}$$

- UPEP upper bound of the TC-SM is calculated as

$$\begin{aligned}\Pr(\mathbf{x} \rightarrow \hat{\mathbf{x}}) &\leq \frac{1}{2}\int_{\mathbf{h}} \pi^{-n_T} \exp\left(-\frac{\gamma}{4}\mathbf{h}^H\mathbf{S}\mathbf{h}\right) \exp\left(-\mathbf{h}^H\mathbf{h}\right) d\mathbf{h} \\ &= \frac{1}{2}\int_{\mathbf{h}} \pi^{-n_T} \exp\left(-\mathbf{h}^H\mathbf{C}^{-1}\mathbf{h}\right) d\mathbf{h}\end{aligned}$$

where $\mathbf{C}^{-1} = \left[\frac{\gamma}{4}\mathbf{S} + \mathbf{I}\right]$ and $\mathbf{I}$ is the $n_T \times n_T$ identity matrix.

- Since $\mathbf{C}$ is a Hermitian and positive definite complex covariance matrix, UPEP upper bound is obtained as

$$\Pr(\mathbf{x} \rightarrow \hat{\mathbf{x}}) \leq \frac{1}{2}\det(\mathbf{C}) = \frac{1}{2\det\left(\frac{\gamma}{4}\mathbf{S} + \mathbf{I}\right)}$$

- On the other hand, for an error event with path length N, the matrix $\mathbf{S}$ has $(n_T)^{2N}$ possible realizations which correspond to all of the possible transmitted and detected antenna indices along this error event.
- However, due to the special structure of $\mathbf{S}$, these $(n_T)^{2N}$ possible realizations can be grouped into a small number of distinct types having the same UPEP upper bound which is mainly determined by the number of degrees of freedom (DOF) of the error event.

Definition

For an error event with path length N, the number of degrees of freedom (DOF) is defined as the total number of different channel fading coefficients in $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ sequences.

Example

For $N = 2$, $\boldsymbol{\alpha} = (\alpha_1, \alpha_2)$ and $\boldsymbol{\beta} = (\beta_1, \beta_2)$, DOF $= 3$ if $\alpha_1 = \beta_1 \neq \alpha_2 \neq \beta_2$.

- η and $\tilde{\eta}$ being the sets of all n for which $\alpha_n = \beta_n$ and $\alpha_n \neq \beta_n$, respectively, and $n(\eta) + n(\tilde{\eta}) = N$, let us rewrite the CPEP expression for the TC-SM scheme as

$$\Pr\left(\mathbf{x} \rightarrow \hat{\mathbf{x}} \middle| \boldsymbol{\alpha}, \boldsymbol{\beta}\right) \leq \frac{1}{2} \exp\left(-\frac{\gamma}{4}\left[\sum\nolimits_{\eta} \left|\alpha_n\right|^2 d_{E_n}^2 + \sum\nolimits_{\tilde{\eta}} \left|\alpha_n s_n - \beta_n \hat{s}_n\right|^2\right]\right)$$

Note

Besides the DOF, $n(\eta)$ and $n(\tilde{\eta})$ also affects the UPEP of the TC-SM scheme.

- η and $\tilde{\eta}$ being the sets of all n for which $\alpha_n = \beta_n$ and $\alpha_n \neq \beta_n$, respectively, and $n(\eta) + n(\tilde{\eta}) = N$, let us rewrite the CPEP expression for the TC-SM scheme as

$$\Pr\left(\mathbf{x} \to \hat{\mathbf{x}} \middle| \boldsymbol{\alpha}, \boldsymbol{\beta}\right) \leq \frac{1}{2} \exp\left(-\frac{\gamma}{4}\left[\sum\nolimits_{\eta} |\alpha_n|^2 d_{E_n}^2 + \sum\nolimits_{\tilde{\eta}} |\alpha_n s_n - \beta_n \hat{s}_n|^2\right]\right)$$

TCM Term

Note

Besides the DOF, $n(\eta)$ and $n(\tilde{\eta})$ also affects the UPEP of the TC-SM scheme.

- η and $\tilde{\eta}$ being the sets of all n for which $\alpha_n = \beta_n$ and $\alpha_n \neq \beta_n$, respectively, and $n(\eta) + n(\tilde{\eta}) = N$, let us rewrite the CPEP expression for the TC-SM scheme as

$$\Pr\left(\mathbf{x} \rightarrow \hat{\mathbf{x}} \middle| \boldsymbol{\alpha}, \boldsymbol{\beta}\right) \leq \frac{1}{2} \exp\left(-\frac{\gamma}{4}\left[\sum\nolimits_{\eta} |\alpha_n|^2 d_{E_n}^2 + \sum\nolimits_{\tilde{\eta}} |\alpha_n s_n - \beta_n \hat{s}_n|^2\right]\right)$$

TCM Term — SM Term

Note

Besides the DOF, $n(\eta)$ and $n(\tilde{\eta})$ also affects the UPEP of the TC-SM scheme.

UPEP values for $N = 2$

Case	PEP(high SNR)
$n(\eta) = 0$, DOF $= 2^*$	$4/(1-\cos\theta)\gamma^2$
$n(\eta) = 0$, DOF $= 3$	$8/3\gamma^2$
$n(\eta) = 0$, DOF $= 3$	$2/\gamma^2$
$n(\eta) = 1$, DOF $= 2$	$8/d_{E_m}^2\gamma^2$
$n(\eta) = 1$, DOF $= 3$	$4/d_{E_m}^2\gamma^2$

UPEP values for $N = 3$

Case	PEP(high SNR)
$n(\eta) = 0$, DOF $= 3$	$16/\left(1-\cos\tilde{\theta}\right)\gamma^3$
$n(\eta) = 0$, DOF $= 3^*$	$16/(1-\cos\theta)\gamma^3$
$n(\eta) = 0$, DOF $= 4$	$8/\gamma^3$
$n(\eta) = 0$, DOF $= 4^*$	$8/(1-\cos\theta)\gamma^3$
$n(\eta) = 0$, DOF $= 5$	$16/3\gamma^3$
$n(\eta) = 0$, DOF $= 6$	$4/\gamma^3$
$n(\eta) = 1$, DOF $= 2^*$	$4/\left(1+d_{E_m}^2-\cos\theta\right)\gamma^2$
$n(\eta) = 1$, DOF $= 3$	$32/d_{E_m}^2\gamma^3$
$n(\eta) = 1$, DOF $= 3^*$	$16/(1-\cos\theta)d_{E_m}^2\gamma^3$
$n(\eta) = 1$, DOF $= 4$	$32/3d_{E_m}^2\gamma^3$
$n(\eta) = 1$, DOF $= 5$	$8/d_{E_m}^2\gamma^3$

- M-PSK constellation is assumed.
- $\theta = \pm\Delta\theta_1 \pm \Delta\theta_2$, $\tilde{\theta} = \pm\Delta\theta_1 \pm \Delta\theta_2 \pm \Delta\theta_3$, $\Delta\theta_n = \theta_n - \hat{\theta}_n, n = 1,2,3$ and $s_i = e^{j\theta_i}, \hat{s}_i = e^{j\hat{\theta}_i}$ with $\theta_i, \hat{\theta}_i \in \left\{\frac{2\pi r}{M}, r = 0, \cdots, M-1\right\}$ and $m \in [1, N]$. $d_{E_m}^2 = |s_m - \hat{s}_m|^2$.
- The asterisk for DOF values means the considered UPEP value is dependent on θ.
- As seen from these tables, for an error event with path length N, a diversity order of N is achieved if DOF $\geq N$.

TC-SM Code Design Criteria

Theorem

In case of an error event with path length N, in order to achieve a diversity order of N (an UPEP upper bound of a/γ^N for $\gamma \gg 1$ and $a \in \mathbb{R}^+$), a necessary condition is DOF $\geq N$.
Proof: It is shown that rank$(\mathbf{S}) = N$ only for DOF $\geq N$.

Diversity gain criterion

For a trellis code with minimum error event length N, to achieve a diversity order of N, DOF must be greater than or equal to N for all error events with path length greater than or equal to N.

Coding gain criterion

After ensuring maximum diversity gain, the distance spectrum of the TC-SM should be optimized by considering the calculated UPEP values.

TC-SM Design Examples

State	$k = 2$ bits/s/Hz	$k = 3$ bits/s/Hz	$k = 4$ bits/s/Hz
4	$\begin{bmatrix} 0 & 3 & 0 & 1 \\ 1 & 0 & 2 & 0 \end{bmatrix}$	-	-
8	$\begin{bmatrix} 0 & 2 & 4 & 2 \\ 3 & 4 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \\ 1 & 0 & 0 & 2 & 0 & 0 \end{bmatrix}$	-
16	$\begin{bmatrix} 5 & 1 & 3 & 0 \\ 1 & 4 & 0 & 3 \end{bmatrix}^*$	$\begin{bmatrix} 0 & 4 & 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 4 & 0 & 2 \\ 3 & 0 & 5 & 0 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 2 & 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 2 \end{bmatrix}$

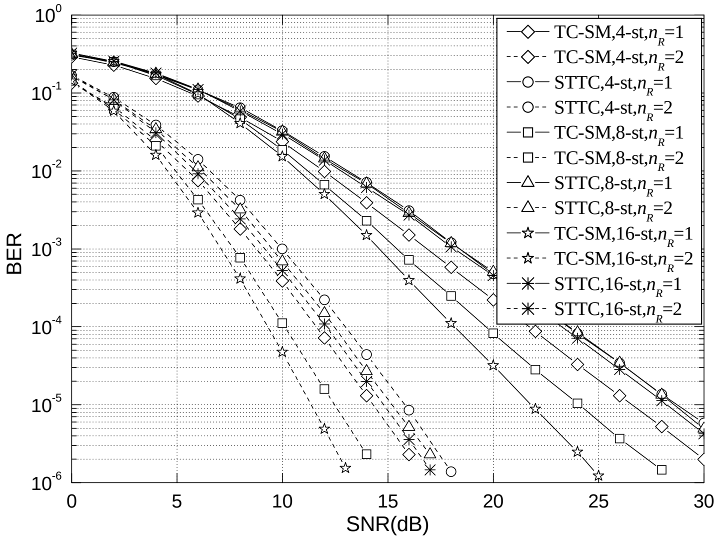
*Time diversity order of three is achieved, since DOF ≥ 3

- All codes are designed according to the TC-SM design criteria.
- 2 bits/s/Hz transmission $\Longrightarrow n_T = 4$, QPSK, $R = 2/4$ trellis encoder
- 3 bits/s/Hz transmission $\Longrightarrow n_T = 8$, 8-PSK, $R = 3/6$ trellis encoder
- 4 bits/s/Hz transmission $\Longrightarrow n_T = 8$, 8-PSK, $R = 4/6$ trellis encoder

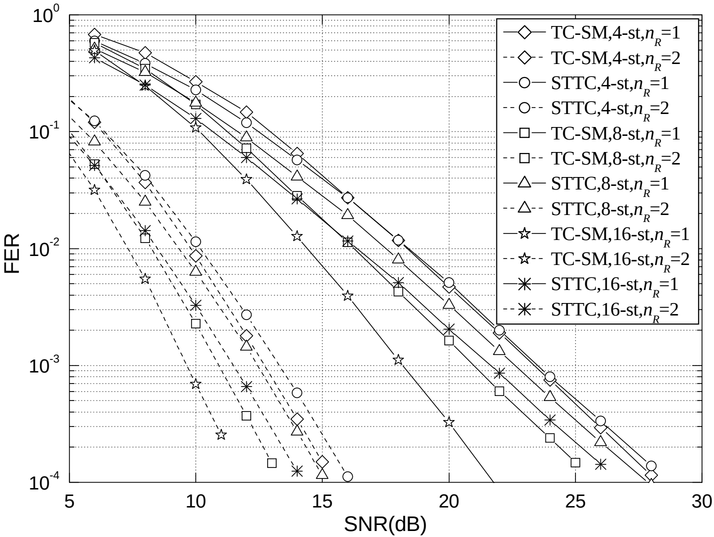
Simulation Resuts and Comparisons

- We present simulation results for the TC-SM scheme with different configurations and make comparisons with the following reference systems:
  - STTC: The optimal space-time trellis codes for $n_T = 2$
  - scheme-Mesleh et al.: The suboptimum trellis coded SM scheme proposed by Mesleh et al.
  - coded V-BLAST-I: Vertically encoded (single coded) V-BLAST with hard decision Viterbi decoder
  - coded V-BLAST-II: Coded V-BLAST with soft decision Viterbi decoder
- Frame length was chosen as $20k$ bits for both the TC-SM and the reference systems operating at $k = 2, 3$ and 4 bits/s/Hz spectral efficiencies.
- Quasi-static Rayleigh fading was assumed.

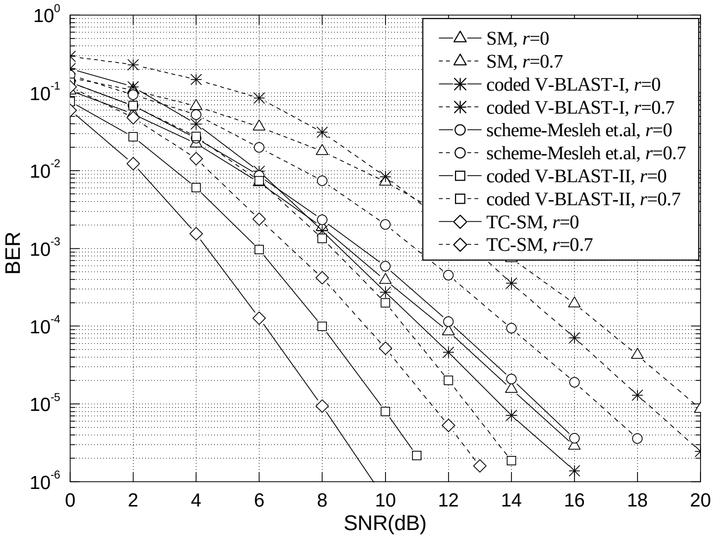
BER performance for 4-,8- and 16-state TC-SM and STTC schemes at 2 bits/s/Hz



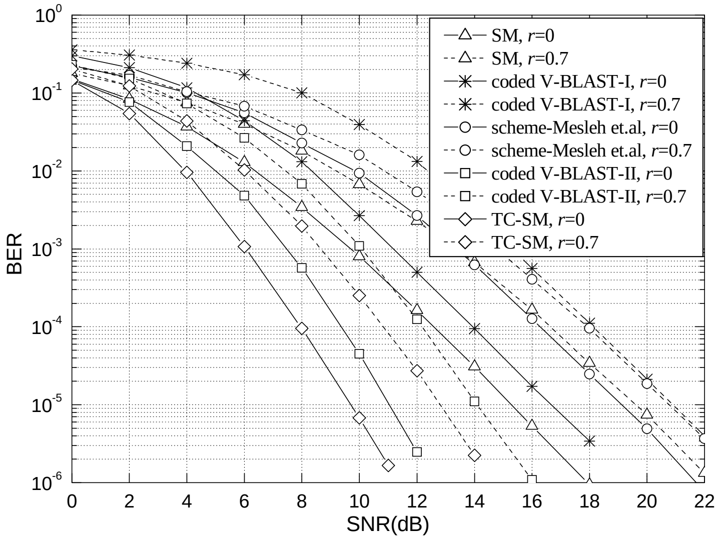
FER performance for 4-,8- and 16-state TC-SM and STTC schemes at 2 bits/s/Hz



BER comparison at 3 bits/s/Hz for uncorrelated and spatially correlated channels, $n_R = 4$



BER comparison at 4 bits/s/Hz for uncorrelated and spatially correlated channels, $n_R = 4$



Complexity Comparison

- For a given spectral efficiency and number of trellis states, it is observed that the number of metric calculations performed by the soft decision Viterbi decoder is the same as TC-SM codes and STTCs.
- However, since only one transmit antenna is active in our scheme, contrary to the reference STTCs with the same trellis structure in which two antennas transmit simultaneously, TC-SM provides 25% and 33% reductions in the number of real multiplications and real additions per single branch metric calculation of the Viterbi decoder, respectively, for 2 bits/s/Hz.
- These values increase to 30% and 37.5% for 3 bits/s/Hz.
- From an implementation point of view, unlike the STTCs, our scheme requires only one RF chain at the transmitter, even if we have a higher number of transmit antennas, and requires no synchronization between them.

Conclusions

- We have introduced a novel coded MIMO transmission scheme which directly combines trellis coding and SM.
- Although one transmit antenna is active during transmission, for quasi-static fading channels, we benefit from the time diversity provided by the SM-TC mechanism, which is forced by our code design criteria to create a kind of virtual interleaving by switching between the transmit antennas of a MIMO link.
- We have proposed some new TC-SM codes which offer significant error performance improvements over its counterparts while having a lower decoding complexity for $2, 3$ and 4 bits/s/Hz transmissions.
- The price is paid by the increased number of transmit antennas.

Further Reading

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