Trellis Coded Space-Shift Keying Modulation

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Abstract—A new multiple-input multiple-output (MIMO) transmission scheme named trellis coded space-shift keying (TC-SSK) is proposed in this paper. This scheme is obtained by the combination of trellis coding and SSK for a given spectral efficiency, number of transmit antennas and trellis states. A soft decision Viterbi decoder is employed at the receiver to decode the transmitted SSK symbols. Using systematic methods, new TC-SSK schemes are obtained for 1 and 2 bits/s/Hz spectral efficiencies. An upper bound for the pairwise error probability (PEP) of the TC-SSK scheme is calculated for quasi-static Rayleigh fading channels. Computer simulation results show that the proposed TC-SSK schemes achieve significantly better error performance than the uncoded SSK, and the theoretical error probability curves are in close match with the computer simulation results. Furthermore, TC-SSK provides an attractive trade-off between complexity and performance compared to the spatial modulation with trellis coding.

I. INTRODUCTION

Multiple-input multiple-output (MIMO) transmission techniques have been implemented in many practical applications, due to their benefits over single antenna systems. Recently, a novel concept known as spatial modulation (SM), which uses the spatial domain to convey information in addition to the classical signal constellations, has emerged as a promising MIMO transmission technique [1], [2], [3]. The SM technique has been proposed as an alternative to existing MIMO transmission strategies such as Vertical Bell Laboratories layered space-time (V-BLAST) and space-time coding which are widely used in today’s wireless standards. The fundamental principle of SM is an extension of two dimensional signal constellations (such as M-ary phase shift keying (M-PSK) and M-ary quadrature amplitude modulation (M-QAM), where M is the constellation size) to a new third dimension, which is the spatial (antenna) dimension. Therefore, in the SM scheme, the information is conveyed both by the amplitude/phase modulation techniques and by the antenna indices. Since only one transmit antenna is active during each symbol transmission, inter-carrier interference is completely eliminated in SM and this results in much lower (linear) decoding complexity. Furthermore, SM does not require synchronization between the transmit antennas of the MIMO link and only one radio frequency (RF) chain is needed at the transmitter.

In [4], by removing M-ary constellations and transmitting the data using the indices of the transmit antennas only, a space-shift keying (SSK) scheme has been proposed. Compared to SM, detection complexity is considerably lowered and the transceiver requirements are reasonably relaxed for SSK by the elimination of amplitude and phase modulations. In [5], SM is combined with trellis coding to obtain additional diversity and coding gains. However, no method for designing reliable trellis coded space-shift keying codes systematically has yet been discovered, and the combination of SSK with trellis coding remains an open problem.

In this study, a new MIMO transmission strategy called trellis coded space-shift keying (TC-SSK) is proposed by directly combining trellis coding and SSK. In this scheme, the trellis encoder and the SSK mapper are jointly designed and a soft decision Viterbi decoder aided by an SSK detector is employed at the receiver. Using systematic methods, new TC-SSK schemes with 2, 4 and 8 states, are proposed for 1 and 2 bits/s/Hz spectral efficiencies. The conditional pairwise error probability (CPEP) upper bound of TC-SSK is derived, then, by averaging over channel fading coefficients for Rayleigh fading, the unconditional PEP (UPEP) of TC-SSK scheme is obtained to calculate its average bit error probability (ABEP). It is shown by computer simulations that the proposed TC-SSK schemes achieve significantly better error performance than the uncoded SSK, and the theoretical ABEP curves are in close match with the computer simulation results.

The organization of the paper is as follows. In Section II, we give our system model and introduce the new TC-SSK scheme. In Section III, PEP upper bound for the TC-SSK scheme is derived. Simulation results are given in Section IV. Finally, Section V includes the main conclusions of the paper.

II. SYSTEM MODEL

The considered TC-SSK system model operating over a MIMO system having nT transmit and nR receive antennas is depicted in Fig. 1. At the transmitter, after the encoding of independent and identically distributed (i.i.d.) binary information sequence u with a rate \( R = k/n \) trellis (convolutional) encoder, the encoded bit sequence v is processed by the

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Notation: Bold lowercase and uppercase letters are used for column vectors and matrices, respectively. \((.)^T\) and \((.)^H\) denote transposition and Hermitian transposition, respectively. \(\|\cdot\|\), \(\|\cdot\|_F\) and \(\otimes\) stand for Euclidean norm, Frobenius norm and the Kronecker product, respectively. det(A) denote the determinant of A and \(\mathbf{I}_n\) is the \(n \times n\) identity matrix. vec(A) is the operator which creates a column vector from A by stacking its column vectors. \(\mathbb{R}\) and \(\mathbb{C}\) denote the ring of real and complex numbers, respectively. The probability of an event is denoted by \(Pr(\cdot)\). Probability distribution function (p.d.f.) of a random vector x is denoted by \(f(x)\).
SSK modulator (mapper). The number of transmit antennas is adjusted according to \( n = \log_2(n_T) \) so that \( n \) coded bits are transmitted in a transmission interval. According to the SSK principle, only one transmit antenna is active at each transmission interval, and the activated antenna transmits a unit energy signal without \( M \)-ary amplitude or phase modulation. Natural mapping is considered at the SSK transmitter to determine the index of the active transmit antenna according to incoming bit sequence \( v \). Owing to the redundancy provided by the trellis coding, the overall spectral efficiency of the TC-SSK becomes \( k \) bits/s/Hz. Let us denote the signal generated by SSK mapper (SSK symbol) at the \( t \)th transmission interval by \( i_t \) where \( i_t \in \{1, \ldots, n_T\} \). After the generation of the SSK transmission vectors whose all entries are zero except their \( i_t \)th entries, which is equal to one for the \( t \)th transmission interval, these vectors are transmitted over the MIMO channel which experiences flat Rayleigh fading. At the receiver, first, the corresponding branch metrics are calculated by the SSK decoder, then, a soft decision Viterbi decoder is employed to determine the most likely information sequence \( \hat{u} \) that could be transmitted.

We introduce the TC-SSK transmission concept by the trellis code example given in Fig. 2, where \( 0/00/(1) \) indicates that for the corresponding state transition, the related input bit, the coded output bits, and the SSK symbol are given as 0, 00 and 1, respectively. This 4-state trellis code has a transmission rate of \( R = 1/2 \) and uses \( n_T = 4 \) transmit antennas. We consider a \( R = 1/2 \) trellis encoder with the generator matrix \( [D \quad 1 + D^2] \), where \( D \) denotes the unit delay operator. At each transmission interval, two coded bits determine the index of the active transmit antenna according to the natural mapping as shown in Fig. 2.

According to the system model given in Fig. 1, we provide the system parameters and generator matrices of the TC-SSK codes in Table I for different numbers of transmit antennas, transmission rates and trellis states. Different SSK symbols are assigned to the branches of the trellis so that a catastrophic code is avoided. In the spatial modulation with trellis coding (SM-TC) scheme [5], it was not possible to obtain optimal trellis codes systematically and the corresponding trellis codes were obtained after the optimization of the distance spectra of the trellis codes, which is a difficult task. However, in the TC-SSK scheme, all SSK symbols are equidistant to each other, and this simplifies the code design considerably. Therefore, for a given trellis state number and transmission rate, it is possible to obtain the optimal TC-SSK scheme easily by assigning the corresponding SSK symbols to the transitions of the trellis. Furthermore, due to the symmetrical design and equidistant SSK symbols, there is not a single optimal code, and it is possible to obtain optimal trellis codes by many different, but equivalent trellis transition assignments. As an example, consider the SSK symbols assigned to the trellis transitions of the TC-SSK code given in Fig. 2 as \( \{(1, 2), (3, 4), (2, 1), (4, 3)\} \), the following three trellis codes are identical to this one in terms of the ABEP: \( \{(3, 4), (1, 2), (4, 3), (2, 1)\}, \{(1, 3), (2, 4), (3, 1), (4, 2)\}, \) and \( \{(1, 4), (2, 3), (4, 1), (3, 2)\} \).

### Table I

<table>
<thead>
<tr>
<th>System Parameters</th>
<th>Generator Matrix ( G(D) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_T = 4, R = 1/2, 2\text{-state} )</td>
<td>( \begin{bmatrix} D &amp; 1 \end{bmatrix} )</td>
</tr>
<tr>
<td>( n_T = 4, R = 1/2, 4\text{-state} )</td>
<td>( \begin{bmatrix} D &amp; 1 + D^2 \end{bmatrix} )</td>
</tr>
<tr>
<td>( n_T = 8, R = 1/3, 8\text{-state} )</td>
<td>( \begin{bmatrix} D^2 &amp; D &amp; 1 + D^3 \end{bmatrix} )</td>
</tr>
<tr>
<td>( n_T = 8, R = 2/3, 4\text{-state} )</td>
<td>( \begin{bmatrix} 0 &amp; 1 + D &amp; D &amp; D \end{bmatrix} )</td>
</tr>
<tr>
<td>( n_T = 8, R = 2/3, 8\text{-state} )</td>
<td>( \begin{bmatrix} 0 &amp; 1 &amp; D &amp; D &amp; D &amp; 1 + D^2 \end{bmatrix} )</td>
</tr>
</tbody>
</table>

### III. Pairwise-Error Probability (PEP) Derivation of the TC-SM Scheme

In this section, first, the CPEP of the TC-SSK scheme is derived, and then by averaging CPEP over channel fading coefficients, the UPEP of the TC-SSK scheme is obtained. Let us assume that \( x = (i_1, i_2, \ldots, i_N) \) is the sequence of the transmitted SSK symbols from the transmitter, where \( i_n \in \{1, \ldots, n_T\} \) for \( n = 1, \ldots, N \) represents the index of the activated transmit antenna at the \( n \)th transmission interval. Alternatively, the transmitted SSK symbols can be expressed in transmission matrix form as \( X = [i_1^T \ i_2^T \ \cdots \ i_N^T]^T \), where \( i_n, n = 1, \ldots, N \) is an \( 1 \times n_T \) all-zero vector whose \( i_n \)th entry is one and \( X \in \mathbb{R}^{N \times n_T} \). As an example, assume that
\( x = (1, 2, 4) \) for \( n_T = 4 \) and \( N = 3 \). For this case, \( X \) is given as
\[
X = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.
\]
The input-output relationship of the proposed scheme can be expressed in matrix form as
\[
Y = XH + W
\]
where \( H \) and \( W \) denote the \( n_T \times n_R \) channel matrix and \( N \times n_R \) noise matrix, respectively. The entries of \( H \) and \( W \) are assumed to be i.i.d. complex Gaussian random variables with zero means, with unit and \( N_0 \) variances, respectively. We assume that \( H \) remains constant during the transmission of a frame and takes independent values from one frame to another. We further assume that \( H \) is known at the receiver, but not at the transmitter. A pairwise error event of length \( N \) occurs when the Viterbi decoder decides in favor of \( \hat{X} \) when \( X \) is transmitted. The CPEP for this case is given by
\[
Pr(X \rightarrow \hat{X} \mid H) = Pr\left( \|Y - XH\|_F^2 < \|Y - XH\|_F^2 \right).
\]
After some matrix algebra, the CPEP can be expressed as [6]
\[
Pr(X \rightarrow \hat{X} \mid H) = Q \left( \sqrt{\frac{\gamma \|X - \hat{X}H\|_F^2}{2}} \right)
\]
where \( \gamma = 1/N_0 \) is the received signal-to-noise ratio (SNR) at each receive antenna. Using the well-known bound \( Q(x) \leq \frac{1}{2} e^{-x^2/2} \), the CPEP of the proposed scheme can be upper bounded by
\[
Pr(X \rightarrow \hat{X} \mid H) \leq \frac{1}{2} \exp \left( -\frac{\gamma \|X - \hat{X}H\|_F^2}{4} \right)
\]
where \( A = I_{n_R} \otimes (X - \hat{X}) \in \mathbb{R}^{n_T n_R \times n_T n_R} \), \( h = \text{vec}(H) \in \mathbb{C}^{n_T n_R \times 1} \). Considering \( \|Ah\|^2 = h^H A^H A h \) and multivariate complex Gaussian p.d.f. of \( h \) which is given as
\[
f(h) = \pi^{-n_T n_R} \exp \left( -\frac{h^H h}{4} \right),
\]
the UPEP upper bound of the proposed scheme can be calculated as follows:
\[
Pr(X \rightarrow \hat{X}) \leq \frac{1}{2} \int \pi^{-n_T n_R} \exp \left( -\frac{\gamma}{4} [h^H A^H A h] \right) \exp \left( -\frac{h^H h}{4} \right) dh
\]
\[
= \frac{1}{2} \int \pi^{-n_T n_R} \exp \left( -\frac{h^H \Sigma^{-1} h}{4} \right) dh
\]
where \( \Sigma^{-1} = \frac{1}{4} A^H A + I_{n_T n_R} \). We observe that \( \Sigma \) is a Hermitian and positive definite complex covariance matrix; therefore, we obtain the following result from the integrand of the multivariate complex Gaussian p.d.f. given in (6):
\[
Pr(X \rightarrow \hat{X}) \leq \frac{1}{2} \text{det} \left( \frac{1}{4} S + I_{n_T n_R} \right)
\]
where \( S = A^H A \) [5]. Considering that \( S = I_{n_R} \otimes D \) where \( D = (X - \hat{X})^H (X - \hat{X}) \), we obtain the following final result:
\[
Pr(X \rightarrow \hat{X}) \leq \frac{1}{2} \text{det} \left( \frac{1}{4} D + I_{n_T n_R} \right)
\]
After the calculation of the UPEP upper bound of the TC-SSK scheme, the approximate BEP performance of the TC-SSK scheme is calculated by considering error events with lengths up to a predetermined finite \( N \) value as, [7],
\[
P_b \approx \frac{1}{c} \sum_{x} \left[ \frac{1}{k} \sum_{\tilde{x} \neq x} e(\tilde{x}, \hat{X}) \Pr(X \rightarrow \hat{X}) \right],
\]
where \( k \) is the number of input bits per trellis transition, \( c \) is the total number of different transmission matrices and \( e(\tilde{x}, \hat{X}) \) is the number of bit errors associated with the corresponding error event. In the following, we give a generic example to show the accuracy of (9) for our scheme.

**Example** (TC-SSK, \( n_T = 4, n_R = 1, R = 1/2, 4\)-state): For this trellis code, the minimum error event path length is equal to \( N = 3 \) with \( k = 3 \), and due to symmetrical design, if we only consider the error events originating from the first state, we have \( c = 8 \) different realizations of \( x \). As an example, for the correct transmission path \( x = (1, 1, 1) \), we may have a single erroneous path \( \tilde{x} = (2, 3, 2) \) as indicated in Fig. 3. In order to implement (9), we have to consider the following error events: \( x = (1, 1, 2) \) and \( \tilde{x} = (2, 3, 1) \), \( x = (1, 2, 3) \) and \( \tilde{x} = (2, 4, 3) \) and \( x = (1, 2, 4) \) and \( \tilde{x} = (2, 4, 3) \). For the remaining four error events, \( x \) and \( \tilde{x} \) sequences swap their values, therefore, no new calculations will be required. For the first two error events, the \( D \) matrix is calculated as follows:
\[
D = \begin{bmatrix} 3 & -2 & -1 \ 0 & 2 & 0 \ -1 & 0 & 1 \ 0 & 0 & 0 \ \end{bmatrix}
\]
which yields the following UPEP upper bound from (8),
\[
\Pr(X \rightarrow \hat{X})_{1,2} \leq \frac{1/2}{1 + \frac{3\gamma}{2} + \frac{3\gamma^2}{8}},
\]
while the corresponding values for the third and fourth error events are as follows:
\[
D = \begin{bmatrix}
1 & -1 & 0 & 0 \\
-1 & 2 & 0 & -1 \\
0 & 0 & 1 & -1 \\
0 & -1 & -1 & 2
\end{bmatrix}
\]
\[
\Pr(X \rightarrow \hat{X})_{3,4} \leq \frac{1/2}{1 + \frac{3\gamma}{2} + \frac{5\gamma^2}{8} + \frac{5\gamma^3}{80}}.
\]
Considering that \(e(X, \hat{X}) = 1\) for all error events, from (9), we obtain the approximate ABEP of the considered TC-SSK as
\[
P_b \approx \frac{\Pr(X \rightarrow \hat{X})_{1,2} + \Pr(X \rightarrow \hat{X})_{3,4}}{2}
\]
which is given at the top of Table II.

Using similar procedures, the ABEP of two other TC-SSK schemes is calculated for \(N = 2\), which considers the most dominant error events, and given in Table II for \(n_R = 1\). As seen from the results given in Table II, the considered three TC-SSK schemes achieve a time diversity order of two since their ABEP decay with the second order of the SNR.

**Remark 1 (performance comparison):** We can compare the relative performances of these three TC-SSK schemes at high SNR (\(\gamma \gg 1\)). Ignoring the other terms, the asymptotic ABEP values of the trellis codes given in Table II are calculated from top to the bottom as \(\frac{1}{2\gamma^2}\), \(\frac{8}{3\gamma^2}\), and \(\frac{14}{3\gamma^2}\), respectively. As expected, even if using \(n_T = 8\) transmit antennas, \(R = 2/3\) code exhibits the highest asymptotic ABEP because of its higher rate, while among \(R = 1/2\) codes, \(4\)-state code achieves a better asymptotic ABEP performance than the \(2\)-state code because of its higher trellis complexity.

**IV. Simulation Results and Comparisons**

In this section, we present computer simulation results for the TC-SSK scheme with different configurations and make comparisons with the reference systems. The bit error rate (BER) performance of these schemes was evaluated via Monte Carlo simulations for various transmission rates and numbers of trellis states as a function of the average SNR per receive antenna. In all cases, the decision depth of the Viterbi decoder was chosen to be 20, which corresponds to a frame length of 20k bits for a TC-SSK scheme employing a \(R = k/n\) trellis code. For convenience, we assume \(n_R = 1\). However, increasing \(n_R\) will increase the system error performance by linearly increasing the diversity order, the price paid being an increase in decoding complexity.

In Fig. 4, the BER performance of three different TC-SSK schemes is shown with the theoretical results given in Table II. As seen from Fig. 4, for the \(n_T = 4, R = 1/2, 2\)-state code, there is an approximately 2 dB SNR gap between the theoretical and computer simulation curves for a fixed BER value, which can be explained by this code’s trellis structure. In other words, for this trellis code, error events with higher lengths (\(N > 2\)) have a comparable probability as those of error events with path length \(N = 2\). On the other hand, as seen from Fig. 4, the theoretical curve matches very well with the computer simulation curve for the \(n_T = 4, R = 1/2, 4\)-state code. This can be explained by the fact that for this code, the minimum error event path length is \(N = 3\) and error events with path length \(N = 3\) are the most dominant among all error events. Finally, we provide the theoretical and computer simulation curves for the \(n_T = 8, R = 2/3, 4\)-state code. As seen from Fig. 4, the theoretical ABEP curve is reasonably close to the computer simulation curve for this code.

In Fig. 5, we compare the performance of three different TC-SSK schemes with the uncoded SSK for 1 bits/s/Hz transmission. For this transmission rate, uncoded SSK uses \(n_T = 2\) transmit antennas, while TC-SSK schemes uses \(R = 1/2\) or \(R = 1/3\) trellis encoders. As seen from Fig. 5, the proposed TC-SSK schemes exhibit considerably better BER performance compared to the uncoded SSK scheme. As an example, for a BER value of \(10^{-4}\), the proposed \(n_T = 4, R = 1/2, 2\)-state and \(n_T = 4, R = 1/2, 4\)-state TC-SSK schemes provide 16.6 dB and 18 dB better BER performance than the uncoded SSK. It is important to note that the proposed \(n_T = 8, R = 1/3, 8\)-state scheme achieves a time diversity order of three due to its trellis structure, which is verified by the computer simulation results given in Fig. 5. For this trellis code, the minimum error event path is \(N = 4\), and it can be easily shown that all error events provide a diversity order of at least three.

We extend our studies to 2 bits/s/Hz transmission in Fig.
TABLE II
ABEP OF TC-SSK SCHEMES WITH DIFFERENT PARAMETERS

<table>
<thead>
<tr>
<th>Code Parameters</th>
<th>ABEP</th>
<th>Asymptotic ABEP (γ ≫ 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>n_T = 4, R = 1/2, 4-st</td>
<td>1/4 [1 + \frac{3\gamma^2}{2} + \frac{3\gamma^2}{8} + \frac{5\gamma^6}{32}]</td>
<td>[2 \frac{8}{3\gamma^2}]</td>
</tr>
<tr>
<td>n_T = 4, R = 1/2, 2-st</td>
<td>1/2 [1 + \gamma + \frac{3\gamma^2}{16}]</td>
<td>[8 \frac{8}{3\gamma^2}]</td>
</tr>
<tr>
<td>n_T = 8, R = 2/3, 4-st</td>
<td>[1/2 \frac{1 + \gamma + \frac{3\gamma^2}{16}}{1 + \gamma + \frac{\gamma^2}{4}}]</td>
<td>[14 \frac{14}{3\gamma^2}]</td>
</tr>
</tbody>
</table>

Fig. 5. BER performance for different TC-SSK schemes at 1 bits/Hz

Fig. 6. BER performance for different TC-SSK schemes at 2 bits/Hz

6. As seen from Fig. 6, the proposed TC-SSK schemes provide superior performance compared to the uncoded SSK. As expected, the proposed n_T = 8, R = 2/3, 8-state code outperforms n_T = 8, R = 2/3, 4-state code due to its higher trellis complexity. In Fig. 6, we also show the BER performance of the 4-state SM-TC scheme [5] with n_T = 4 at the same transmission rate, which has the same trellis structure as that of n_T = 8, R = 2/3, 4-state TC-SSK code. As seen from Fig. 6, 4-state SM-TC scheme achieves 1.2 dB better BER performance than the 4-state TC-SSK scheme. However, due to the simple structure of SSK and using a less total number of different symbols on trellis transitions for the 4-state TC-SSK scheme, per each step of the Viterbi decoder, 4-state SM-TC scheme requires 96 real multiplications (RMs) and 80 real additions (RAs), while these values are equal to 16 and 24 for the 4-state TC-SSK scheme, respectively. Therefore, TC-SSK scheme provides an attractive tradeoff between complexity and performance compared to the SM-TC by reducing the number of RMs and RAs by 83.3% and 70%, respectively.

V. CONCLUSIONS

A novel trellis coded MIMO scheme has been proposed in this paper. The proposed TC-SSK concept combines trellis coding with the SSK principle to obtain simple yet effective trellis codes in a systematic way. The PEP of the proposed TC-SSK scheme has been derived and it has been shown by computer simulations that the proposed scheme exhibits considerably better BER performance than the uncoded SSK.

REFERENCES