Orthogonal Frequency Division Multiplexing with Index Modulation

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Outline

Introduction

- Multicarrier transmission has become a key technology for wideband digital communications in recent years and has been included in many wireless standards.
- OFDM has been the most popular multicarrier transmission technique in wireless communications.
- Similarly, multiple-input multiple-output (MIMO) transmission techniques have been implemented in many practical applications, due to their benefits over single antenna systems.
- Spatial modulation (SM), which uses the spatial domain to convey information in addition to the classical signal constellations, has emerged as a promising MIMO transmission technique.

Motivation: Application of SM Principle to OFDM

- The application of the SM principle to the subcarriers of an OFDM system has been proposed by Abu-alhiga and Haas[1].
- However, in this scheme, the number of active OFDM subcarriers varies for each OFDM block, and furthermore, a kind of perfect feedforward is assumed from the transmitter to the receiver via the excess subcarriers to explicitly signal the mapping method for the subcarrier index selecting bits.
- In this work, we present a novel OFDM scheme by taking a different approach than Abu-alhiga and Haas's work.

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[1] R. Abu-alhiga and H. Haas, "Subcarrier-index modulation OFDM," in Proc. IEEE Int. Sym. Personal, Indoor and Mobile Radio Commun., Tokyo, Japan, Sep. 2009, pp. 177–181.

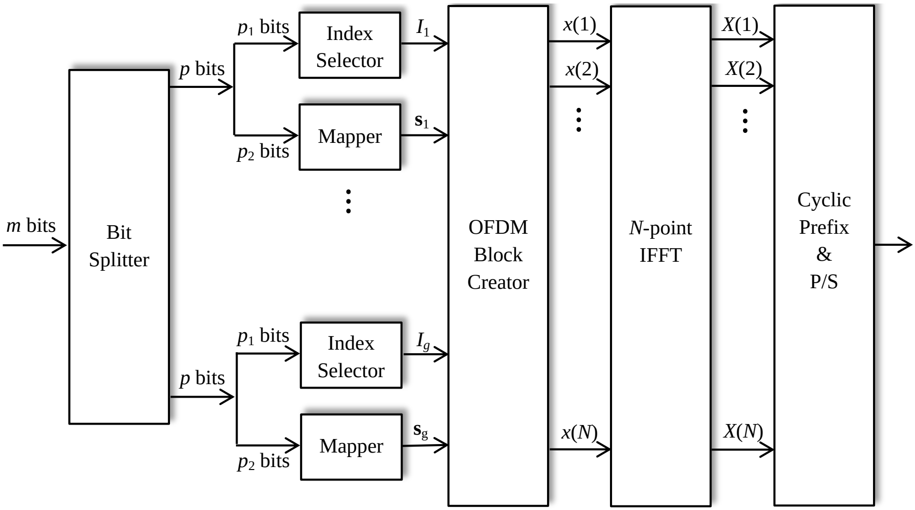
Orthogonal Frequency Division Multiplexing with Index Modulation

- In this scheme, information is conveyed not only by M-ary signal constellations as in classical OFDM, but also by the indices of the subcarriers, which are activated according to the incoming information bits.
- A general method, by which the number of active subcarriers can be adjusted, and the incoming bits can be systematically mapped to these active subcarriers, is presented for the OFDM-IM scheme.
- Different mapping and detection techniques are proposed for the new scheme.

Orthogonal Frequency Division Multiplexing with Index Modulation cont'd

- First, a simple *look-up table* is implemented to map the incoming information bits to the subcarrier indices and an ML detector is employed at the receiver.
- Then, a simple yet effective technique based on *combinatorial number system* is used to map the information bits to the subcarrier indices, and a *log-likelihood ratio (LLR) detector* is employed at the receiver to determine the most likely active subcarriers as well as corresponding constellation symbols.
- A theoretical error performance analysis based on *pairwise error probability (PEP)* calculation is provided for the new scheme operating under ideal channel conditions.
- It is shown via computer simulations that the proposed scheme achieves significantly better bit error rate (BER) performance than the classical OFDM.

Block Diagram of the OFDM-IM Transmitter



OFDM-IM Transmission

- A total of m information bits enter the OFDM-IM transmitter for the transmission of each OFDM block.
- These m bits are then split into g groups each containing p bits, i.e., $m = pg$.
- Each group of p-bits is mapped to an *OFDM subblock* of length n, where $n = N/g$ and N is the number of OFDM subcarriers.
- Contrary to the classical OFDM, this mapping operation is not only performed by means of the modulated symbols, but also by the indices of the subcarriers.

OFDM-IM Transmission cont'd

- Inspiring from the SM concept, additional information bits are transmitted by a subset of the OFDM subcarrier indices.
- For each subblock, only k out of n available indices are employed for this purpose and they are determined by a selection procedure from a predefined set of active indices, based on the first p_1 bits of the incoming p-bits sequence.
- We set the symbols corresponding to the inactive subcarriers to zero.
- The remaining $p_2 = k \log_2 M$ bits of this sequence are mapped on to the M-ary signal constellation to determine the data symbols that modulate the subcarriers having active indices, therefore, we have $p = p_1 + p_2$.

The Main Idea of OFDM-IM

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In the OFDM-IM scheme, the information is conveyed by both of the M-ary constellation symbols and the indices of the subcarriers that are modulated by these constellation symbols.

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Due to the fact that we do not use all of the available subcarriers, we compensate for the loss in the total number of transmitted bits by transmitting additional bits in the spatial domain of the OFDM block.

OFDM-IM Subblock Concept

- For each subblock β, the incoming p_1 bits are transferred to the index selector, which chooses k active indices out of n available indices, where the selected indices are given by

$$I_\beta = \{i_{\beta,1}, \ldots, i_{\beta,k}\}$$

where $i_{\beta,\gamma} \in [1, \ldots, n]$ for $\beta = 1, \ldots, g$ and $\gamma = 1, \ldots, k$.

- Therefore, for the total number of information bits carried by the spatial position of the active indices in the OFDM block, we have

$$m_1 = p_1 g = \lfloor \log_2 \left(C \left(n, k \right) \right) \rfloor g.$$

- In other words, I_β has $c = 2^{p_1}$ possible realizations.

OFDM-IM Subblock Concept cont'd

- On the other hand, the total number of information bits carried by the M-ary signal constellation symbols are given by

$$m_2 = p_2 g = k \left(\log_2 \left(M \right) \right) g$$

- The total number of active subcarriers is $K = kg$ in our scheme.
- Consequently, a total of

$$m = m_1 + m_2$$

bits are transmitted by a single block of the OFDM-IM scheme.

The Forming of the Main OFDM Block

- The vector of the modulated symbols at the output of the M-ary mapper (modulator), which carries p_2 bits, is given by

$$\mathbf{s}_\beta = \left[s_\beta(1) \ \ldots \ s_\beta(k) \right]$$

  where $s_\beta(\gamma) \in \mathcal{S}$, $\beta = 1, \ldots, g$, $\gamma = 1, \ldots, k$.
- The OFDM block creator creates all of the subblocks by taking into account I_β and $\mathbf{s}_\beta$ for all β first and it then forms the $N \times 1$ main OFDM block

$$\mathbf{x}_F = \left[x(1) \ x(2) \ \cdots \ x(N) \right]^T$$

  where $x(\alpha) \in \{0, \mathcal{S}\}, \alpha = 1, \ldots, N$, by concatenating these g subblocks.
- Unlike the classical OFDM, in our scheme $\mathbf{x}_F$ contains some zero terms whose positions carry information.

OFDM Transmission

- After this point, the same procedures as those of classical OFDM are applied.

- Inverse FFT (IFFT): $\mathbf{x}_T = \frac{N}{\sqrt{K}} \text{IFFT}\left\{\mathbf{x}_F\right\} = \frac{1}{\sqrt{K}} \mathbf{W}_N^H \mathbf{x}_F$
- Cyclic Prefix (CP) of length: $\left[X\left(N-L+1\right) \ \cdots \ X\left(N-1\right) \ X\left(N\right)\right]^T$
- Frequency-selective Rayleigh fading channel: $\mathbf{h}_T = \left[h_T\left(1\right) \ \ldots \ h_T\left(\nu\right)\right]^T$

where
$\mathbf{W}_N$: discrete Fourier transform (DFT) matrix with $\mathbf{W}_N^H \mathbf{W}_N = N\mathbf{I}_N$
$N/\sqrt{K}$: normalization term
L: CP length
ν: tap length
$h_T\left(\sigma\right), \sigma = 1, \ldots, \nu \sim \mathcal{CN}\left(0, \frac{1}{\nu}\right)$

OFDM Transmission cont'd

- The equivalent frequency domain input-output relationship of the OFDM scheme is given by

$$y_F(\alpha) = x(\alpha)\, h_F(\alpha) + w_F(\alpha), \quad \alpha = 1, \ldots, N$$

  - $y_F(\alpha)$: received signals
  - $h_F(\alpha) \sim \mathcal{CN}(0, 1)$: the channel fading coefficients
  - $w_F(\alpha) \sim \mathcal{CN}(0, N_{0,F})$: noise samples
- Noise variance in the frequency domain

$$N_{0,F} = (K/N)\, N_{0,T}$$

- Signal-to-noise ratio (SNR):

$$\rho = \frac{E_b}{N_{0,T}} = \frac{(N+L)/m}{N_{0,T}}$$

OFDM-IM Receiver Structures

- The receiver's task is to detect the indices of the active subcarriers and the corresponding information symbols by processing $y_F(\alpha), \alpha = 1, \ldots, N$.
- Unlike the classical OFDM, a simple maximum likelihood (ML) decision on $x(\alpha)$ cannot be given by considering only $y(\alpha)$ in our scheme due to the spatial information carried by the OFDM-IM subblocks.
- We investigate two different type of mapping&detection algorithms for the OFDM-IM scheme:
  - Look-Up Table Method - Maximum Likelihood (ML) Detector
  - Combinadics Method - Log-likelihood Ratio (LLR) Detector

Some Notes on Implementation of OFDM-IM

- Note that OFDM-IM scheme can be implemented without using a bit splitter at the beginning, i.e., by using a single group ($g = 1$) which results in $n = N$.
- However, in this case, $C(n, k)$ can take very large values which make the implementation of the overall system difficult.
- Therefore, instead of dealing with a single OFDM block with higher dimensions, we split this block into smaller subblocks to ease the index selection and detection processes at the transmitter and receiver sides, respectively.

Look-up Table Method

- A look-up table of size c is created to use at both transmitter and receiver sides.
- At the transmitter, this look-up table provides the corresponding indices for the incoming p_1 bits for each subblock, and it performs the opposite operation at the receiver.

A look-up table example for $n = 4, k = 2$ and $p_1 = 2$

Bits	Indices	subblocks
$[0 \;\; 0]$	$\{1, 2\}$	$[s_\chi \;\; s_\zeta \;\; 0 \;\; 0]^T$
$[0 \;\; 1]$	$\{2, 3\}$	$[0 \;\; s_\chi \;\; s_\zeta \;\; 0]^T$
$[1 \;\; 0]$	$\{3, 4\}$	$[0 \;\; 0 \;\; s_\chi \;\; s_\zeta]^T$
$[1 \;\; 1]$	$\{1, 4\}$	$[s_\chi \;\; 0 \;\; 0 \;\; s_\zeta]^T$

- Since $C(4, 2) = 6$, two combinations out of six are discarded.

Notes on Look-up Table Method

- Although a very efficient and simple method for smaller c values, this mapping method is not feasible for higher values of n and k due to the size of the table.
- We employ this method with the *ML detector* since the receiver has to know the set of possible indices for ML decoding, i.e., it requires a look-up table.

ML Detector

- The ML detector for the OFDM-IM scheme considers all possible subblock realizations by searching for all possible subcarrier index combinations and signal constellation points.
- It makes a joint decision on the active indices and the constellation symbols for each subblock by

$$\left(\hat{I}_\beta, \hat{\mathbf{s}}_\beta\right) = \arg \min_{I_\beta, \mathbf{s}_\beta} \sum_{\gamma=1}^{k} \left| y_F^\beta\left(i_{\beta,\gamma}\right) - h_F^\beta\left(i_{\beta,\gamma}\right) s_\beta\left(\gamma\right) \right|^2$$

where $y_F^\beta\left(\xi\right)$ and $h_F^\beta\left(\xi\right)$ for $\xi = 1, \ldots, n$ are the received signals and the corresponding fading coefficients for the subblock β.
- The total number of metric calculations performed here is cM^k since I_β and $\mathbf{s}_\beta$ have c and M^k different realizations, respectively.
- Therefore, this ML detector becomes impractical for larger values of c and k due to its exponentially growing decoding complexity.

Combinadics Method

- The combinational number system (combinadics) provides a one-to-one mapping between natural numbers and k-combinations, for all n and k.
- It maps a natural number to a strictly decreasing sequence

$$J = \{c_k, \ldots, c_1\}$$

where $c_k > \cdots > c_1 \geq 0$.

- In other words, for fixed n and k, all $Z \in [0, C(n,k) - 1]$ can be presented by a sequence J of length k, which takes elements from the set $\{0, \ldots, n-1\}$ according to the following equation:

$$Z = C(c_k, k) + \cdots + C(c_2, 2) + C(c_1, 1).$$

Combinadics: A Mapping Example

- As an example, for $n = 8, k = 4$, $C(8,4) = 70$, the following J sequences can be calculated:

$$\begin{aligned}
69 &= C(7,4)+C(6,3)+C(5,2)+C(4,1) \rightarrow J=\{7,6,5,4\} \\
68 &= C(7,4)+C(6,3)+C(5,2)+C(3,1) \rightarrow J=\{7,6,5,3\} \\
&\vdots \\
32 &= C(6,4)+C(5,3)+C(4,2)+C(1,1) \rightarrow J=\{6,5,4,1\} \\
31 &= C(6,4)+C(5,3)+C(4,2)+C(0,1) \rightarrow J=\{6,5,4,0\} \\
&\vdots \\
1 &= C(4,4)+C(2,3)+C(1,2)+C(0,1) \rightarrow J=\{4,2,1,0\} \\
0 &= C(3,4)+C(2,3)+C(1,2)+C(0,1) \rightarrow J=\{3,2,1,0\}.
\end{aligned}$$

Combinadics Method cont'd

- The algorithm, which finds the lexicographic ordered J sequences for all n, can be explained as follows:
  1. Start by choosing the maximal c_k that satisfies $C(c_k, k) \leq Z$,
  2. Choose the maximal c_{k-1} that satisfies $C(c_{k-1}, k-1) \leq Z - C(c_k, k)$ and so on[2].
- In our scheme, for each subblock, we first convert the p_1 bits entering the index selector to a decimal number Z.
- Then feed this decimal number to the combinadics algorithm to select the active indices as $J + 1$.

[2] J. D. McCaffrey, "Generating the mth lexicographical element of a mathematical combination," MSDN Library, Jul. 2004.

Notes on Combinadics Method

- At the receiver side, after determining active indices, we can easily get back to the decimal number $\hat{Z}$ using

$$\hat{Z} = C\left(\hat{c}_k, k\right) + \cdots + C\left(\hat{c}_2, 2\right) + C\left(\hat{c}_1, 1\right).$$

- We then apply this number to a p_1-bit decimal-to-binary converter.
- We employ this method with the *LLR detector* for higher c values to avoid look-up tables.
- However, it can give a catastrophic result at the exit of the decimal-to-binary converter if $\hat{Z} \geq c$; nevertheless, we use this detector for the increased bit-rate.

Combinadics Example, $n = 32, k = 16$

- $C\left(32, 16\right) = 601,080,390$
- $\log_2\left(601,080,390\right) = 29.16$
- $p_1 = 29$ bits
  0 1 0 0 0 0 1 1 0 1 0 0 0 1 1 0 0 0 1 1 1 0 1 1 0 0 0 1 0
- Bin-2-Dec Conversion: $141,084,514$
- Combinadics (active indices) :
  29, 28, 27, 26, 25, 19, 17, 16, 15, 14, 13, 12, 11, 8, 4, 2

Log-Likelihood Ratio (LLR) Detector

- The LLR detector of the OFDM-IM scheme provides the logarithm of the ratio of a posteriori probabilities of the frequency domain symbols by considering the fact that their values can be either non-zero or zero.
- This ratio, which is given below, gives information on the active status of the corresponding index for $\alpha = 1, \ldots, N$:

$$\lambda(\alpha) = \ln \frac{\sum_{\chi=1}^{M} P\left(x(\alpha) = s_\chi \,|\, y_F(\alpha)\right)}{P\left(x(\alpha) = 0 \,|\, y_F(\alpha)\right)}$$

where $s_\chi \in \mathcal{S}$ is the constellation symbol.

- In other words, a larger $\lambda(\alpha)$ value means it is more probable that index α is selected by the index selector at the transmitter, i.e., it is active.

LLR Detector cont'd

- Using Bayes' formula and considering that $\sum_{\chi=1}^{M} p\left(x\left(\alpha\right)=s_{\chi}\right)=k/n$ and $p\left(x\left(\alpha\right)=0\right)=\left(n-k\right)/n$, we obtain

$$\lambda\left(\alpha\right)=\ln\left(k\right)-\ln\left(n-k\right)+\frac{\left|y_{F}\left(\alpha\right)\right|^{2}}{N_{0,F}} + \ln\left(\sum_{\chi=1}^{M}\exp\left(-\frac{1}{N_{0,F}}\left|y_{F}\left(\alpha\right)-h_{F}\left(\alpha\right)s_{\chi}\right|^{2}\right)\right).$$

- In order to prevent numerical overflow, the Jacobian logarithm can be used in this calculation.

LLR Detector cont'd

- For the case of $k = n/2$ and binary-phase shift keying (BPSK) modulation, using Jacobian logarithm, LLR values can be calculated as

$$\lambda(\alpha) = \max(a, b) + \ln\left(1 + \exp\left(-|b - a|\right)\right) + \frac{\left|y_F(\alpha)\right|^2}{N_{0,F}}$$

where

$$a = -\left|y_F(\alpha) - h_F(\alpha)\right|^2 / N_{0,F}$$
$$b = -\left|y_F(\alpha) + h_F(\alpha)\right|^2 / N_{0,F}.$$

- After calculation of the N LLR values, for each subblock, the receiver decides on k active indices out of them which have maximum LLR values.

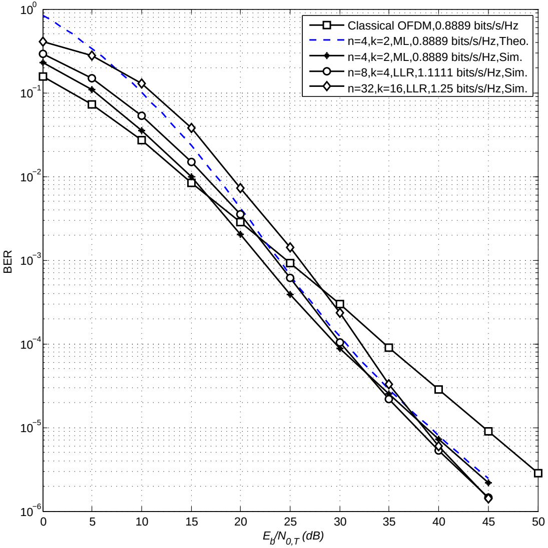
Notes on LLR Detector

- The decoding complexity of this detector is *linearly* proportional to M and comparable to that of classical OFDM.
- This detector is classified as *near-ML* since the receiver does not know the possible values of I_β.
- Although this is a desired feature for higher values of n and k, the detector can decide on a catastrophic set of active indices which is not included in I_β since $C\left(n,k\right) > c$ for $k > 1$, and $C\left(n,k\right) - c$ index combinations are unused at the transmitter.
- On the other hand, a look-up table cannot be used with the *LLR detector* since the receiver cannot decide on active indices if the detected indices do not exist in the table.

Simulation Results for Ideal Channel Conditions

- We present simulation results for the OFDM-IM scheme with different configurations and make comparisons with classical OFDM.
- BER performance of these schemes was evaluated via Monte Carlo simulations.
- We investigate the error performance of OFDM-IM scheme under ideal channel conditions without Doppler spread.
- We assume a simple frequency selective Rayleigh fading channel, which is described earlier.
- In all simulations, we assumed the following system parameters: $N = 128$, $\nu = 10$ and $L = 16$.
- Furthermore, we employed a BPSK constellation ($M = 2$) for all systems.

BER Performance Comparisons



Results-I

- At a BER value of 10^{-5} our new scheme with $n = 4, k = 2$ achieves approximately 6 dB better BER performance than classical OFDM operating at the same spectral efficiency thanks to the bits transmitted by the antenna indices.
- The theoretical curve becomes very tight with the computer simulation curve with increasing SNR values.
- We observe that despite their increased data rates, $n = 8, k = 4$ and $n = 32, k = 16$ OFDM-IM schemes exhibits close BER performance to the low-rate $n = 4, k = 2$ OFDM scheme.
- This can be explained by the fact that for high SNR, the error performance of the OFDM-IM scheme is dominated by the PEP events with $r = 1$.

Conclusions

- A novel OFDM scheme, which uses the indices of the active subcarriers to transmit data, has been proposed in this paper.
- In this scheme, inspiring from the recently proposed SM concept, the incoming information bits are transmitted with the OFDM block in an unique fashion to improve the error performance as well as to increase spectral efficiency.
- Different transceiver structures are presented for the proposed scheme.
- It has been shown that the proposed scheme achieves significantly better BER performance than the classical OFDM.
- As future research, we believe that the implementation of different transceiver structures could be realized for the OFDM-IM scheme to increase the data rate as well as to improve the error performance.

THANK YOU !

any questions ?