Performance analysis of space shift keying for AF relaying with relay selection

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ABSTRACT

In this paper, we propose an amplify-and-forward multiple-input multiple-output (MIMO) relaying scheme, which combines space shift keying (SSK) with the best and partial relay selection. In this scheme, SSK transmission is considered by using the source (S) transmit antennas. Besides the direct link transmission, a relay, which is selected according to the best or partial relay selection techniques, amplifies the data received from S and forwards it to the destination (D). Theoretical error probability expressions of the proposed cooperative SSK systems are derived and an asymptotic diversity analysis is also performed to demonstrate the achievable diversity orders of the systems. It is shown that the proposed SSK systems outperform the conventional cooperative single-input multiple-output (SIMO) systems. It is also revealed that there is an interesting trade-off between SSK with the best and the partial relay selection in terms of error performance and complexity as in conventional cooperative SIMO systems. However, it is shown that the partial relay selection provides an almost identical error performance compared to the best relay selection with a considerably lower complexity when the number of relays is less than or equal to the number of receive antennas at D in the cooperative SSK system.

1. Introduction

During the recent years, multiple-input multiple-output (MIMO) wireless communication systems have provided high data rates and improved error performance at the cost of using multiple radio-frequency (RF) chains at the transmitter. Due to the use of multiple antennas equipped with multiple RF chains, not only inter-channel interference (ICI) occurs at the receiver but also inter-antenna synchronization (IAS) among the transmit antennas becomes a requirement and the transceiver complexity increases. In this context, new techniques, namely spatial modulation (SM) and space shift keying (SSK), provide promising solutions [1–7]. Due to the one-to-one mapping between the information bits and transmit antenna indices, SM and SSK typically activate only one transmit antenna. Hence, the ICI is entirely avoided, the requirement of IAS is eliminated and the transceiver complexity is reduced considerably in SM and SSK. Compared to SM, SSK further decreases the transceiver complexity since mapping of the information bits to M-ary symbols is not performed [7,8].

Cooperative relaying improves the transmission reliability and extends the coverage of the wireless networks [9–13]. Furthermore, the fading effect of the wireless channels is mitigated efficiently; as a result, the error performance is improved. However, orthogonal sub-channel allocation for each of the relaying links and the synchronization between the relays are required for cooperative diversity to be effective [13]. The activation of all available relays by using orthogonal sub-channels limits the throughput of the network. Therefore, there is a trade-off between data rate and the error performance in cooperative networks. In this context, relay selection, which provides improved error performance compared to single-relay transmission and avoids the reduction in the spectral efficiency at the same time, can be an efficient solution [14,15].

Many studies have combined SM and SSK schemes with cooperative networks in recent years. In [16], a new concept of SSK, namely cooperative space-time shift keying, has been proposed. In [17], a SSK scheme with dual-hop amplify-and-forward (AF) relays has been proposed. The error performance of SM with multiple decode-and-forward (DF) relays and SSK with both AF and DF relays has been analyzed in [18,19], respectively. Moreover, in [20], the performance of source transmit antenna selection for SSK with multiple DF relays is investigated. The outage probability of both classical SM and cooperative SM systems has been investigated.
In [21], a new cooperative spectrum sharing protocol, which employs SM at the secondary transmitter, has been proposed for overlay cognitive radio networks. A cooperative SSK scheme with DF relays, which considers the decoding errors at the relays, has been proposed in [23]. On the other hand, studies on the SM and SSK schemes with relay selection is considerably limited. The performance of threshold-based best relay selection in SSK with DF relaying is reported in [24]. The opportunistic relay selection in a SSK with AF relaying scheme is reported in [25]. However, in [25], the direct link transmission is not considered and the number of receive antennas at the destination is restricted to one.

In this paper, we propose an SSK scheme with cooperative AF relaying in which two different relay selection methods, namely the best and the partial relay selection, are performed. Our contributions are summarized as follows. First, a novel cooperative SSK scheme with the best relay selection is proposed. Our system model differs as that of [25] in the following aspects: (i) The direct link transmission is considered; (ii) A new and more general cooperative MIMO scheme is proposed for multiple receive antennas. Second, a partial relay selection-based cooperative SSK scheme is proposed. To the best of our knowledge, partial relay selection is applied to an SSK-based cooperative scheme for the first time in this paper. Finally, we derive the exact bit error rate (BER) of the proposed system when the source is equipped with two transmit antennas and provide a sufficiently tight upper-bound on the BER of the system for $2c$ antennas as indicated in Fig. 1. The overall transmission consists of two phases. In the first phase, due to its simplicity, satisfactory error performance and low complexity, the SSK technique is applied at S, where an SSK signal is transmitted from S to the relays and D. In the second phase, the relay, which is selected based on one of the two selection techniques, namely the best or the partial relay selection, forwards the received signal to D by following the AF protocol. The transmission is performed in two orthogonal channels. D uses the maximum likelihood (ML) detection principle to determine the index of the activated antenna. Perfect channel state information at D is assumed to be available. With $k \in \{1, 2, \ldots, N\}$ denoting the index of a relay, we define the vector of channel coefficients between S and $R_k$ and $R_k$ and D as $h_{sk}$ and $h_{sk}$, whose dimensions are given as $1 \times N_s$ and $N_s \times 1$, respectively. Since we define the index of the selected relay as $p$, the vector of channel coefficients between the selected relay $R_p$ and D is represented as $h_{sk}^p$ (see Fig. 1). On the other hand, the matrix of channel coefficients between S and D with dimensions $N_s \times N_s$ is denoted by $H_{SD}$. We assume that all the channel fading coefficients are independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance and all receivers are exposed to additive white Gaussian noise (AWGN) samples of variance $N_0$.

With $l \in \{1, 2, \ldots, N\}$ denoting the index of the activated antenna at S, the received signal at $R_l$ and the received signal vector at D can be expressed as

$$y_{sk} = \sqrt{E_s} h_{sk} + n_{sk}$$

$$y_{sk} = \sqrt{E_s} h_{sk}^p + n_{sk}^p$$

where $E_s$ is the transmitted signal energy of S and $k = 1, 2, \ldots, N$. $h_{sk}$ and $h_{sk}^p$ denote the $l$th column of $H_{SD}$ and the $l$th element of $h_{sk}$. Additionally, $n_{sk}$ and $n_{sk}^p$ are the Gaussian noise sample and $N_s \times 1$ Gaussian noise vector at $R_k$ and D, respectively.

In the second phase, the selected relay, whose index is $p$, forwards the received signal to D by following the AF protocol and the received signal vector at D can be written as

$$y_{sk}^p = \sqrt{E_s} h_{sk}^p h_{sk}^{p,D} + \sqrt{E_s} h_{sk}^p n_{sk}^p + n_{sk}^p$$

where $E_s$ is the transmitted signal energy of $R_p$ and $h_{sk}$, $p \in \{1, \ldots, N\}$, is the $l$th element of $h_{sk}$, which is the vector of channel coefficients between S and $R_p$. Here, $n_{sk}^p$ is an $N_s \times 1$ Gaussian noise vector at D and $C = 1/\sqrt{E_s + N_0}$ is the amplification coefficient to fix the transmit energy at $R_p$. It is assumed that the perfect CSI is available at D. Furthermore, by applying noise normalization at D as in [17], the received signal vector can be expressed as

$$y_{sk}^p = \sqrt{E_s} h_{sk}^p h_{sk}^{p,D} + n_{sk}^p$$

where $A = \frac{1}{\sqrt{E_s + N_0}}$ and $n_{sk}^p$ is an $N_s \times 1$ Gaussian noise vector whose elements are distributed with $CN(0, N_0)$. From (2) and (4), the decision rule for the source antenna index based on the ML principle can be expressed as

$$I = \arg \min_{1 \leq q \leq N_s} \left\{ \| y_{sk} - \sqrt{E_s} h_{sk}^p \|^2 + \| y_{sk} - \sqrt{E_s} h_{sk}^p \|^2 \right\}.$$ (5)

In the following subsections, we describe the considered relay selection protocols.

2.1. Best relay selection

In the best relay selection, the relay is selected by considering the channel fading coefficients among all relaying paths between S, $R_l$ and D. We define the "best" relay as the one which maximizes the worst case pairwise error probability (PEP) of the relaying link.
Hence, the index of the selected relay according to the best relay selection can be determined as [25]

$$\rho = \arg \max_{1 \leq k \leq N} \left\{ \min_{1 \leq j \leq 1, 2, \ldots, N_k} \left( \frac{\gamma_{R_j-D} \gamma_{S_j-R_k}}{\gamma_{R_k-D} + C_5} \right) \right\}$$

where \( \gamma_{R_j-R_k} = \frac{\mathbf{h}_{R_j-R_k}^H \mathbf{h}_{R_j-R_k}}{N_0} \), \( \gamma_{R_k-D} = \frac{\mathbf{h}_{R_k-D}^H \mathbf{h}_{R_k-D}}{N_0} \) and \( C_5 = \left( \frac{G^2 N_d N_0}{N_0} \right)^{-1} \). Note that such definition of \( \gamma_{R_k-D} \) is used for the ease of operation. The selected relay index \( \rho \) is fed back from \( D \) to the relays.

2.2. Partial relay selection

In this technique, the relay that has the best link between \( S \) and \( R_k \) is selected. Hence, in the partial relay selection, since the selection is performed by considering only the channel fading coefficients between \( S \) and \( R_k \), no feedback is required from \( D \) to the relays and the overall system complexity will be lower than the system with the best relay selection due to less signaling overhead [28]. The index of the selected relay according to the partial relay selection can be determined as

$$\rho = \arg \max_{1 \leq k \leq N} \left\{ \min_{1 \leq j \leq 1, 2, \ldots, N_k} \left( \frac{\gamma_{S_j-R_k}}{\gamma_{R_k-D} + C_5} \right) \right\}.$$  

3. Performance analysis

In this section, we analyze the error performance of the cooperative SSK system for the best and partial relay selection techniques, respectively.

3.1. Pairwise error probability

3.1.1. Best relay selection

\( P(l \rightarrow i) \) is the PEP associated with the erroneous detection of the transmit antenna index \( l \) as \( i \) at \( D \). From (5), the PEP conditioned on the channel coefficients can be calculated as

$$P(l \rightarrow i | \mathbf{h}_{SR}^R, \mathbf{h}_{RD}^D, \mathbf{H}^D) = Q\left( \sqrt{\frac{\gamma_{R_j-D} \gamma_{S_j-R_k}}{\gamma_{R_k-D} + C_5}} \right)$$

where \( BRS \) stands for the best relay selection and

$$\gamma_{BRS}^{S_R-D} = \frac{\mathbf{h}_{BRS-S_R-D}^H \mathbf{h}_{BRS-S_R-D}}{N_0} \text{, } \gamma_{BRS}^{R_k-D} = \frac{\mathbf{h}_{BRS-R_k-D}^H \mathbf{h}_{BRS-R_k-D}}{N_0} \text{, } \gamma_{BRS}^{S-R_k} = \frac{\mathbf{h}_{BRS-S-R_k}^H \mathbf{h}_{BRS-S-R_k}}{2N_0} \text{, } \gamma_{S-D} = \frac{\mathbf{h}_{S-D}^H \mathbf{h}_{S-D}}{2N_0}.$$  

Note that unlike conventional \( M \)-ary modulation schemes such as \( M \)-PSK and \( M \)-QAM, information is conveyed by the index of activated transmit antenna in SSK and the channel fading coefficients corresponding to the transmit antennas are used to estimate active transmit antenna index as in (5). Therefore, the PEP expression of the proposed SSK system depends on the Euclidean distances between the channel fading coefficients corresponding to \( S-R_k \) and \( S-D \) links since SSK transmission is conducted over these links; however, the PEP for the \( M \)-PSK systems is dependent on the Euclidean norm of channel fading coefficients and the Euclidean distances between the \( M \)-PSK symbols. Considering the constellation diagram of SSK for the \( S-R_k \) link, the total number of different squared Euclidean distances in the diagram is equal to \( \binom{N_k}{2} \) and these distances are statistically dependent. Since the selection criterion given in (6) uses these Euclidean distances, it is not feasible to provide the average PEP directly from (8). Hence, we use

$$\gamma_{BRS}^{S_R-D} = \frac{\mathbf{h}_{BRS-S_R-D}^H \mathbf{h}_{BRS-S_R-D}}{N_0} \text{, } \gamma_{BRS}^{R_k-D} = \frac{\mathbf{h}_{BRS-R_k-D}^H \mathbf{h}_{BRS-R_k-D}}{N_0} \text{, } \gamma_{BRS}^{S-R_k} = \frac{\mathbf{h}_{BRS-S-R_k}^H \mathbf{h}_{BRS-S-R_k}}{2N_0} \text{, } \gamma_{S-D} = \frac{\mathbf{h}_{S-D}^H \mathbf{h}_{S-D}}{2N_0}.$$  

which is calculated by considering the minimum of all possible \( \binom{N_k}{2} \) Euclidean distances belonging to \( S-R_k \) link, instead of \( \gamma_{BRS}^{S_R-D} \) in (8) for mathematical tractability. Then, the conditional PEP can be upper-bounded by

$$P(l \rightarrow i | \mathbf{h}_{SR}^R, \mathbf{h}_{RD}^D, \mathbf{H}^D) \leq Q\left( \sqrt{\frac{\gamma_{BRS}^{S_R-D} + \gamma_{S-D}}{4}} \right).$$

Since \( \gamma_{BRS}^{S_R-D} = \gamma_{BRS}^{S-R_k} \) for \( N_k = 2 \), the right-hand side of (10) gives the exact value of \( P(l \rightarrow i | \mathbf{h}_{SR}^R, \mathbf{h}_{RD}^D, \mathbf{H}^D) \) for this case. Averaging (10) over \( \mathbf{h}_{SR}^R, \mathbf{h}_{RD}^D \) and \( \mathbf{H}^D \), and using the moment generation function (MGF) approach [29], the unconditional PEP upper-bound is obtained as
where $M_{\gamma}(\cdot)$ stands for the MGF of $\gamma$. Moreover, the probability density function (PDF) of $\gamma_{S-D}$ is given by [8]

$$f_{\gamma_{S-D}}(x) = \frac{x^{\alpha-1} e^{-\frac{x^\alpha}{\alpha}}}{(P_s)^{\frac{N}{\alpha}} I(N_d)}$$

where $P_s = E_s/N_d$. Then, the MGF of $\gamma_{S-D}$ can be expressed as

$$M_{\gamma_{S-D}}(s) = (1 + sP_s)^{-N_d}.$$  

(13)

In addition, since only $\gamma_{S_D}$ depends on $i$ and $j$ in (9), $\gamma_{S_D}$ can be rewritten as

$$\gamma_{S_D} = \min_{i,j} \{ \gamma_{S_D} \}.$$  

(14)

where $\gamma_{S_D} = \min_{i,j} \{ \gamma_{S_D} \}$. With the help of [30, (2.1.2)], the cumulative distribution function (CDF) of $\gamma_{S_D}$ can be given as

$$F_{\gamma_{S_D}}(x) = 1 - e^{\frac{-x}{\alpha}}$$

(15)

where $F_{\gamma_{S_D}}(x)$ is the CDF of $\gamma_{S_D}$, which is a chi-square random variable [32] and $P_s = E_s/N_d$. By substituting the CDF of $\gamma_{S_D}$ into (15), the CDF of $\gamma_{S_D}$ can be rewritten as

$$F_{\gamma_{S_D}}(x) = \left(1 - e^{\frac{-x}{\alpha}}\right)^N.$$  

(16)

With the help of [27, (3.371.9)], the CDF of $\gamma_{S_D}$ can be derived as

$$F_{\gamma_{S_D}}(x) = \left(1 - \left(\frac{\alpha}{\alpha}k\right)^\frac{N}{\alpha}\right)\left(1 - e^{\frac{-x}{\alpha}}\right)^N.$$  

(17)

where $D_s = N_sC_s/P_s$. Then, the MGF of $\gamma_{S_D}$ can be expressed as

$$M_{\gamma_{S_D}}(s) = s \int_0^\infty e^{-sx} F_{\gamma_{S_D}}(x) dx.$$  

(18)

By substituting (13) and (18) into (11) and calculating numerical integrals with the help of common mathematical software, an upper-bound to $P(1-i)$ can be easily found.

### 3.2. Partial relay selection

The partial relay selection criterion given in (7) uses the metrics that contain dependent Euclidean distances as in the best relay selection. Hence, the PEP conditioned on the channel coefficients can be upper-bounded by

$$P(I \rightarrow \tilde{l}) \leq \frac{1}{\pi} \int_0^{\pi/2} M_{\gamma_{S-D}} \left( \frac{1}{2 \sin^2 \theta} \right) M_{\gamma_{S-D}} \left( \frac{1}{2 \sin^2 \theta} \right) d\theta$$

(19)

where $P_{\gamma}(\cdot)$ stands for the partial relay selection and $\gamma_{S-D} = \min_{i \in [1, 2, \ldots, N_s]} \{ \gamma_{S_D} \}$ and $\gamma_{S-D} = \max_{i \in [1, 2, \ldots, N_s]} \{ \min_{j \in [1, 2, \ldots, N_s]} \{ \gamma_{S_D} \} \}$. Averaging (19) over $h_s^a$, $h_{s_D}^a$, and $h_D^a$ and using the MGF approach [29], the unconditional PEP upper-bound is obtained as

$$P(I \rightarrow \tilde{l}) \leq \frac{1}{\pi} \int_0^{\pi/2} M_{\gamma_{S-D}} \left( \frac{1}{2 \sin^2 \theta} \right) M_{\gamma_{S-D}} \left( \frac{1}{2 \sin^2 \theta} \right) d\theta.$$  

(20)

Since the relay selection is performed by considering only the relaying links as in the best relay selection, statistics of the S-D link are the same as the previous subsection and the MGF of $\gamma_{S-D}$ is given in (13). Hence, in order to obtain the PEP given in (20), we need to find the distribution of the random variable $\gamma_{S-D}$. The CDF of $\gamma_{S-D}$ can be given as $F_{\gamma_{S-D}}(x) = \left(1 - e^{\frac{-x}{\alpha}}\right)^N$ [30, (2.1.2)]. Then, the CDF of $\gamma_{S-D}$ can be written as

$$F_{\gamma_{S-D}}(x) = \int_0^\infty f_{\gamma_{S-D}} \left( \frac{y+c_s}{y} \right) f_{\gamma_{S-D}}(y) dy = \int_0^\infty \left(1 - e^{\frac{-x}{\alpha}}\right)^N y^{N_d-1} \frac{N_d}{N_s} e^{-\frac{y}{\alpha}} dy.$$  

(21)

By using binomial expansion and [27, (3.371.4) and (3.471.9)], the CDF of $\gamma_{S-D}$ can be derived as

$$F_{\gamma_{S-D}}(x) = 1 + \sum_{a=0}^{N_d} \left( \frac{N_s}{N_d} \right)^a (-1)^a \exp \left( \frac{aTx}{P_s} \right) \frac{2(a+\alpha)\alpha}{\Gamma(N_d)} K_{N_d}(2\sqrt{aTx})$$

(22)

where $D_s$ is defined as in the previous subsection. Then, the MGF of $\gamma_{S-D}$ can be expressed as

$$M_{\gamma_{S-D}}(s) = s \int_0^\infty e^{-sx} F_{\gamma_{S-D}}(x) dx.$$  

(23)

By substituting (22) into (23) and using [27, (6.643.3)], the closed-form expression for $M_{\gamma_{S-D}}(s)$ can be derived as

$$M_{\gamma_{S-D}}(s) = 1 + sN_s \sum_{a=0}^{N_d} \left( \frac{N_s}{N_d} \right)^a (-1)^a \frac{(T_{a-T})^{N_d-1}}{\Gamma(N_d)} \left( \frac{TaD_s}{25 + \frac{2a}{\alpha}} \right) \times \left( \frac{25 + \frac{2a}{\alpha}}{S + \frac{2a}{\alpha}} \right).$$  

(24)

By substituting (13) and (24) into (20) and evaluating the integrals numerically with the help of common mathematical softwares, an upper-bound to $P(I \rightarrow \tilde{l})$ can be calculated.

### 3.3. Average bit error probability

A tight upper bound on the average BER is given by the well-known union bound [32]

$$P_b \leq \frac{1}{N_{\log_2(N_s)}} \sum_{l=1}^{N_s} N(I \rightarrow \tilde{l}) P(I \rightarrow \tilde{l})$$

(25)

where $N(I \rightarrow \tilde{l})$ is the number of bits in error for the corresponding pairwise error event. For the case of $N_s$ being equal to a power of two, (25) becomes

$$P_b \leq \frac{N_s}{2} P(I \rightarrow \tilde{l})$$

(26)

where the equality holds for $N_s = 2$ [33].

### 4. Diversity order analysis

In this section, we analyze the achievable diversity order of the cooperative SSK system for the best and partial relay selection techniques, respectively.

#### 4.1. Best relay selection

If we evaluate the PDF given in (12) at high SNR values, the MGF of $\gamma_{S-D}$ can be approximated as

$$f_{\gamma_{S-D}}(x) \approx \frac{x^{\alpha-1} e^{-\frac{x^\alpha}{\alpha}}}{(P_s)^{\frac{N}{\alpha}} I(N_d)}$$

(12)
\[ M_{T_d, \sigma} (s) = \frac{s^{-N_d}}{(P_S/C_0)^{N_d}}. \]  
(27)

With the help of [27, (1.211)] and [27, (8.446)], the CDF of \( \frac{T_{\text{PRS}}}{P_S} - \text{BER} \) given in (17) can be expressed for high SNR values and \( N_d > 1 \) as

\[ F_{X(\text{PRS})} (x) \approx x^N \left( \frac{T}{P_S} + \frac{TD}{(N_d - 1)} \right)^N. \]  
(28)

Hence, the MGF of \( \frac{T_{\text{PRS}}}{P_S} - \text{BER} \) can be given as

\[ M_{T_{\text{PRS}}, \sigma} (s) = \frac{N!}{s^N} \left( \frac{T}{P_S} + \frac{TD}{(N_d - 1)} \right)^N, \quad P_S > 1. \]  
(29)

By substituting (27) and (29) into (11) and evaluating numerical integrals with the help of common mathematical software, the upper-bound on the BER expression given in (25) can be calculated numerically for high SNR values. It can easily be seen from exponential power of \( s \) in (27) and (29) that the diversity order of the system with the best relay selection is \( N_d + N \). Note that the diversity order analysis given in this subsection is valid for \( N_d > 1 \) and it is shown in [25] that the diversity order of the relaying part of the system with the best relay selection for \( N_d = 1 \) is \( N \).

### 4.2. Partial relay selection

It is proved in [34] that if the PDF of \( \frac{T_{\text{PRS}}}{P_S} - \text{BER} \) can be written as

\[ f_{X(\text{PRS})} (x) = \omega x^a + o(x^a) \]  
for \( x \rightarrow 0^+ \), the diversity order of the system is obtained as \( t = 1 \). By using Taylor series expansion and definition of the modified Bessel function of the second kind [27, (8.446)], the CDF of \( \frac{T_{\text{PRS}}}{P_S} - \text{BER} \) in (21) can be rewritten as

\[ F_{X(\text{PRS})} (x) = \sum_{n=0}^{N-1} \frac{N!}{a^n n!} a^n \sum_{d=1}^{N_d} \phi_d x^d \sum_{d=1}^{N-1} \frac{N!}{a^n n!} a^n \ln a \]  
(30)

where \( \delta_d \) and \( \phi_d \) are the coefficients in which the variables \( a \) and \( x \) are not included. Interested readers are referred to the appendix for the proof of (30). By deriving (30) with respect to \( x \), the PDF of \( \frac{T_{\text{PRS}}}{P_S} - \text{BER} \) can be obtained as

\[ f_{X(\text{PRS})} (x) = \sum_{n=0}^{N-1} \frac{N!}{n!} \phi_d x^d \sum_{d=1}^{N_d} \phi_d x^d \sum_{d=1}^{N-1} \frac{N!}{a^n n!} a^n \ln a. \]  
(31)

Then, the PDF of \( \frac{T_{\text{PRS}}}{P_S} - \text{BER} \) around the origin, \( x \rightarrow 0^+ \), can be written as

\[ f_{X(\text{PRS})} (x) = \begin{cases} e_1 x^{N_d - 1} + o(x^{N_d - 1}), & N_d < N \\ e_2 x^{N_d - 1} + o(x^{N_d - 1}), & N_d \geq N \end{cases} \]  
(32)

where \( e_1 \) and \( e_2 \) denote the coefficients of the lowest order terms. Since \( e_1 \) and \( e_2 \) is very complicated, the asymptotic error performance analysis of the partial relay selection cannot be performed easily. However, considering (27) and (32), the diversity order of the system can be calculated as \( 2N_d \) for \( N_d < N \) and \( N_d + N \) for \( N_d \geq N \).

### 5. Simulation results and analysis

In this section, analytical expressions given in the previous sections are verified through Monte Carlo simulations. We provide BER results for the cooperative SSK scheme with the best and the partial relay selection techniques. Furthermore, the performance comparisons between the proposed SSK system and conventional cooperative scheme, which transmits M-PSK symbol from S, are performed. Results are plotted as a function of \( E_t/N_0 \) where \( E_t = E_s + E_e \) is the total transmitted energy in the network. For simplicity, we assume \( E_s = E_e \). In the figures, \( (N_s, N/1, N_d) \) and \( (N/1, N_d)(M - \text{PSK}) \) stand for the SSK and conventional M-PSK systems, respectively, where \( N_s \) and \( N_d \) transmit and receive antennas are available at S and D, respectively. One relay is selected among \( N \) relays and \( M \) stands for the size of the PSK constellation.

Fig. 2 depicts the BER performance of the proposed SSK system with the best relay selection \( (N_s, 4/1, N_d) \) for \( N_s \in \{2, 4\} \) and \( N_d \in \{2, 4\} \). As can be observed from Fig. 2, the theoretical results exactly match with the computer simulation results for \( N_s = 2 \) and the proposed upper-bound is sufficiently accurate for \( N_s = 4 \). In addition to these results, Fig. 2 also shows that the asymptotic BER curves derived in Section 4.1 approaches to the exact BER curves at high SNR values. According to the asymptotic analysis, the asymptotic diversity orders of the curves corresponding to the proposed SSK systems \( (2, 4/1, 4), (2, 4/1, 2) \) and \( (4, 4/1, 2) \) are calculated as \( N_d + N = 8, 6 \) and 6, respectively. It can be observed from the slopes of the BER curves given in Fig. 2 that these values are consistent with the computer simulation results.

Fig. 3 compares the BER performances of the proposed SSK system with the best relay selection \( (N_s, 4/1, 4) \) and the conventional cooperative M-PSK system \( (1, 4/1, 4)(M - \text{PSK}) \), which applies the best relay selection technique, for \( N_s, M \in \{4, 8\} \). Fig. 3 clearly indicates that the proposed SSK system outperforms the conventional cooperative M-PSK system when the data rate increases. It is important to note that this improvement is achieved with no increase in transceiver complexity; however, with the cost of employing multiple antennas at S without increasing the number of RF chains.

In Fig. 4, BER performances of the proposed SSK system \( (N_s, 2/1, 4) \) with the partial relay selection and conventional cooperative M-PSK system \( (1, 2/1, 4)(M - \text{PSK}) \), which applies the partial relay selection technique, are compared for \( N_s, M \in \{4, 8\} \). Fig. 4 shows that the conventional cooperative M-PSK system \( (4, 2/1, 4) \) outperforms the proposed SSK system \( (4, 2/1, 4) \) by approximately 1.33 dB; however, the proposed SSK system \( (8, 2/1, 4) \) outperforms the conventional cooperative M-PSK system \( (1, 2/1, 4)(M - \text{PSK}) \) by approximately 2.67 dB at a SER value of 10\(^{-4}\). Hence, the proposed SSK system outperforms the conventional cooperative M-PSK system when the data rate increases as in the proposed SSK scheme with the best relay selection. Note that since the proposed SSK system with the partial relay selection provides the same diversity gain and almost the same BER performance as that of the proposed scheme with the best relay selection for \( (N_s, 4/1, 4) \) and \( N_s \in \{4, 8\} \), we choose a different scenario as the number of relays \( N_d = 2 \) in this figure unlike Fig. 3.

In Fig. 5, computer simulation and theoretical BER performance results of the SSK system with the partial relay selection \( (N_s, N/1, 3) \) are given for \( N_s \in \{2, 4\} \) and \( N \in \{2, 5, 10\} \). Fig. 5 clearly indicates that the computer simulation results match with the theoretical results as well as the diversity order results given in Section 4.2 and the BER performance of the system is improved when the number of relays increases. As seen from Fig. 5, the theoretical results exactly match with the computer simulation results for \( N_s = 2 \) and the proposed upper-bound is sufficiently accurate for \( N_s = 4 \). Since the diversity order of the system is \( 2N_d \) for \( N_s < N \) and \( N_d + N \) for \( N_d \geq N \) as given in Section 4.2, the improvement in BER performance decreases when \( N_d = 3 < N \). According to the diversity order analysis, the diversity orders of the curves corresponding to the proposed SSK systems \( (N_s, N/1, 3) \) are calculated as \( 5, 6 \) and \( 6 \) for \( N = \{2, 5\} \) and 10, respectively. It can be observed from the slopes of the BER curves given in Fig. 5 that these values are consistent with the computer simulation results.
Fig. 6 compares the BER performance of the cooperative SSK schemes applying the best and partial relay selection techniques for \(N_s = 2, N \in \{1, 2, 5\}\) and \(N_d = 2\). Here, the system \((2, 1, 1, 2)\) (No Relay Selection) corresponds to the conventional cooperative SSK system without relay selection. As seen from Fig. 6, SSK systems with the best and partial relay selection considerably outperform the SSK system without relay selection by introducing additional diversity gain to the system. Moreover, the best relay selection exhibits a better error performance by providing a constant diversity order of \(N_d + N\) regardless of the system parameters. However, the partial relay selection provides a diversity gain that depends on the number of receive antennas \(N_d\) in D and the number of available relays \(N\); therefore, it provides a worse error performance than the best relay selection. On the other hand, in the best relay selection, since the selection is based on the channel fading coefficients corresponding to S-R and R-D links, a feedback channel between D and the relays is required. Hence, the signaling overhead of the system with partial relay selection is lower than...
that of the best relay selection. As a result, we observe an interesting trade-off between the error performance and system complexity by the best and the partial relay selection methods. However, since the partial relay selection gives almost an identical BER performance compared to the best relay selection when $N_d \geq N$, it can be a more effective solution for the case of $N_d \geq N$. On the other

![Fig. 4. The BER performance comparison of the proposed SSK system $(N_s, 2/1.4)$ and conventional cooperative M-PSK system $(1, 2/1.4)(M – PSK)$ with the partial relay selection for $N_s, M \in \{4, 8\}$.](image1)

![Fig. 5. The BER performance of the proposed SSK system $(N_s, N/1.3)$ with the partial relay selection for $N_s \in \{2, 4\}$ and $N \in \{2, 5, 10\}$.](image2)
hand, the best relay selection would be a better choice for $N_d < N$ with the price of increased complexity.

6. Conclusion

A novel cooperative AF-SSK scheme, which applies the best and partial relay selection methods, has been proposed in this paper. It has been shown that the proposed SSK system outperforms the conventional cooperative SIMO system with relay selection for high data rates and sufficient number of receive antennas at D. It has been also demonstrated that the proposed system provides an interesting trade-off between complexity and error performance by the best and partial relay selection; however, this trade-off no longer exists when the number of receive antennas at D is less than or equal to number of relays and the partial relay selection becomes the preferable option. The exact average BER of the system for $N_d = 2$ and a substantially accurate upper-bound expression on the BER of the system for $N_d = 2^c$, where $c > 1$ is an arbitrary integer, have been obtained. Extensive computer simulation results have been provided to verify the theoretical analysis and show the superiority of the proposed schemes. We conclude that SSK provides a promising solution for cooperative networks employing relay selection techniques.

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Appendix A. Derivation of (30)

By using Taylor series expansion [27], the term $e^{\frac{aT}{P_S}x}$ given in (22) can be expanded as

$$e^{\frac{aT}{P_S}x} = 1 - \frac{aT}{P_S}x + \left(\frac{aT}{P_S}x\right)^2 \frac{1}{2!} - \left(\frac{aT}{P_S}x\right)^3 \frac{1}{3!} + \ldots \quad (33)$$

Then, by using the definition of the modified Bessel function of the second kind [27, (8.446)], the right-hand side term in (22) can be rewritten as

$$2\left(\frac{aT}{P_S}x\right)^r \frac{C_m}{\Gamma(N_d)} \sum_{l=0}^{N_d-1} \frac{(-1)^l(N_d-1)!}{v!} \left(\frac{(aT\delta x)^{N_d+v}}{v!}\right)$$

$$\times \left(ln(aT\delta x) + \Psi(v + N_d + 1) - \Psi(v + 1)\right) \quad (34)$$

Since only the powers of a and x determine the diversity order, (34) can be further simplified as [26, (4.1.26)]

$$2\left(\frac{aT}{P_S}x\right)^r \frac{C_m}{\Gamma(N_d)} \sum_{l=0}^{N_d-1} \frac{(-1)^l(N_d-1)!}{v!} \left(\frac{(aT\delta x)^{N_d+v}}{v!}\right)$$

$$\times \left(ln(aT\delta x) + \Psi(v + N_d + 1) - \Psi(v + 1)\right) \quad \text{for } r \leq 5$$

(35)

where $\beta_v$ and $\alpha_v$ are the coefficients in which the variables a and x are not included. Substituting (33) and (35) into (22), the CDF of $\gamma_{PS}^{N_d+1}$ can be written as

$$F_{\gamma_{PS}^{N_d+1}}(x) = 1 + \sum_{v=0}^{N_d-1} \frac{N}{a} (-1)^v \left(1 + \frac{aT}{P_S}x + \left(\frac{aT}{P_S}x\right)^2 \frac{1}{2!} \ldots \right)$$

$$\times \left[1 + \sum_{v=0}^{\infty} \beta_v(ax)^v + \sum_{v=0}^{\infty} \alpha_v(ax)^v \ln a\right] \quad (36)$$

After some simple mathematical manipulations and transformation of v into $b = v + N_d$, (36) can be expressed as

$$F_{\gamma_{PS}^{N_d+1}}(x) = \sum_{v=0}^{\infty} \delta_v x^v \sum_{a=0}^{N} \left(\frac{N}{a}\right)(-1)^v a^v + \sum_{b=N_d}^{\infty} \phi_b x^b \sum_{a=0}^{N} \left(\frac{N}{a}\right)(-1)^v b^v \ln a$$

where $\delta_v$ and $\phi_b$ are the coefficients in which the variables a and x are not included. Finally, with the help of [27, (0.154.3)], (30) can be obtained.
References