AKM204E Fluid Mechanics
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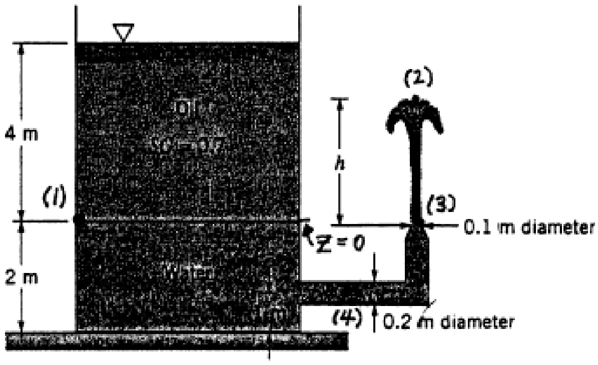
HOMEWORK 2 ***S O L U T I O N S***

3.29

3.29 A large open tank contains a layer of oil floating on water as shown in Fig. P3.29. The flow is steady and inviscid. (a) Determine the height, h, to which the water will rise. (b) Determine the water velocity in the pipe. (c) Determine the pressure in the horizontal pipe.



■ FIGURE P3.29

(a) $$\frac{p_1}{\gamma}+z_1+\frac{V_1^2}{2g}=\frac{p_2}{\gamma}+z_2+\frac{V_2^2}{2g} \qquad (1)$$

where

$z_1=0,\ p_2=0,\ V_1=V_2=0,\ z_2=h$, and $p_1=4m\,(\gamma_{oil})$

Thus, with $\gamma_{oil}=SG\,\gamma_{H_2O}=0.7\left(9.80\,\frac{kN}{m^3}\right)=6.86\,\frac{kN}{m^3}$

and from Eq. (1)

$\frac{p_1}{\gamma}=z_2$ or $p_1=\gamma h$ so that

$$h=\frac{4m\,\gamma_{oil}}{\gamma}=4m\,\frac{6.86\,\frac{kN}{m^3}}{9.80\,\frac{kN}{m^3}}=\underline{\underline{2.80m}}$$

(b) $V_4A_4=V_3A_3$ or $V_4=\frac{A_3}{A_4}V_3=\frac{\frac{\pi}{4}(0.1m)^2}{\frac{\pi}{4}(0.2m)^2}V_3=\frac{1}{4}V_3$

But from the Bernoulli equation,

$$V_3=\sqrt{2gh}=\sqrt{2(9.81\,m/s^2)(2.80m)}=7.41\,\frac{m}{s}$$

Thus,

$$V_4=\frac{1}{4}\left(7.41\,\frac{m}{s}\right)=\underline{\underline{1.85\,\frac{m}{s}}}$$

(c) $$\frac{p_4}{\gamma}+z_4+\frac{V_4^2}{2g}=\frac{p_2}{\gamma}+z_2+\frac{V_2^2}{2g}$$

where

$z_4=-1m,\ V_4=1.85\,\frac{m}{s},\ p_2=0,\ z_2=2.8m,\ V_2=0$

Thus,

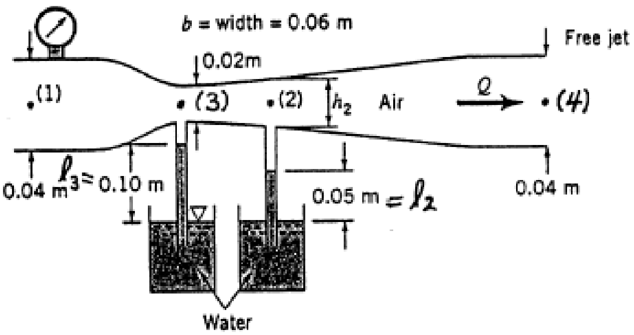
$$\frac{p_4}{\gamma}-1m+\frac{(1.85\,\frac{m}{s})^2}{2(9.81\,\frac{m}{s^2})}=2.8m \quad \text{or} \quad \frac{p_4}{\gamma}=3.63\,m$$

Thus,

$$p_4=3.63\,m\left(9.80\,\frac{kN}{m^3}\right)=\underline{\underline{35.5\,kPa}}$$

3.51

3.51 Air flows through a Venturi channel of rectangular cross section as shown in Video V3.6 and Fig. P3.51 The constant width of the channel is 0.06 m and the height at the exit is 0.04 m. Compressibility and viscous effects are negligible. (a) Determine the flowrate when water is drawn up 0.10 m in a small tube attached to the static pressure tap at the throat where the channel height is 0.02 m. (b) Determine the channel height, h_2, at section (2) where, for the same flowrate as in part (a), the water is drawn up 0.05 m. (c) Determine the pressure needed at section (1) to produce this flow.



■ FIGURE P3.51

(a) For steady, inviscid, incompressible flow: $(\gamma = 12.0 \frac{N}{m^3})$

(1) $\frac{p_3}{\gamma} + \frac{V_3^2}{2g} = \frac{p_4}{\gamma} + \frac{V_4^2}{2g}$ where $p_4 = 0$, $p_3 = -\gamma_{H_2O}\ell_3 = 9.80 \times 10^3 \frac{N}{m^3}(0.1m)$
$= -980 \frac{N}{m^2}$

Also, $A_3 V_3 = A_4 V_4$ so that $V_3 = \frac{(0.04m \times 0.06m)}{(0.02m \times 0.06m)} V_4 = 2V_4$

Thus, Eqn. (1) becomes

$$\frac{-980 \frac{N}{m^2}}{12.0 \frac{N}{m^3}} + \frac{4V_4^2}{2(9.81\frac{m}{s^2})} = \frac{V_4^2}{2(9.81\frac{m}{s^2})} \quad \text{or} \quad V_4 = 23.1 \frac{m}{s}$$

Hence,

$Q = A_4 V_4 = (0.04m \times 0.06m)(23.1 \frac{m}{s}) = \underline{\underline{0.0554 \frac{m^3}{s}}}$

(2) (b) $\frac{p_2}{\gamma} + \frac{V_2^2}{2g} = \frac{p_4}{\gamma} + \frac{V_4^2}{2g}$ where $p_4 = 0$, $p_2 = -\gamma_{H_2O}\ell_2 = 9.80 \times 10^3 \frac{N}{m^3}(0.05m)$
$= -490 \frac{N}{m^2}$

From part (a), $V_4 = 23.1 \frac{m}{s}$

Thus, Eqn. (2) becomes

$$\frac{-490 \frac{N}{m^2}}{12.0 \frac{N}{m^3}} + \frac{V_2^2}{2(9.81\frac{m}{s^2})} = \frac{(23.1 \frac{m}{s})^2}{2(9.81\frac{m}{s^2})} \quad \text{or} \quad V_2 = 36.5 \frac{m}{s}$$

But $V_2 A_2 = V_4 A_4$ so that

$(36.5 \frac{m}{s})(0.06m)h_2 = (23.1 \frac{m}{s})(0.06m)(0.04m)$ or $h_2 = \underline{\underline{0.0253 m}}$

(3) (c) Also, $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_4}{\gamma} + \frac{V_4^2}{2g}$ where $p_4 = 0$ and $A_1 V_1 = A_4 V_4$

But since $A_1 = (0.04m \times 0.06m) = A_4$ then $V_1 = V_4$ and Eqn. (3) gives

$p_1 = p_4 = \underline{\underline{0}}$

4,23

4.23 As a valve is opened, water flows through the diffuser shown in Fig. P4.23 at an increasing flowrate so that the velocity along the centerline is given by $\mathbf{V} = u\hat{\mathbf{i}} = V_0(1 - e^{-ct})(1 - x/\ell)\hat{\mathbf{i}}$, where u_0, c, and ℓ are constants. Determine the acceleration as a function of x and t. If $V_0 = 10$ ft/s and $\ell = 5$ ft, what value of c (other than $c = 0$) is needed to make the acceleration zero for any x at $t = 1$ s? Explain how the acceleration can be zero if the flowrate is increasing with time.

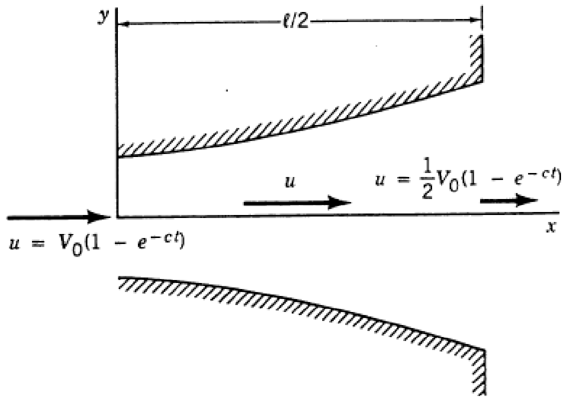


FIGURE P4.23

$\vec{a} = \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V}$ With $u = u(x,t)$, $v = 0$, and $w = 0$ this becomes

$\vec{a} = (\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x})\hat{i} = a_x\hat{i}$, where $u = V_0(1 - e^{-ct})(1 - \frac{x}{\ell})$

Thus,

$$a_x = V_0(1 - \frac{x}{\ell})c\,e^{-ct} + V_0^2(1 - e^{-ct})^2(1 - \frac{x}{\ell})(-\frac{1}{\ell})$$

or

$$\underline{\underline{a_x = V_0(1 - \frac{x}{\ell})\left[c\,e^{-ct} - \frac{V_0}{\ell}(1 - e^{-ct})^2\right]}}$$

If $a_x = 0$ for any x at $t = 1$ s we must have

$\left[c\,e^{-ct} - \frac{V_0}{\ell}(1 - e^{-ct})^2\right] = 0$ With $V_0 = 10$ and $\ell = 5$

$c\,e^{-c} - \frac{10}{5}(1 - e^{-c})^2 = 0$ The solution (root) of this equation is $\underline{\underline{c = 0.490\ \frac{1}{s}}}$

For the above conditions the local acceleration ($\frac{\partial u}{\partial t} > 0$) is precisely balanced by the convective deceleration ($u\frac{\partial u}{\partial x} < 0$).

The flowrate increases with time, but the fluid flows to an area of lower velocity.

4.7

4.7 The velocity field of a flow is given by $u = -V_0 y/(x^2 + y^2)^{1/2}$ and $v = V_0 x/(x^2 + y^2)^{1/2}$, where V_0 is a constant. Where in the flow field is the speed equal to V_0? Determine equation of the streamlines and discuss the various characteristics of this flow.

$$u = -V_0 \frac{y}{(x^2+y^2)^{1/2}} \quad , \quad v = V_0 \frac{x}{(x^2+y^2)^{1/2}} \quad \text{so that}$$

$$V = \sqrt{u^2+v^2} = \left[\frac{V_0^2 (y^2+x^2)}{(x^2+y^2)} \right]^{1/2} = V_0$$

Thus, $\underline{\underline{V = V_0 \text{ throughout the entire flow field}}}$

Streamlines are given by

$$\frac{dy}{dx} = \frac{v}{u} = \frac{x}{-y} \quad \text{or} \quad -y\,dy = x\,dx \quad \text{which can be integrated}$$

to give $\underline{\underline{x^2 + y^2 = \text{const.}}}$

Thus, the fluid flow with circular streamlines and the speed is constant throughout.

4.15

4.15 A three-dimensional velocity field is given by $u = x^2$, $v = -2xy$, and $w = x + y$. Determine the acceleration vector.

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$
$$= x^2(2x) = 2x^3$$

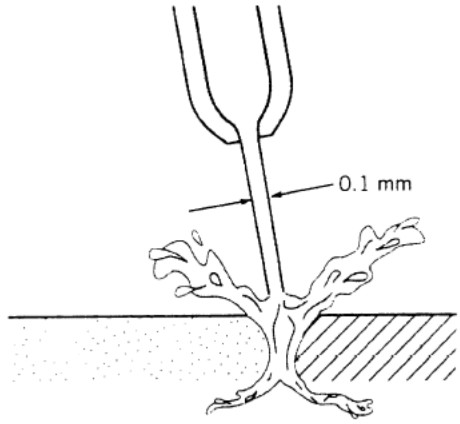
$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$
$$= x^2(-2y) + (-2xy)(-2x) = 2x^2 y$$

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$
$$= x^2(1) + (-2xy)(1) = x^2 - 2xy$$

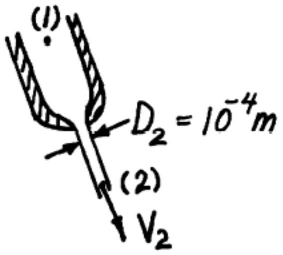
Thus,

$$\vec{a} = \underline{\underline{2x^3\,\hat{i} + 2x^2 y\,\hat{j} + (x^2 - 2xy)\,\hat{k}}}$$

3.26 Small-diameter, high-pressure liquid jets can be used to cut various materials as shown in Fig. P3.26. If viscous effects are negligible, estimate the pressure needed to produce a 0.10-mm-diameter water jet with a speed of 700 m/s. Determine the flowrate.



■ FIGURE P3.26



$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 \quad \text{where } V_1 \approx 0,\ z_1 \approx z_2, \text{ and } p_2 = 0$$

Thus $p_1 = \frac{1}{2}\frac{\gamma}{g}V_2^2 = \frac{1}{2}\rho V_2^2 = \frac{1}{2}\left(999\frac{kg}{m^3}\right)\left(700\frac{m}{s}\right)^2 = \underline{\underline{2.45 \times 10^5 \frac{kN}{m^2}}}$

Also,

$$Q = V_2 A_2 = 700\frac{m}{s}\left[\frac{\pi}{4}(10^{-4}m)^2\right] = \underline{\underline{5.50 \times 10^{-6}\frac{m^3}{s}}}$$