Michelle Mizuno-Wiedner
Dept. of Astronomy and Space Physics
Uppsala University

Comment on the gas constant

One of the annoying things about converting between cgs and SI units is that you must be especially careful whenever using a formula involving the gas constant,

$$\mathcal{R} = 8.3143 \text{ J K}^{-1}\text{mol}^{-1} \qquad \text{or} \qquad 8.3143 \times 10^7 \text{ erg K}^{-1}\text{mol}^{-1} \tag{1}$$

As an example, let us look at the ideal gas law (or perfect gas law),

$$P = nkT = \frac{\rho}{\mu m_u} kT \tag{2}$$

where P is the pressure, T the temperature, n the number density of particles, ρ the mass density, μ the mean molecular weight and $m_u = 1.66 \times 10^{-27}$ kg the atomic mass unit. The gas constant $\mathcal{R}$ is related to the Boltzmann constant k by

$$\mathcal{R} = kN_A \tag{3}$$

where $N_A = 6.022 \times 10^{23}$ mol^{-1} is Avogadro's number. The ideal gas law then becomes

$$P = \frac{\mathcal{R}}{\mu m_u N_A} \rho T \tag{4}$$

Both Eqs. 2 and 4 are valid regardless of whether you use cgs or SI units. In terms of dimensions, if m_u is regarded as a mass, then μ is dimensionless.

One important point is that $m_u N_A = 1$ (g mol^{-1}), *but only in cgs units.* This is because 1 mole is defined in terms of grams and not kilograms. Thus many books write the ideal gas law as

$$P = \frac{\mathcal{R}}{\mu} \rho T \quad [\text{cgs}] \tag{5}$$

Please note: omitting the factor $m_u N_A$ means that μ is no longer dimensionless but now has the units of g mol^{-1}. This could lead to problems if you use SI units but still assume that μ is dimensionless, because then there will be this missing factor of $m_u N_A = 0.001$ kg mol^{-1}. For example, if you plug in the correct SI values for ρ and $\mathcal{R}$ into Eq. 5 but take the mean molecular weight μ of an element equal to its atomic weight $\mathcal{A}$, then your numerical result will be 1000 times too small.

The course textbook states the gas constant not as $\mathcal{R} = kN_A$ but as $\mathcal{R} = k/m_u$, again assuming that $m_u N_A = 1$. This gives the gas constant in terms of kilograms rather than moles (i.e., $\mathcal{R} = 8.3143 \times 10^3$ J kg^{-1} K^{-1}), which yields the correct numerical result in cases like Eq. 5. Personally, I find the resulting units somewhat inconsistent, e.g., the atomic mass unit $m_u = 1/N_A = 1.66 \times 10^{-24}$ mol^{-1} is in inverse moles rather than in (kilo)grams.

Of course there may be a simpler way to understand the hassle with the units, and if so, please tell me about it! In the meantime, my advice is: if you want to avoid cgs units, stick with the Boltzmann constant k rather than the gas constant $\mathcal{R}$.