Karman Pohlhausen integral yöntemi

θ için birinci dereceden Taylor açılımı ile

$$\theta(x+\Delta x)=\theta(x)+\left(\frac{d\theta}{dx}\right)_x \Delta x$$

Buradaki $d\theta/dx$ türevi *von Karman integral denklemi*nden hesaplanabilir.

$$\frac{d\theta}{dx}=\frac{C_f}{2}-\frac{\theta}{U_e}\frac{dU_e}{dx}(2+H)$$

$$H=\frac{\delta^*}{\theta}$$

Ancak, $x = 0$ için $U_e = 0$ ve $C_f \to \infty$ olup

$$\frac{d\theta}{dx}=\infty-\infty$$

Belirsizliği gidermek için, $Z=\frac{\theta^2}{\nu}$ şeklinde bir değişken dönüşümü faydalı olur

$$\frac{dZ}{dx}=\frac{C_f \mathrm{Re}_\theta-\Lambda(2+H)}{U_e}$$

Burada

$$\mathrm{Re}_\theta=\frac{U_e\theta}{\nu}$$

$$\Lambda=\frac{\theta^2}{\nu}\frac{dU_e}{dx}=Z\frac{dU_e}{dx}$$

Holstein-Bohlen basınç gradyantı parametresi

Bu durumda Z için birinci dereceden Taylor açılımı ile

$$Z(x+\Delta x)=Z(x)+\left(\frac{dZ}{dx}\right)_x \Delta x$$

Karman Pohlhausen integral yöntemi

Pohlhausen hız profilleri

$$\frac{u}{U_e} = 2\zeta - 2\zeta^3 + \zeta^4 + \frac{\lambda}{6}\zeta(1-\zeta)^3$$

$$\zeta = \frac{y}{\delta}$$

$$\lambda = \frac{\delta^2}{\nu}\frac{dU_e}{dx}$$

Pohlhausen basınç gradyantı parametresi

$$\frac{C_f}{2}\frac{U_e\delta}{\nu} = 2 + \frac{\lambda}{6}$$

$$H = \frac{\delta^*}{\theta} = \frac{\delta^*/\delta}{\theta/\delta}$$

$$\Lambda = \frac{\theta^2}{\nu}\frac{dU_e}{dx}$$

$$\lambda = \frac{\delta^2}{\nu}\frac{dU_e}{dx}$$

$$\Lambda = \lambda(\theta/\delta)^2$$

$$C_f \mathrm{Re}_\theta = 2\frac{\theta}{\delta}\left(2 + \frac{\lambda}{6}\right)$$

$$\Lambda(2+H) = \lambda\frac{\theta}{\delta}\left(2\frac{\theta}{\delta} + \frac{\delta^*}{\delta}\right)$$

$$C_f \mathrm{Re}_\theta - \Lambda(2+H) = f(\lambda) = 2\frac{\theta}{\delta}\left(2 + \frac{\lambda}{6}\right) - \lambda\frac{\theta}{\delta}\left(2\frac{\theta}{\delta} + \frac{\delta^*}{\delta}\right)$$

$$\frac{\delta^*}{\delta} = \frac{3}{10} - \frac{\lambda}{120}$$

$$\frac{\theta}{\delta} = \frac{37}{315} - \frac{\lambda}{945} - \frac{\lambda^2}{9072}$$

$$f(\lambda) = 2\left(\frac{37}{315} - \frac{\lambda}{945} - \frac{\lambda^2}{9072}\right)\left[2 - \frac{116}{315}\lambda + \left(\frac{2}{945} + \frac{1}{120}\right)\lambda^2 + \frac{2}{9072}\lambda^3\right]$$

Karman Pohlhausen integral yöntemi

Başlangıç koşulları

$$\boxed{\frac{dZ}{dx}=\frac{C_f\,\mathrm{Re}_\theta-\Lambda(2+H)}{U_e}} \Rightarrow \boxed{f(\lambda)=C_f\,\mathrm{Re}_\theta-\Lambda(2+H)} \Rightarrow \boxed{\frac{dZ}{dx}=\frac{f(\lambda)}{U_e}}$$

$$f(\lambda)=2\left(\frac{37}{315}-\frac{\lambda}{945}-\frac{\lambda^2}{9072}\right)\left[2-\frac{116}{315}\lambda+\left(\frac{2}{945}+\frac{1}{120}\right)\lambda^2+\frac{2}{9072}\lambda^3\right]$$

Sınır tabakanın başlangıç noktasında $U_e = 0$ olup dZ/dx türevinin tekillik göstermemesi için $f(\lambda)=0$ olmalı $\Rightarrow$ $\boxed{\lambda=7.052}$

$\Downarrow$

Ayrıca $$\Lambda=\lambda\left(\frac{\theta}{\delta}\right)^2=\lambda\left(\frac{37}{315}-\frac{\lambda}{945}-\frac{\lambda^2}{9072}\right)^2 \Rightarrow \boxed{\Lambda=0.0770}$$

Bu durumda $$\left(\frac{dZ}{dx}\right)_{x=0}=\frac{0}{0} \Rightarrow$$ Limit alınarak $$\boxed{\left(\frac{dZ}{dx}\right)_{x=0}=-0.0652\frac{(d^2U_e/dx^2)_{x=0}}{(dU_e/dx)^2_{x=0}}}$$

Ayrıca $$\Lambda=Z\frac{dU_e}{dx} \Rightarrow Z=\frac{\Lambda}{dU_e/dx} \Rightarrow \boxed{Z_{x=0}=\frac{0.0770}{(dU_e/dx)_{x=0}}}$$

Karman Pohlhausen integral yöntemi

Dairesel silindir üzerinde hız dağılımı $$U_e = 2U_\infty \sin\theta = 2U_\infty \sin\frac{x}{R}$$

Türev alınarak $$\frac{dU_e}{dx} = 2\frac{U_\infty}{R}\cos\frac{x}{R} \Rightarrow \left(\frac{dU_e}{dx}\right)_{x\to 0} = 2\frac{U_\infty}{R}$$

$$\frac{d^2U_e}{dx^2} = 2\frac{U_\infty}{R^2}\sin\frac{x}{R} \Rightarrow \left(\frac{d^2U_e}{dx^2}\right)_{x\to 0} = 0$$

Buna göre $$\boxed{\left(\frac{dZ}{dx}\right)_{x=0} = 0}$$

Herhangi küt bir cisim için de hücum kenarı eğrilik yarıçapı R_0 olmak üzere

$$\boxed{\left(\frac{dZ}{dx}\right)_{x=0} \cong 0} \quad \boxed{Z_{x=0} = \frac{0.0770}{2U_\infty / R_0}}$$

Karman Pohlhausen integral yöntemi

Başlangıç koşulları

$$x = 0 \qquad \left(\frac{dZ}{dx}\right)_{x=0} \cong 0 \qquad Z_{x=0} = \frac{0.0770}{2U_\infty / R_0} \qquad \theta = \sqrt{\nu Z}$$

$$x = x + \Delta x$$

$$Z(x+\Delta x) = Z(x) + \left(\frac{dZ}{dx}\right)_x \Delta x$$

$$\theta = \sqrt{\nu Z}$$

$$\Lambda = Z \frac{dU_e}{dx}$$

$$\lambda$$

$$\Lambda = \left(\frac{37}{315} - \frac{\lambda}{945} - \frac{\lambda^2}{9072}\right)^2 \lambda$$

$$\lambda = -12$$

$$\frac{dZ}{dx} = \frac{f(\lambda)}{U_e}$$

$$f(\lambda) = C_f \,\mathrm{Re}_\theta - \Lambda(2 + H)$$

$$C_f \,\mathrm{Re}_\theta = 2(\theta/\delta)(2 + \lambda/6)$$

$$\delta^* = H \cdot \theta$$

$$H = \frac{\delta^*}{\theta} = \frac{\delta^*/\delta}{\theta/\delta}$$

$$\delta^*/\delta = 3/10 - \lambda/120$$

$$\theta/\delta = 37/315 - \lambda/945 - \lambda^2/9072$$

Karman Pohlhausen integral yöntemi

$$\Lambda = \left(\frac{37}{315} - \frac{\lambda}{945} - \frac{\lambda^2}{9072} \right)^2 \lambda$$

denkleminin Λ verildiğinde λ için çözümü

$$\lambda^{k+1} = \frac{\Lambda}{\left(\frac{37}{315} - \frac{\lambda^k}{945} - \frac{(\lambda^k)^2}{9072} \right)^2}$$

Şeklinde basit iterasyonla gerçekleştirilebilir

λ	Λ
8	0.083134166
-12	-0.156734694
-11	-0.14741125
-10	-0.136935401
-9	-0.125434028
-8	-0.113042997
-7	-0.099905696
-6	-0.08617158
-5	-0.071994709
-4	-0.057532295
-3	-0.04294324
-2	-0.028386679
-1	-0.014020523
0	0
1	0.013523803
2	0.026405398
3	0.038506354
4	0.049696755
5	0.059856658
6	0.068877551
7	0.076663812
8	0.083134166
9	0.088223143
10	0.091882537
11	0.094082863
12	0.094814815
13	0.094090726

HESAPLA