

Circuit and System Analysis

EEF 232E

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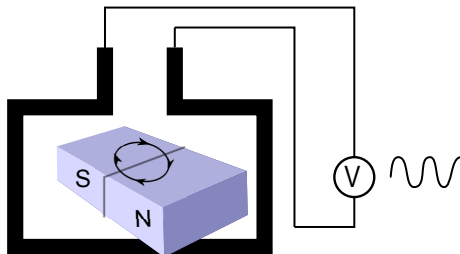
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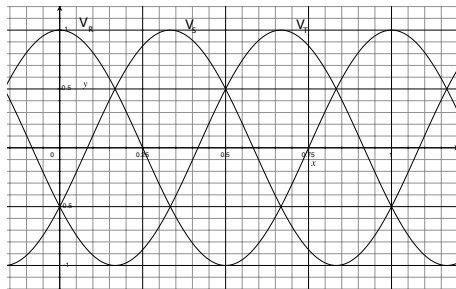
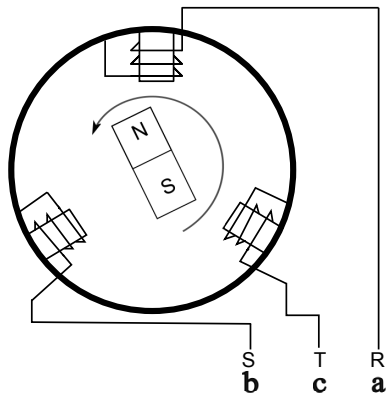
1 Three-Phase Systems

- Y-connected source
- Line-to-line voltage
- Delta-connected source
- Load in Three Phase Circuit
- Δ -to-Y
- Power calculation in balanced three-phase circuits

Three-Phase Systems

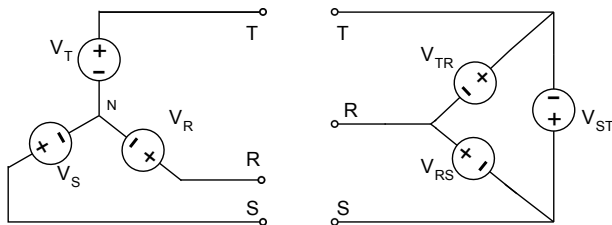


Three-Phase Sources



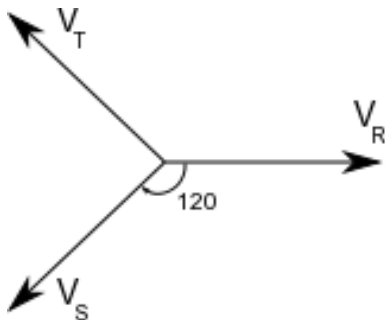
Three-Phase Sources

There are two ways of interconnecting the separate phase windings to form a three phase source in either a wye (Y) or a delta (Δ) configuration.



Phase Voltages of Y-connected source are the line-to-neutral voltages.
Phase Voltages of Δ -connected source are the line-to-line voltages.

Y-connected source



Phase Voltage of Y-connected source (The line-to-neutral voltages)

$$v_a(t) = v_R(t) = V_{RM} \cos(\omega t - \theta_R)$$

$$v_b(t) = v_S(t) = V_{SM} \cos(\omega t - \theta_S)$$

$$v_c(t) = v_T(t) = V_{TM} \cos(\omega t - \theta_T)$$

Balanced Three-Phase Voltage Sources



$$V_{RM} = V_{SM} = V_{TM}$$



$$\theta_S = \theta_R - 120^\circ$$

$$\theta_T = \theta_S - 120^\circ$$

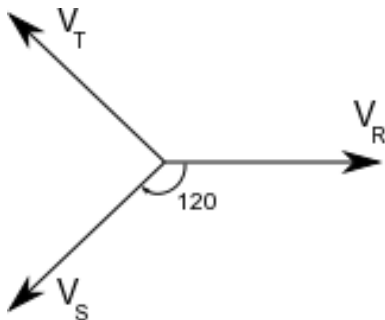
$$V_R = V_m e^{j\theta_R}$$

$$V_S = V_m e^{j\theta_S}$$

$$V_T = V_m e^{j\theta_T}$$

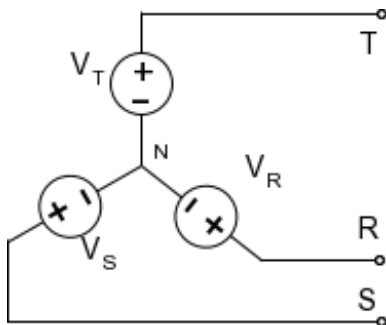
Phasor Diagrams

RST or abc phase sequence !



$$V_R + V_S + V_T = ?$$

Line Voltage



Line Voltages:

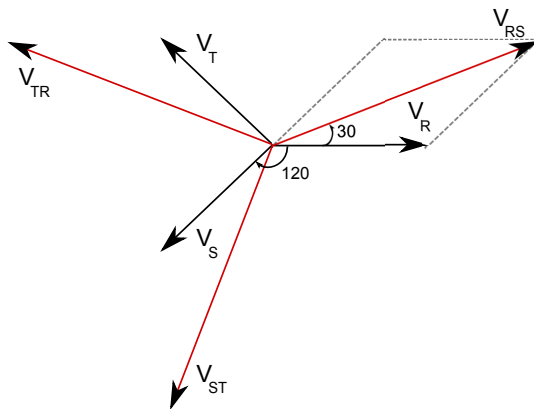
$$V_{RS} = V_R - V_S = V_m e^{j\theta_R} (1 - e^{-j120}) = V_m \sqrt{3} e^{j(\theta_R + 30)}$$

$$V_{ST} = V_S - V_T = V_m \sqrt{3} e^{j(\theta_R - 90)}$$

$$V_{TR} = V_T - V_R = V_m \sqrt{3} e^{j(\theta_R - 210)}$$

Line voltage refers to the voltage across any pair of lines.

Phasor Diagrams

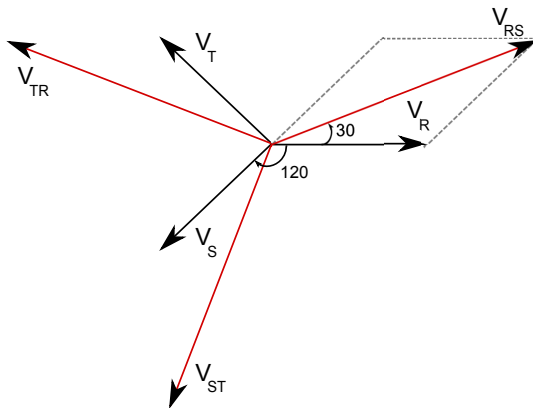


$$|V_{RS}| = \sqrt{3}|V_R|$$

$$|V_{ST}| = \sqrt{3}|V_S|$$

$$|V_{TR}| = \sqrt{3}|V_T|$$

Phasor Diagrams

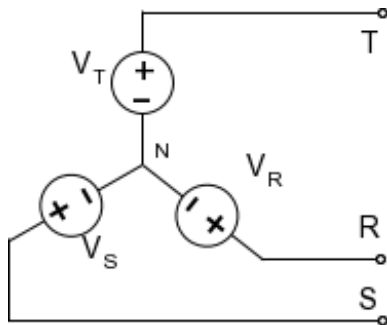


$$V_{RS} = \sqrt{3}V_R e^{30j}$$

$$V_{ST} = \sqrt{3}V_S e^{30j}$$

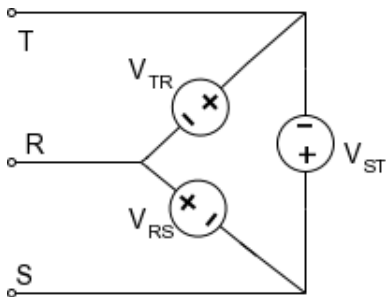
$$V_{TR} = \sqrt{3}V_T e^{30j}$$

Line Voltage



In Y-connection: line voltage and phase voltage are not identical !

Δ -connected source



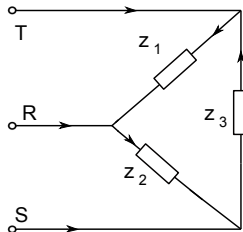
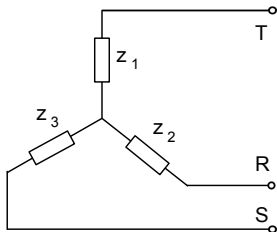
$$V_{RS} = V_M e^{\theta}$$

$$V_{ST} = V_M e^{\theta - 120}$$

$$V_{TR} = V_M e^{\theta - 240}$$

In Δ -connection: line voltage and phase voltage are identical !

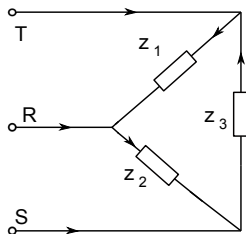
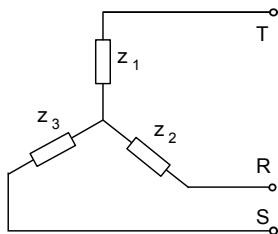
Load in Three Phase Circuit



Balanced load : $Z_1 = Z_2 = Z_3$

In Y-connected Load: Phase currents and line current are identical. In Δ -connected they are not!

Y-connected Load



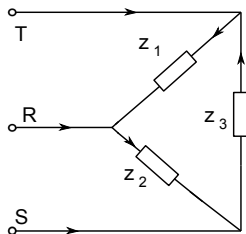
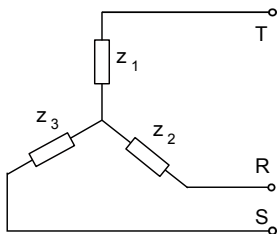
$$|V_R| = |V_S| = |V_T| \text{ and}$$

$$Z_1 = Z_2 = Z_3 = Z_Y = |Z_Y|e^{j\theta_Y}$$

Phase currents (line current);

$$I_R = \frac{V_R}{Z_Y}, \quad I_S = \frac{V_R}{Z_Y}e^{-120j}, \quad I_T = \frac{V_R}{Z_Y}e^{-240j}$$

Δ -connected Load



Phase currents !:

$$I_2 = \frac{V_{RS}}{Z_2} = \frac{V_{RS}}{Z_{\Delta}}, \quad I_3 = \frac{V_{ST}}{Z_3} = \frac{V_{ST}}{Z_{\Delta}}, \quad I_1 = \frac{V_{TR}}{Z_1} = \frac{V_{TR}}{Z_{\Delta}}$$

Line currents:

$$I_R = I_2 - I_1 = \frac{V_{RS} - V_{TR}}{Z_{\Delta}}, \quad I_S = I_3 - I_2 = \frac{V_{ST} - V_{RS}}{Z_{\Delta}},$$

$$I_T = I_1 - I_3 = \frac{V_{TR} - V_{ST}}{Z_{\Delta}}$$

Δ -connected Load

$$\begin{aligned}V_{RS} - V_{TR} &= \sqrt{3}V_R e^{30j} - \sqrt{3}V_T e^{30j} \\ &= \sqrt{3}e^{30j}(V_R - V_T) \\ &= \sqrt{3}e^{30j}(\sqrt{3}V_R e^{-30j}) \\ &= 3V_R\end{aligned}$$

Line currents;

$$I_R = \frac{3V_R}{Z_\Delta}, I_S = \frac{3V_S}{Z_\Delta}, I_T = \frac{3V_T}{Z_\Delta}.$$

$|I_R| = |I_S| = |I_T| = |I_L|$ and $|I_1| = |I_2| = |I_3| = |I_\Delta|$ where

$$|I_L| = \sqrt{3}|I_\Delta|$$

Example

In a balanced δ -connected load, line-to-line voltage is 600V at @50Hz.(a) Calculate phase current and (b) line current.

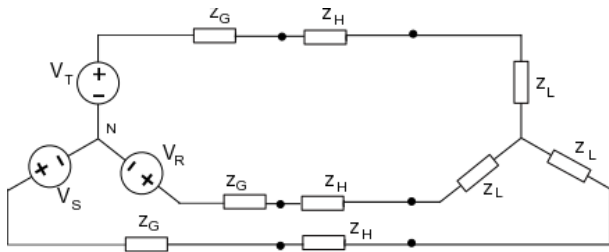
Phase current :

$$I_{\Delta} = \frac{600}{120} = 5A$$

Line current:

$$I_L = \sqrt{3}I_{\Delta} = 8.66A$$

Example

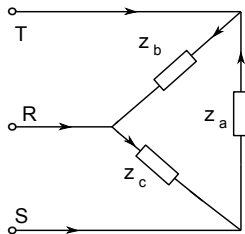
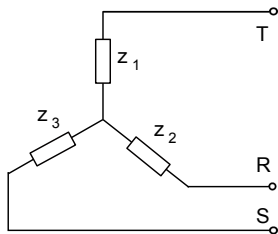


Line current (T):

$$I_L = \frac{V_T}{Z_G + Z_H + Z_L}$$

The phase voltage at the load (T): $V_{TL} = I_L Z_L$ The line voltage at the load (T): $V_{TR} = \sqrt{3} V_{TL} e^{-30j}$

Δ -to-Y



Y to Δ ;

$$Z_a = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_2}, \quad Z_b = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_3}, \quad Z_c = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_3}{Z_1}$$

Δ to Y;

$$Z_1 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c}, \quad Z_2 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c}, \quad Z_3 = \frac{Z_a Z_c}{Z_a + Z_b + Z_c}$$

PS: if $Z_1 = Z_2 = Z_3 = Z_Y$ and $Z_a = Z_b = Z_c = Z_\Delta$ then

$$Z_Y = \frac{Z_\Delta}{3}$$

Power calculation

Complex power in Y-connected load

$$N_Y = N_R + N_S + N_T$$

the complex power associate the R-phase

$$\begin{aligned} N_R &= \frac{1}{2} V_R \bar{I}_R = \frac{1}{2} V_R \bar{V}_R \bar{Y}_Y \\ &= \frac{1}{2} |V_R|^2 \bar{Y}_Y \end{aligned}$$

$$|V_R| = |V_S| = |V_T| = |V_P| \text{ and } V_{P_{\text{rms}}} = \frac{|V_P|}{\sqrt{2}}$$

$$N_Y = \frac{3}{2} \bar{Y}_Y |V_R|^2 = 3 \bar{Y}_Y V_{P_{\text{rms}}}^2$$

or using phase current;

$$N_Y = 3 Z_Y I_{L_{\text{rms}}}^2.$$

Power calculation

Line-to-line voltage for balanced load

$$V_{RS} = \sqrt{3}V_R e^{-30j}$$

$$|V_{RS}| = |V_{ST}| = |V_{TR}| = |V_{LL}| \text{ and } V_{LL\text{rms}} = \frac{|V_{LL}|}{\sqrt{2}}.$$

Power ;

$$N_Y = \bar{Y}_Y V_{LL\text{rms}}^2$$

Complex power of Y-connected $Z_Y = |Z_Y|e^{j\phi_Y}$ ($\phi_Y = \phi_V - \phi_I$)

$$N_Y = \frac{3}{2}|I_L||V_P|(\cos \phi_Y + j \sin \phi_Y)$$

instead of V_P lets use line-to-line voltage

$$\begin{aligned} N_Y &= \frac{3}{2}|I_L| \frac{|V_{LL}|}{\sqrt{3}} (\cos \phi_Y + j \sin \phi_Y) \\ &= \frac{\sqrt{3}}{2}|I_L||V_{LL}| (\cos \phi_Y + j \sin \phi_Y) \end{aligned}$$

$$N_Y = \sqrt{3}I_{L\text{rms}}V_{LL\text{rms}}(\cos \phi_Y + j \sin \phi_Y)$$

Power calculation in Δ -connected load

$$N_{\Delta} = N_1 + N_2 + N_3$$

For Z_1

$$N_1 = \frac{1}{2} Z_1 |I_1|^2 = \frac{1}{2} Z_{\Delta} |I_1|^2$$

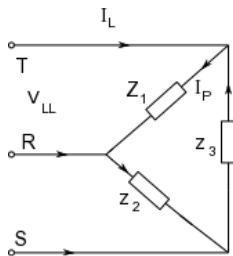
using $|I_1| = |I_2| = |I_3| = |I_P|$

$$N_{\Delta} = 3 Z_{\Delta} |I_{Prms}|^2$$

we have $|I_L| = \sqrt{3} |I_P|$ hence

$$N_{\Delta} = Z_{\Delta} |I_{Lrms}|^2$$

Power calculation



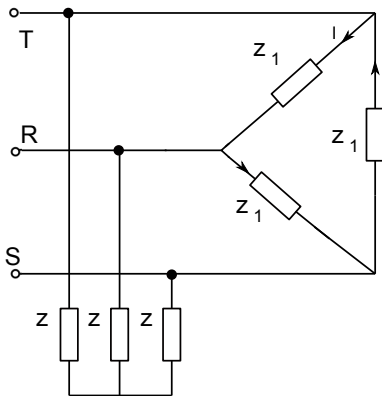
$$Z_{\Delta} = |Z_{\Delta}| e^{j\phi_{\Delta}} \quad (\phi_{\Delta} = \phi_V - \phi_I)$$

$$N_{\Delta} = \frac{3}{2} |I_P| |V_{LL}| (\cos \phi_{\Delta} + j \sin \phi_{\Delta})$$

$$\begin{aligned} N_{\Delta} &= \frac{3}{2} |V_{LL}| \frac{|I_L|}{\sqrt{3}} (\cos \phi_{\Delta} + j \sin \phi_{\Delta}) = \frac{\sqrt{3}}{2} |V_{LL}| |I_L| (\cos \phi_{\Delta} + j \sin \phi_{\Delta}) \\ &= \sqrt{3} I_{L_{\text{rms}}} V_{LL_{\text{rms}}} (\cos \phi_{\Delta} + j \sin \phi_{\Delta}). \end{aligned}$$

Examples

A balanced three-phase source with line-to-line voltage $220\sqrt{6}$ is supplying $25992W$ active and $0VAR$ reactive power to Y and Δ -connected two balanced loads. Δ -connected balanced load is absorbing $12966W$ active and $4332VAR$ reactive power. Calculate Y -connected load.



Examples

Total Power delivered to loads

$$N = N_Y + N_{\Delta} = 25992$$

hence

$$N_Y = 12996 - j43432.$$

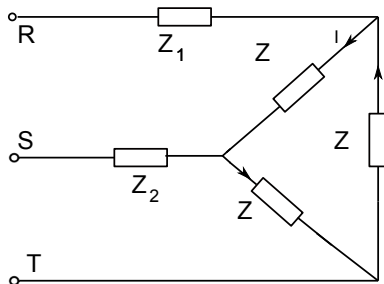
$$V_R = \frac{1}{\sqrt{3}} V_{RS} e^{-j30} \text{ then } V_R = 220\sqrt{2} e^{-j30}.$$

$$N_Y = 3V_{L_{rms}}^2 \bar{Y}_Y$$

using $V_{L_{rms}} = 220$ we obtaine

$$Y_Y = 0.0895 + j0.02982$$

Examples



In a balanced 3-phase system, $Z = 4 + j2$, $Z_1 = \frac{2}{3}Z$ and $Z_2 = \frac{1}{6}Z$ are given. For $V_{RS} = V_{ST} = 20 + j10$, calculate the currents I_R , I_S , I_T .

Examples

Use Δ to Y conversion, then obtaine the empedance connected to each line : $2Z/3 + Z/3$, $Z/6 + Z/3$ and $Z/3$. Then KVL:

$$(Z)I_R - I_S Z/2 = V_{RS}$$

and

$$Z/2 I_R - I_T Z/3 = V_{ST}$$

then obtaine $I_T = -10$ and $I_R = 20/3$ (2 Equ.s and 3 unknown !!).