Circuit and System Analysis
EEF 232E

Prof. Dr. Müştak E. Yalçın

Istanbul Technical University
Faculty of Electrical and Electronic Engineering

mustak.yalcin@itu.edu.tr

Outline I

Transfer function

$H(jw) \in \mathbb{C}$

$$H(jw) = |H(jw)|e^{j\angle H(jw)}$$

$U(jw) \in \mathbb{C}$

$$U(jw) = |U(jw)|e^{j\angle U(jw)}$$

$$\begin{aligned} Y(jw) = H(jw)U(jw) &= |H(jw)|e^{j\angle H(jw)}|U(jw)|e^{j\angle U(jw)} \\ &= |H(jw)||U|e^{j(\angle H(jw)+\angle U_s)} \\ y(t) &= \underbrace{|H(jw)||U|}_{|Y(jw)|}\cos(wt + \underbrace{\angle H(jw) + \angle U}_{\angle Y(jw)}) \end{aligned}$$

Transfer function

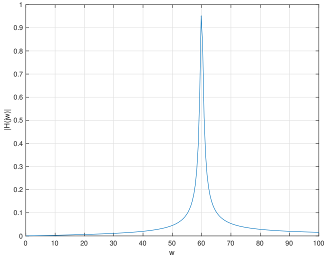
$$H(jw) = \frac{jw/Q}{(jw)^2 + jw/Q + {w_0}^2}$$

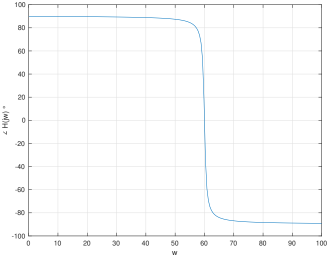
$>>$Q=1;w$_0 = 60$;%*Matlabcode*

$>>$w = linspace(0.1,100,200);

$>>$H=(j*w./Q)./((j*w).$^2 + j * w./Q + w_0^2$);

$>>$plot(w,abs(H));plot(w,unwrap(angle(H))*180/pi);





$w = 60; |H| = 1; \angle = 0^\circ$ and $w = 40; |H| = 0.02; \angle = 88.8542^\circ$

Kirchhoff's Laws in the Frequence Domain: KVL

Lets assuming that $v_1, v_2 ... v_{n_e}$,represent voltages aroud a closed path in a circuit.

KVL requires that

$$\sum_{k=1}^{n_e} v_k(t) = 0$$

We assume that the circuit is operating in a sinusoidal steady state therefore

$$\sum_{k=1}^{n_e} \mathfrak{Re}\left\{ V_k e^{jwt} \right\} = 0$$

Factoring the term e^{jwt} from each term yields

$$\sum_{k=1}^{n_e} V_k = 0.$$

Kirchhoff's Laws in the Frequence Domain: KCL

A similar derivation applies to a set of sinusoidal currents (KCL). Thus if

$$\sum_{k=1}^{n_e} i_k(t) = 0$$

We assume that the circuit is operating in a sinusoidal steady state therefore

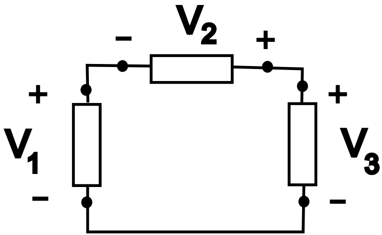
$$\sum_{k=1}^{n_e} \mathfrak{Re}\{I_k e^{\theta_k)}\} = 0$$

Factoring the term e^{jwt} from each term yields

$$\sum_{k=1}^{n_e} I_k = 0.$$

Question 1: Four branches terminates at a common node. The reference direction of each branch current (i_1, i_2, i_3, i_4, is toward the node if
$i_1 = 100\cos(wt + 25^\circ)$A $i_2 = 100\cos(wt + 145^\circ)$A
$i_3 = 100\cos(wt - 95^\circ)$A, find i_4

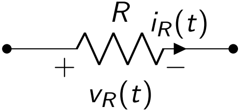
Question 2:

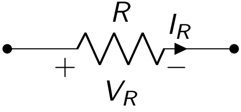


$v_1(t) = 100\cos(wt + 30^\circ)$V $v_2(t) = 100\sin(wt + 45^\circ)$V, find $v_3(t)$

The Passive Circuit Elements in the Frequency Domain

Resistors





From Ohm's law, if the current in a resistor varies sinusoidally with time, the voltage at the terminals of the resistor

$$v(t) = R\,\mathfrak{Re}\{I_R e^{jwt}\} = \mathfrak{Re}\{R I_R e^{jwt}\}$$

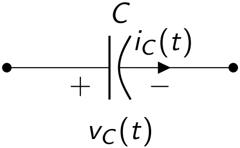
$$\mathrm{Re}\{V_R e^{jwt}\} = \mathfrak{Re}\{R I_R e^{jwt}\}$$

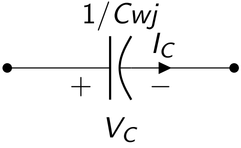
from the properties of phasor

$$V_R = R I_R \text{ or } I_G = G V_G$$

There is no phase shift between the current and voltage of resistor. The signals of voltage and current are said to be in phase.

Capacitor





Substituting the phasor representation of the current and phasor voltage at the terminals of a capacitor into $i = C\frac{dv}{dt}$

$$\mathfrak{Re}\{I_C e^{jwt}\} = C\frac{d\mathfrak{Re}\{V_C e^{jwt}\}}{dt}$$

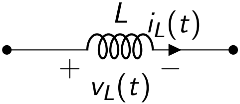
using the properties of phasor

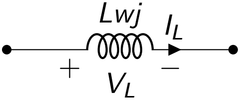
$$\mathfrak{Re}\{I_C e^{jwt}\} = \mathfrak{Re}\{CV_C\frac{de^{jwt}}{dt}\} = \mathfrak{Re}\{CjwV_C e^{jwt}\}$$

we get

$$I_C = jwCV_C$$

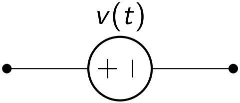
Inductor

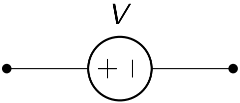




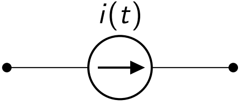
$$V_L = jwLI_L$$

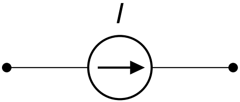
Independent voltage sources





Independent current sources





$$I = I_m e^{j\theta} \mathrm{and V} = \mathrm{V_m e^{j\theta}}$$

$$I_C = jwCV_C = wCV_Ce^{j\frac{\pi}{2}}$$

$\angle I_C = \angle V_C + \frac{\pi}{2}$
The current leads the voltage across the terminals of a capacitor by 90°.

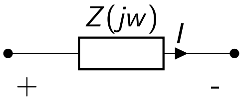
$$V_L = jwL = wLI_Le^{j\frac{\pi}{2}}$$

$\angle V_L = \angle I_L + \frac{\pi}{2}$
The current lags the voltage by 90°.

The Concept of Impedance and Admittance

The driving-point impedance of the one-port at the frequency w to be the ratio of the port-voltage phasor V and the input-current phasor I that is,

$$Z(jw) = \frac{V}{I}.$$



Z represents the impedance of the circuit element

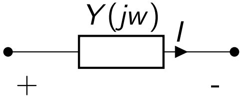
$$Z = \frac{V}{I} = R + jX$$

R, is called resistance and X, is called reactance.
Note

$$Z = \frac{V}{I} = \frac{1}{Y}$$

The Concept of Impedance and Admittance

$$Y(jw) = \frac{I}{V}.$$

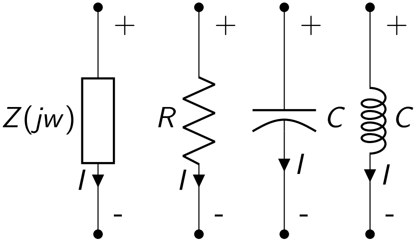


Y represents the admittance of the circuit element

$$Y = \frac{V}{I} = G + jB$$

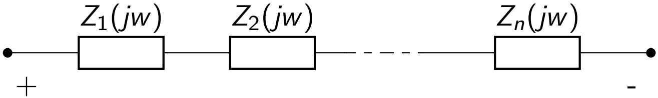
G, is called conductance and B, is called susceptance.

Element	Impedance	Reactance	Admintance	Susceptance
Resistor	R	-	G	-
Capacitor	-j/wC	-1/wC	jwC	wC
Inductor	jwL	wL	-j/wL	-1/wL



Lets $Z = |Z|e^{j\theta}$ if $\theta < 0$, this impedance is Capacitive if $\theta > 0$ this impedance is Inductive !

Combining Impedance in Series and Parallel



Impedances in series can be combined into a single impedance by simply adding the individual impedances.

$$Z = Z_1 + Z_2 + ... + Z_n$$

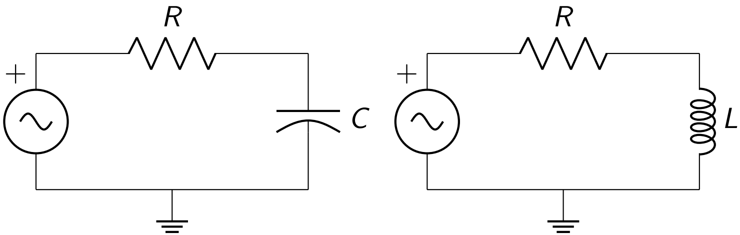
when they in parallel

$$Z = \left\{ \frac{1}{Z_1} + \frac{1}{Z_2} + ... + \frac{1}{Z_n} \right\}^{-1}$$

admittances in parallel:

$$Y = Y_1 + Y_2 + ... + Y_n$$

RC and RL Circuits



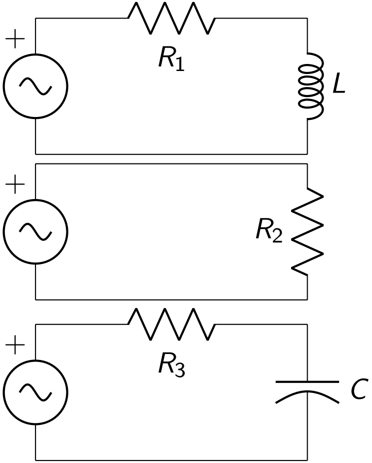
$$I_c = \frac{V_s}{1 + CRjw}, \qquad I_L = \frac{V_s}{R + Ljw}$$

See: Circuits>A/C Circuits>Caps of Various Capacitances; Also :

Circuits>A/C Circuits>Caps w/ of Various Frequencies; Circuits>A/C Circuits>Inductors of Various Inductances;

Circuits>A/C Circuits>Inductors w/ of Various Frequencies

$R_1 = 100\Omega; L = 344.6mH; R_2 = 200\Omega; R_3 = 100\Omega; C = 11.5\mu F; f = 80Hz$



$$Z_1 = R_1 + Ljw = 200e^{j60^\circ}\Omega$$

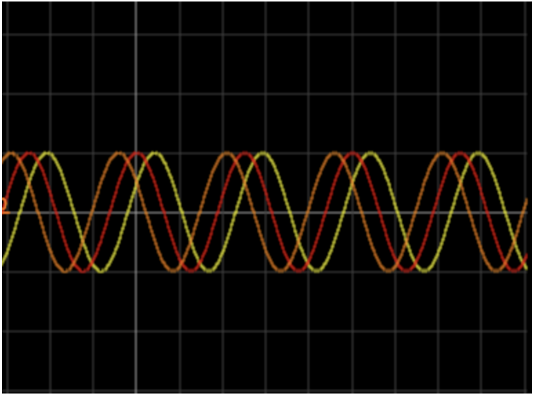
$$Z_2 = R_2 = 200\Omega$$

$$Z_3 = R_3 + \frac{1}{Cjw} = 200e^{-j60^\circ}\Omega$$

$$|Z_1| = 200, \angle Z_1 = 60^\circ,\ |Z_2| = 200, \angle Z_2 = 0^\circ,\ |Z_3| = 200, \angle Z_3 = -60^\circ,$$

Current:

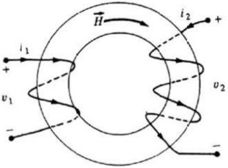
$$I = \frac{V_s}{Z}$$

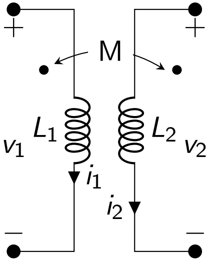


$|Z_1| = 200, \angle Z_1 = 60^\circ,\ |Z_2| = 200, \angle Z_2 = 0^\circ,\ |Z_3| = 200, \angle Z_3 = -60^\circ,$

Circuits>A/C Circuits>Caps of Various Capacitances;

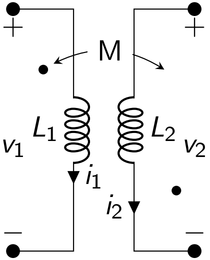
Mutual Inductance





$$\phi_1 = L_1 i_1 + M i_2$$
$$\phi_2 = L_2 i_2 + M i_1$$

$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} - - > V_1 = L_1 jwI_1 + MjwI_2$$
$$v_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} - - > V_2 = L_2 jwI_2 + MjwI_1$$



$$\phi_1 = L_1 i_1 - M i_2$$
$$\phi_2 = L_2 i_2 - M i_1$$

and

$$V_1 = L_1 jwI_1 - MjwI_2$$
$$V_2 = L_2 jwI_2 - MjwI_1$$

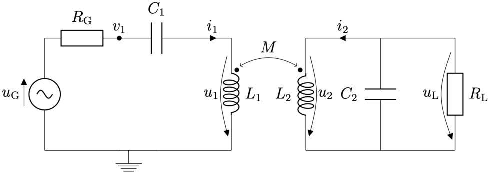
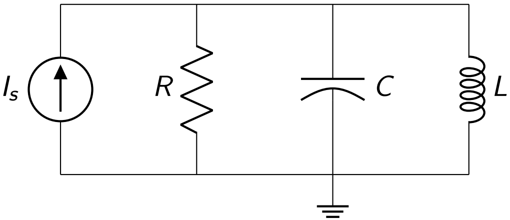


Figure 2.1: Circuit diagram with capacitor C_1 in series with the coil on the primary side. A voltage u_G is applied to the primary circuit on the left, inducing a voltage u_L in the secondary circuit on the right.

Parallel Resonance Circuit

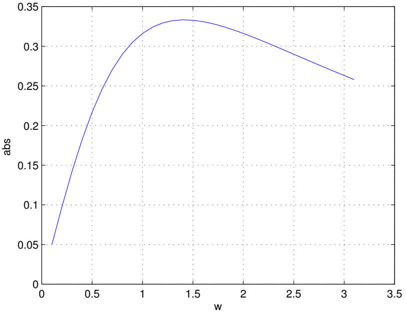
Find the solution for $R = 1/3\Omega$, $C = 1F$, $L = 1/2H$, $i(t) = \cos(wt)$ in SSA.



$$Y_{eq}(j\omega) = \frac{1}{j\omega L} + \frac{1}{R} + j\omega C = \frac{R + j\omega L - \omega^2 RLC}{j\omega RL}$$

$$V_C = Z_{eq}(j\omega) \cdot I_s = \frac{I_s}{Y_{eq}(j\omega)} = \frac{I_s}{Y_{eq}(j\omega)} = \frac{j\omega RL}{R - \omega^2 RLC + j\omega L} I_s$$

Magnitude of $V_C(j\omega)$ is maximun when $\omega = \frac{1}{\sqrt{LC}}$!



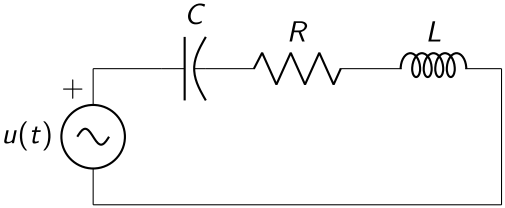
$$Z(j\frac{1}{\sqrt{LC}}) = R$$

Resonance

Resonance occurs at a particular resonance frequency when the imaginary parts of impedances or admittances of circuit elements cancel each other.

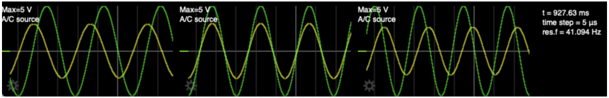
$\omega = \frac{1}{\sqrt{LC}}$ is the resonance frequency.

Serial Resonance Circuit



$$Z = \left(\frac{1}{Cjw} + R + Ljw\right) = \frac{RCw + j(LCw^2 - 1)}{Cw}$$

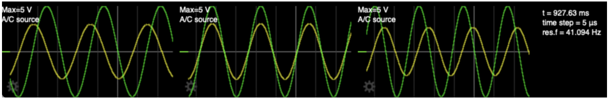
if $w = \frac{1}{\sqrt{(LC)}}$ then $Z = R$.
$R = 10, L = 1H$ and $C = 15\mu F$ then $w_{resonans} = 41Hz$

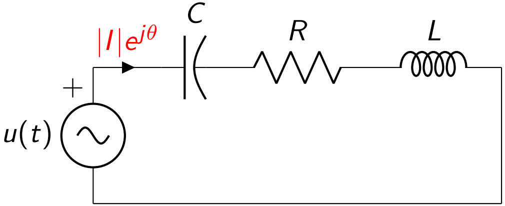


v_u and i_u for @$35Hz$, @$41Hz$ and @$45Hz$

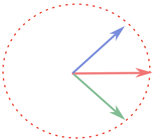
▸ Circuits>A/C Circuits>Series Resonance

Serial Resonance Circuit





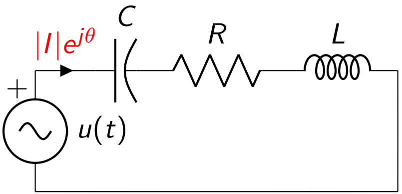
$$I = \frac{U}{Z} = \frac{|U|}{|Z|e^{j\theta}} = \frac{|U|}{|Z|}e^{-j\theta}$$

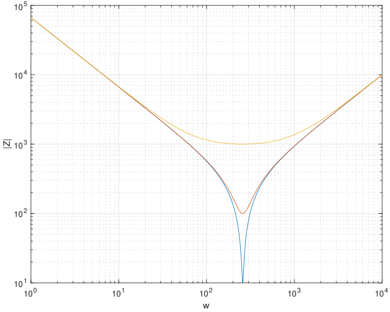


Serial Resonance Circuit

$$V_L = \frac{Ljw_0}{Z(w_0)}U = \frac{Lw_0}{R}jU = QjU, \quad V_C = \frac{1}{Cjw_0}\frac{1}{Z(w_0)}U = -QjU$$

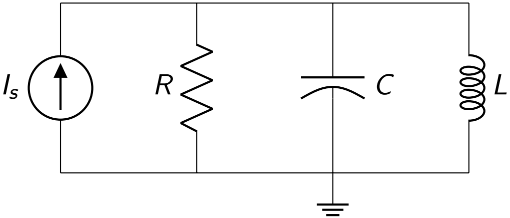
w_0 resonance frequency and Q factor at resonance $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$





$R = 10\Omega, 100\Omega, 1k\Omega$

Parallel Resonance Circuit



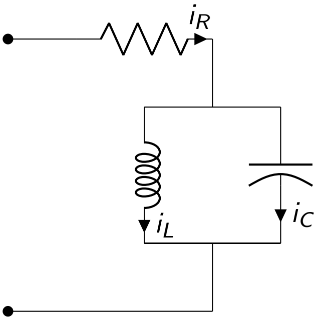
At w_0

$$I_C(jw_0) = I_s \frac{Cjw_0}{Y(jw_0)} = I_s \frac{Cjw_0}{G} = jI_s Q_p$$

$$I_L(jw_0) = I_s \frac{1}{Y(jw_0)} \frac{1}{Ljw_0} = I_s \frac{R}{Ljw_0} = -jI_s Q_p$$

Q_p factor at resonance

Example



Find the impedance Z between the terminals:

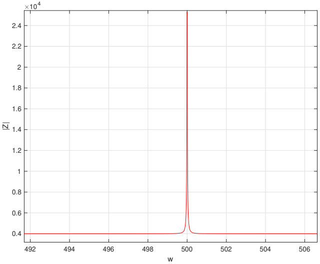
$$Z = R + \{Cjw + \frac{1}{Ljw}\}^{-1} = R + \frac{Ljw}{1 - LCw^2}$$

$$Z = \frac{R - RLCw^2 + Ljw}{1 - LCw^2}$$

Example

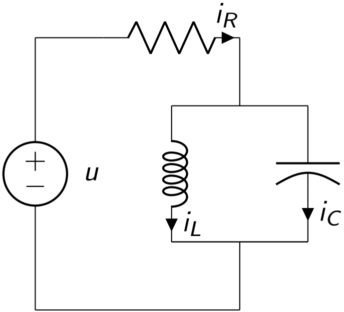
$$Z = \frac{R - RLCw^2 + Ljw}{1 - LCw^2}$$

For $R = 4k\Omega$, $C = 2mF$ ve $L = 2mH$



$w = \frac{1}{\sqrt{LC}} = 500$ and then $Z = \frac{***}{0}$ (open-circuit)!

Example



State equation:

$$\frac{d}{dt}\begin{bmatrix} V_{C1} \\ i_L \end{bmatrix} = \begin{bmatrix} -\frac{1}{RC} & -\frac{1}{C} \\ \frac{1}{L} & 0 \end{bmatrix}\begin{bmatrix} V_{C1} \\ i_L \end{bmatrix} + \begin{bmatrix} \frac{1}{RC} \\ 0 \end{bmatrix} u$$

$$v_R = \begin{bmatrix} -1 & 0 \end{bmatrix}\begin{bmatrix} v_{C1} \\ i_L \end{bmatrix} + u$$

Using the state-equation

$$V_C = [1 \;\; 0] \begin{bmatrix} jw + \frac{1}{RC} & \frac{1}{C} \\ -\frac{1}{L} & jw \end{bmatrix}^{-1} \begin{bmatrix} \frac{1}{RC} \\ 0 \end{bmatrix} e$$

$$= \frac{\frac{jw}{RC}}{\frac{1}{LC} - w^2 + \frac{jw}{RC}} U$$

$$V_R = U - V_C = \left\{ 1 - \frac{\frac{jw}{RC}}{\frac{1}{LC} - w^2 + \frac{jw}{RC}} \right\} U$$

$$= \left\{ \frac{\frac{1}{LC} - w^2}{\frac{1}{LC} - w^2 + \frac{jw}{RC}} \right\} U$$

$$= \frac{R(1 - LCw^2)}{Ljw + R(1 - LCw^2)} U$$

Example

Using the The Concept of Impedance, lets find the V_R

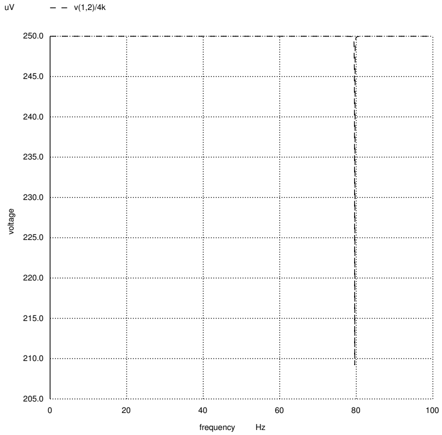
$$V_R = R\frac{U}{Z}$$
$$= R\frac{(1 - LCw^2)}{Ljw + R(1 - LCw^2)}U$$

What happen when $w = \frac{1}{\sqrt{LC}}$?

Example

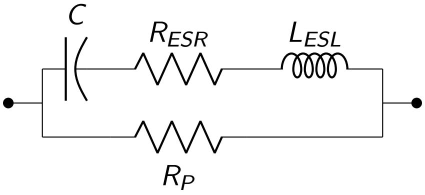
```
Sekil 7.2
v1 1 0 sin(0 .1 10) dc 0 ac 1
r 1 2 4k
l 2 0 2m
c 2 0 2m
.control
ac lin 1000 .1 100
plot v(1,2)/4k
.endc
.end
```

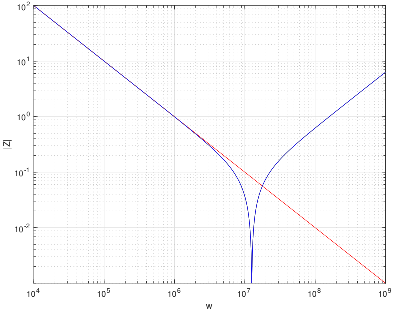
Example



▸ YouTube Video: Example with two sources

Real Life Problem ! The non-ideal behavior of a capacitor is due to imperfections within the capacitor's material that create resistance causing the capacitor to dissipate energy.





PS:red is ideal...

A producer Web Page (Ceramic Cap (SMD))

$$Z = R_P // (\frac{1}{Cjw} + R_{ESR} + L_{ESL} jw)$$

$C = 10\mu F$, $R_{ESR} = 1m\Omega$, $L_{ESL} = 6.33nL$, $R_P = 100M\Omega$.