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Force estimation using vibration data

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ABSTRACT

The use of indirect methods for the estimation of dynamic forces acting on mechanical systems is very attractive as direct measurement of these forces is usually very difficult and sometimes impossible. This paper addresses such an indirect force determination method which is based on utilizing the Frequency Response Functions (FRF) of a system and the measured vibration levels under operating conditions. This procedure requires a generalized inverse of the FRF matrix and the spectrum of the measured response levels. The applicability of the force estimation method is examined using both numerical and experimental case studies. The method is tested using different structures with different types of excitation forces. It is shown that careful inspection of the numerical properties of the data is essential if the unique forces acting on a system are to be determined.

1 INTRODUCTION

The knowledge of the dynamic forces is an essential requirement during the design and optimization stages of mechanical systems. However, the detailed information about these forces is usually not available as the direct measurement of these forces under running conditions is either very difficult or not feasible at all. This makes alternative methods of force estimation necessary and very valuable. Cutting forces of machine tools, supporting forces of bearings, aerodynamic forces on various machine components and excitation forces on rotating machines are some examples to illustrate the need for alternative methods to identify the dynamic forces.

Force estimation using vibration data has attracted a lot of attention from researchers. Hillary and Ewins [1] investigated the problem of sinusoidal load identification of a cantilever beam and determination of impact forces on aircraft engine turbine blades using a least-squares method. Stevens [2] has presented an overview of the force identification process for the case of linear vibration systems. Dascotte and Desanghere [3] described a methodology that allows optimizing a given set of force vectors in order to minimize the difference between the measured and calculated response levels. The forces are obtained using an iterative least-squares or Bayesian force estimation procedure. O'Callahan and Piergentili [4] tried to find the excitation forces by using frequency response functions and operational responses. Regardless of the number of assumed force input locations chosen, the forces were determined exactly if the assumed force input locations were actual force points. Steltzner and Kammer [5] presented a new method for the estimation of structural input forces. This technique uses an Inverse Structural Filter (ISF) that takes as input, the structural

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response data, and returns, as output, an estimate of the input forces. Tao, Liu and Lam [6] presented a new method to identify the engine excitation forces at the c.g. of an engine from the measured velocities at the mounting points. In the study of Guillaume et. al. [7], an iterative weighted pseudoinverse algorithm was presented to estimate the forces that are acting on a structure starting from the measured response spectra and the estimated modal model. Ma, Chang and Lin [8] used an on-line recursive inverse method, which is based on the Kalman Filter and a recursive least-squares algorithm to estimate the input forces of beam structures. Ma and Ho [9] proposed an inverse method to identify input forces of non-linear structural systems in their study, which is an extension of their previous work that was limited to linear structural systems. Wang [10] made an overview of force determination methods and presented a general approach in developing the force prediction model for impact and harmonic forces.

FRF Based Direct Force Estimation Method is used in this study. In this method, numerically calculated or experimentally measured FRFs are used with the system's responses to predict the characteristics of the unknown excitation forces. Both numerical and experimental case studies are carried out to assess the applicability of the method to real world problems.

For the numerical studies, first a structure is modeled using a Finite Element program. Then, certain number of co-ordinates (locations) of the structure is assumed to be "measurement" locations and the full FRF matrix corresponding to those co-ordinates are formed. After that, a dynamic force is applied to and the structural responses are calculated at all selected co-ordinates. By using Fourier Transform, the results in time-domain are transformed into frequency-domain. Then, the FRF matrix is used with the numerical response levels in order to estimate the properties of the dynamic force. The numerical results are then compared with the known forces in order to evaluate the accuracy and the reliability of the force determination method.

For the experimental studies, the elements of the Frequency Response Function matrix and the vector of response levels are measured using a real structure. Modal shakers or modal hammers are used to excite the structure while piezoelectric accelerometers are utilized to measure the system response. Then, the measured data are used directly during the force determination process. Finally, some concluding remarks are given about the accuracy and reliability of the force determination method.

2 THEORY

For a multi degree of freedom system with N degrees of freedom, the governing equations of motions can be written in matrix form as:

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=\{f(t)\} \tag{1}$$

where [M], [C] and [K] are *NxN* mass, damping and stiffness matrices respectively, and *{x(t)}* and *{f(t)}* are *Nx1* vectors of time varying displacements and forces [11]. If the structure is excited sinusoidally by a set of forces all at the same frequency, ω, but with individual amplitudes and phases, then

$$\{f(t)\}=\{F\}e^{i\omega t} \tag{2}$$

and this will lead to a solution of the form:

$$\{x(t)\}=\{X\}e^{i\omega t} \tag{3}$$

where *{F}* and *{X}* are *Nx1* vectors of time independent complex amplitudes. The equation of motion then becomes

$$\left([K]-\omega^2[M]+i\omega[C]\right)\{X\}e^{i\omega t}=\{F\}e^{i\omega t} \tag{4}$$

Rearranging to solve for the unknown responses leads to

$$\{X\}=\left([K]-\omega^2[M]+i\omega[C]\right)^{-1}\{F\} \tag{5}$$

which may be written as

$$\{X\}=[Z]^{-1}\{F\} \tag{6a}$$

where

$$[Z]=\left([K]-\omega^2[M]+i\omega[C]\right) \tag{6b}$$

[Z] is the so-called dynamic stiffness matrix, which can be computed if the stiffness, damping and mass matrices of a structure are known. However, the inverse of the dynamic stiffness matrix is equal to the FRF matrix of a structure and it is possible to measure the elements of this matrix experimentally. If the FRF matrix instead of the inverse of the dynamic stiffness matrix is inserted into Eq. (6a), this gives:

$$\{X(\omega)\}=[H(\omega)]\{F(\omega)\} \tag{7a}$$

where

$$[H(\omega)]=[Z]^{-1}=[K]-\omega^2[M]+i\omega[C] \tag{7b}$$

and, *{F(ω)}* represents the force vector, *{X(ω)}* represents the vibration vector and *[H(ω)]* represents the Frequency Response Function (FRF) matrix. According to this equation, the multiplication of the FRF matrix with the vector of excitation forces yields the response of these forces on a structure. It should be noted that the FRF matrix, the force and the response vectors are all functions of frequency ω. However, the symbol ω is dropped in the rest of the equations for brevity. Eq. (7a) can be written more explicitly as:

$$\begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ \vdots \\ X_n \end{Bmatrix}=\begin{bmatrix} H_{11} & H_{12} & H_{13} & \cdots & H_{1n} \\ H_{21} & H_{22} & \vdots & \cdots & \vdots \\ H_{31} & \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ H_{n1} & H_{n2} & H_{n3} & \cdots & H_{nn} \end{bmatrix}\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ \vdots \\ F_n \end{Bmatrix} \tag{8}$$

By taking the inverse of *[H(ω)]* in the Eq.(8), and multiplying both sides of this equation with this inverse ($[H(\omega)]^{-1}$) will form the foundation of the so-called FRF Based Direct Force Estimation Method. The new equation allows the excitation forces to be determined by using the FRF matrix and vibration response levels as:

$$\{F(\omega)\}=[H(\omega)]^{-1}\{X(\omega)\} \tag{9}$$

In practice, the most difficult part of the "Frequency Response Function Based Force Estimation" is creating the [H] matrix and taking its inverse with acceptable accuracy and reliability. Because FRF matrix represents the dynamic properties of a structure it is crucial to get frequency response functions measured or calculated with high quality. Furthermore, various numerical aspects of the matrix inversion can affect the quality of the results directly. It is therefore very essential to resort to Singular Value Decomposition (SVD) technique to get the most out of the available data for force determination. It should also be noted that number of response levels to be measured (m) and the number of forces to be determined (n) are crucial parameters during the solution of this inverse problem. It is obvious that there must be at least as many response levels measured as there are forces to be determined, i.e., m>n.

3 NUMERICAL SIMULATIONS

In order to investigate the practical aspects and the accuracy of the present approach in estimating the unknown input forces, numerical case studies are performed first, including a case study using a plate structure. The finite element model of the plate and possible excitation points on this plate are shown in Figure 1.

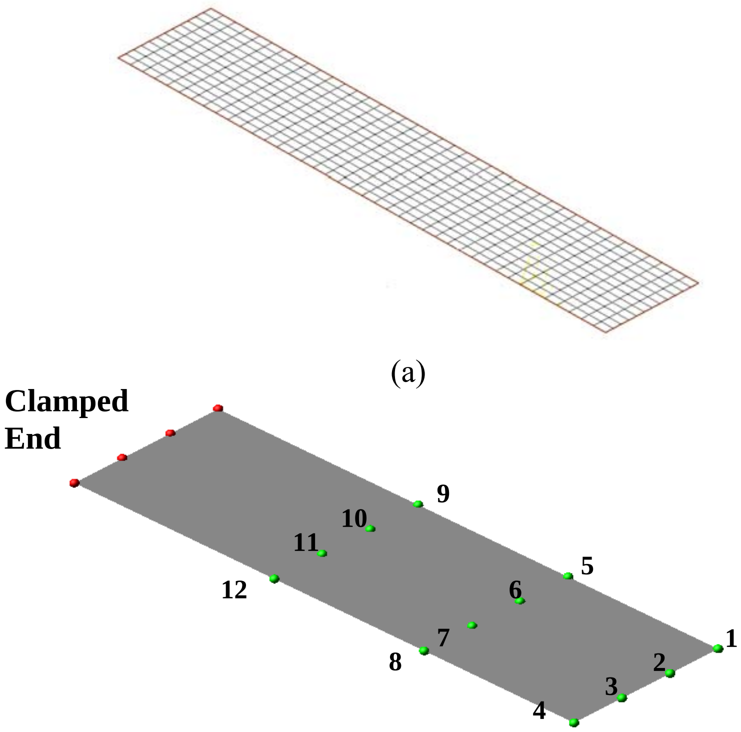


Figure 1: Finite element model of the plate and possible excitation points

In this test case, a periodic force is applied to a point in the numerical model of the plate after it was clamped at one end. Various other types of input forces (i.e., sinusoidal, periodic, impact, chirp) were also examined in the numerical simulations [12]. However only the results corresponding to the periodic force type are included here as the performance of the method for other excitation types was quite similar.

As mentioned, the periodic excitation is applied to one co-ordinate only. However, it is assumed during force determination process that there are 12 possible coordinates where forces can be applied to. In other words, the force estimation method is tested whether it can find that there is actually a non-zero periodic forcing acting on one co-ordinate while the excitations at other co-ordinates are all zero. The spectrums of the predicted forces at 12 coordinates are shown in Figure 2.

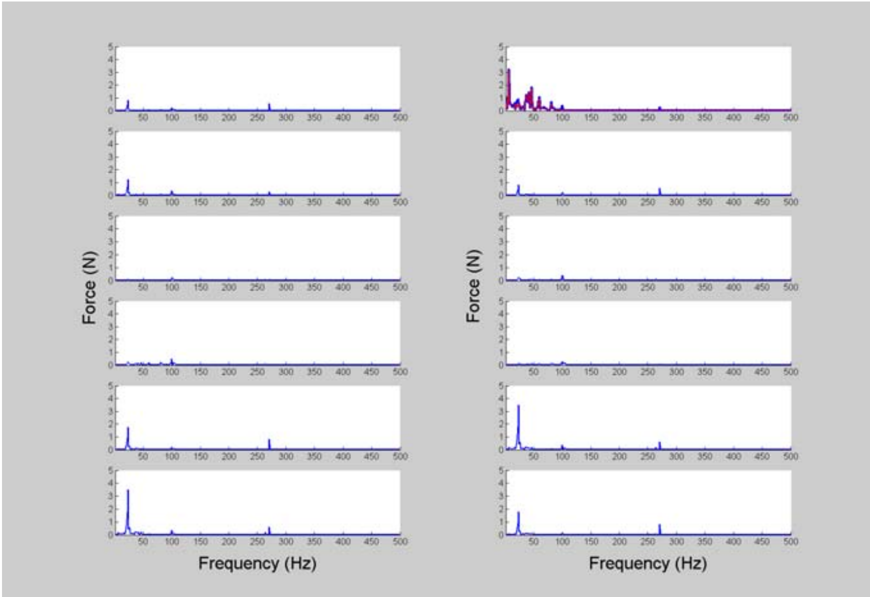


Figure 2: Predicted force spectrums at various co-ordinates,of the plate model. (Actual excitation is at only one co-ordinate)

At the coordinate where the force was actually applied at, predicted force is equal to the real force component. A detailed comparison of the predicted and the actual forces is given in Figure 3 between 0-200 Hz range. It is seen that the predicted and the actual forces are almost the same although there is a bit of discrepancy around 100 Hz.

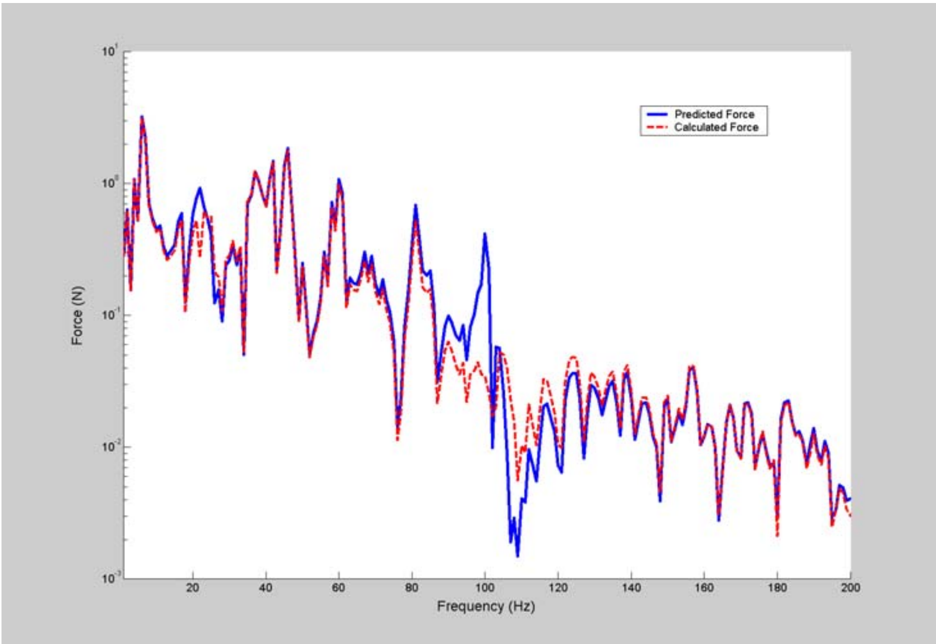


Figure 3: Comparison of the predicted and actual forces

It is worth restating that the actual force is applied from only one location (node 2) However, predicted results in Figure 2 indicate as if there are forces acting at other co-ordinates as well, particularly around 30 Hz. This error has been found to be due to a numerical problem which can be explained by examining the condition number of the *[H(ω)]* matrix as a function of frequency. It is well-known that condition number is the ratio of the largest to smallest singular values of a matrix. A matrix with a low condition number is said to be well-conditioned, while a matrix with a high condition number is said to be ill-conditioned. When the $[H(\omega)]$ matrix is ill-conditioned, the condition number is high and at these frequencies, the results of the force estimation approach tend to be unreliable. The condition number of the FRF matrix for the plate example is plotted as a function of frequency in Figure 4 and this explains the fictitious forces shown in Figure 2 (especially around 30 Hz).

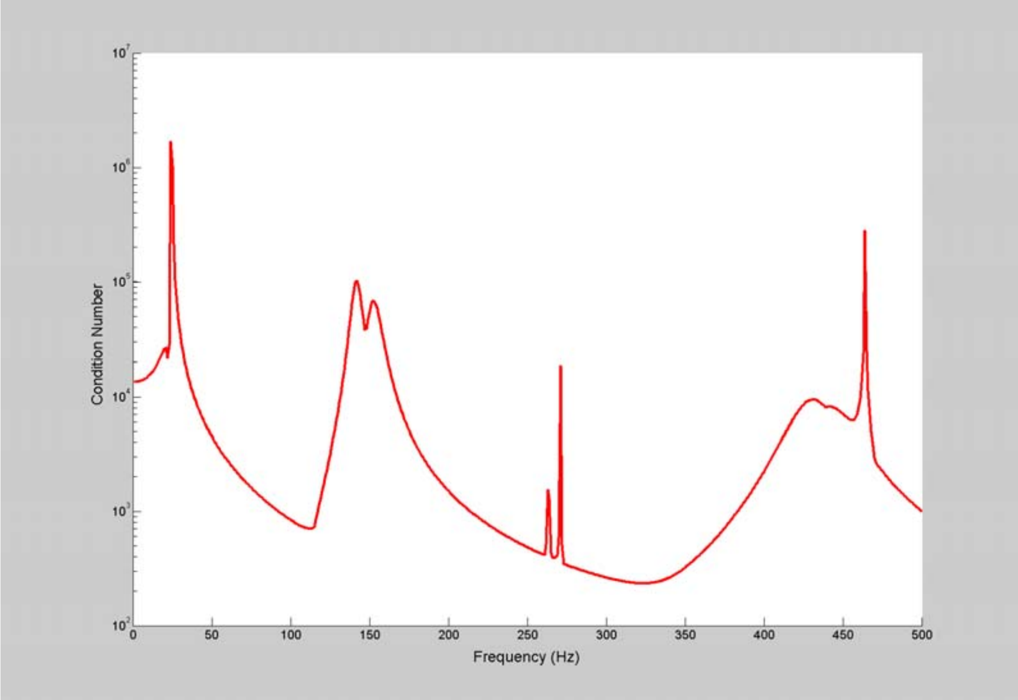


Figure 4: Condition number for the FRF matrix of the plate

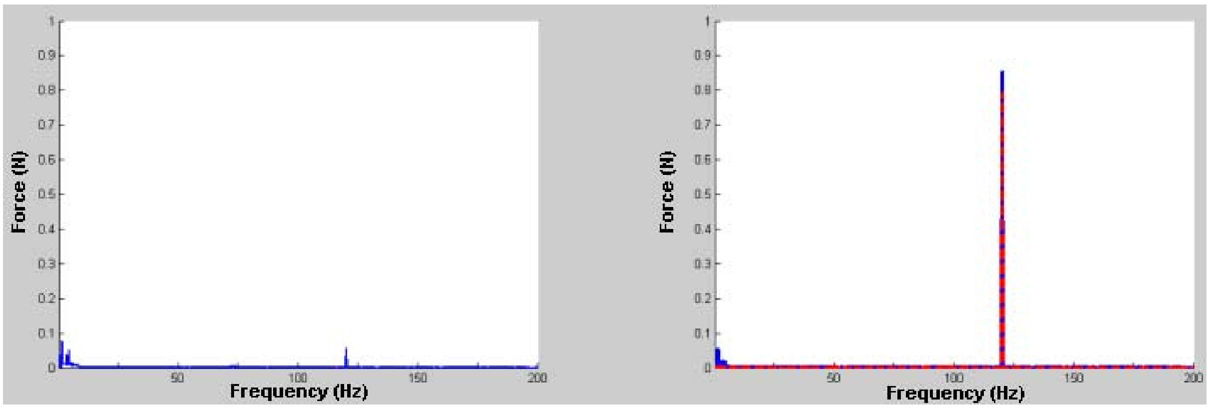
4 EXPERIMENTAL STUDY

In this section, experimental case studies are presented to assess the applicability of the FRF based force estimation method in practice. In this part of the study, instead of the computed FRFs, real FRFs directly measured on a structure are used: the structure is excited with a force function and the resulting vibrations are measured. The input forces and the output vibrations are measured using force gauges and accelerometers, respectively. The measured excitation forces are then assumed to be unknown as they are to be determined by using the measured FRFs and the measured vibrations.

In this experimental case study, an L-shaped plate is used as the test structure. After measuring the required FRF matrix, force determination method is applied when the excitation was either sinusoidal or impact type.

In the sinusoidal excitation case, six response measurement locations are chosen on the structure. Then two columns of the FRF matrix are measured, i.e., 6x2 FRF matrices are obtained. Then, a sinusoidal force signal at 120 Hz is applied from one of the co-ordinates of the structure by using a modal shaker. Simultaneously, the responses caused by that excitation, are measured at all six locations. It is crucial that boundary conditions should be the same for the structure during FRF and responses measurements. Both measurements were done while the plate was standing on the floor in this case study. Finally, the location and the amplitude of the excitation force are assumed to be unknown and they were tried to be determined by using the force estimation method. Six measurements are used to estimate the forces at two different locations, noting that the actual force was applied from one location only.

The spectrums of the predicted forces at assumed two locations are shown in Figure 5. The red plot represents the estimated force while the blue one represents the actual force component. As can be seen from these results, the force in Figure 5b is predicted correctly. As there was no other excitation from any other location, the method should predict zero force at the other location. Inspection of the spectrum in Figure 5a indicates that the predicted force that must be zero is in fact quite small compared to the actual force in Figure 5b. Therefore, the force prediction method can be considered to be fairly successful in this situation.



a) b)

Figure 5: Comparison of predicted (blue) and the actual forces (red) on L-shaped plate

The purpose of another experimental test case was to evaluate the performance of the force prediction method when the forcing was impact type of excitation. In this case, 3 columns of FRF matrix of the L-plate are measured using an instrumented impact hammer in free free condition. The size of the measured FRF matrix was 6x3. The measurement setup is illustrated in Figure 6.

Figure 6: Measurement setup for experimental force estimation study of impact excitation

After the FRF matrix was ready, the structure was excited by an impact force using modal hammer again from one of its co-ordinates. The excitation and the response levels were measured simultaneously. It should be noted that this time the actual force was acting at one location only, but it was assumed to predict the forces at 3 locations. Therefore, the force estimation formula becomes as follows:

$$\{F(\omega)\}_{3x1} = [H(\omega)]^{-1}{}_{3x6}\{X(\omega)\}_{6x1} \tag{10}$$

As before, spectrums of the actual and the predicted forces are compared in Figure 7. It is seen once again that spectrum of the predicted force at the location where there was actual force agrees quite well with the reality. However, the method also predicts some forcing at other locations where there was not any actual force there. This is probably a clear indication that determination of the dynamic forces uniquely is still a quite difficult problem if the number of forces to be determined is not obvious.

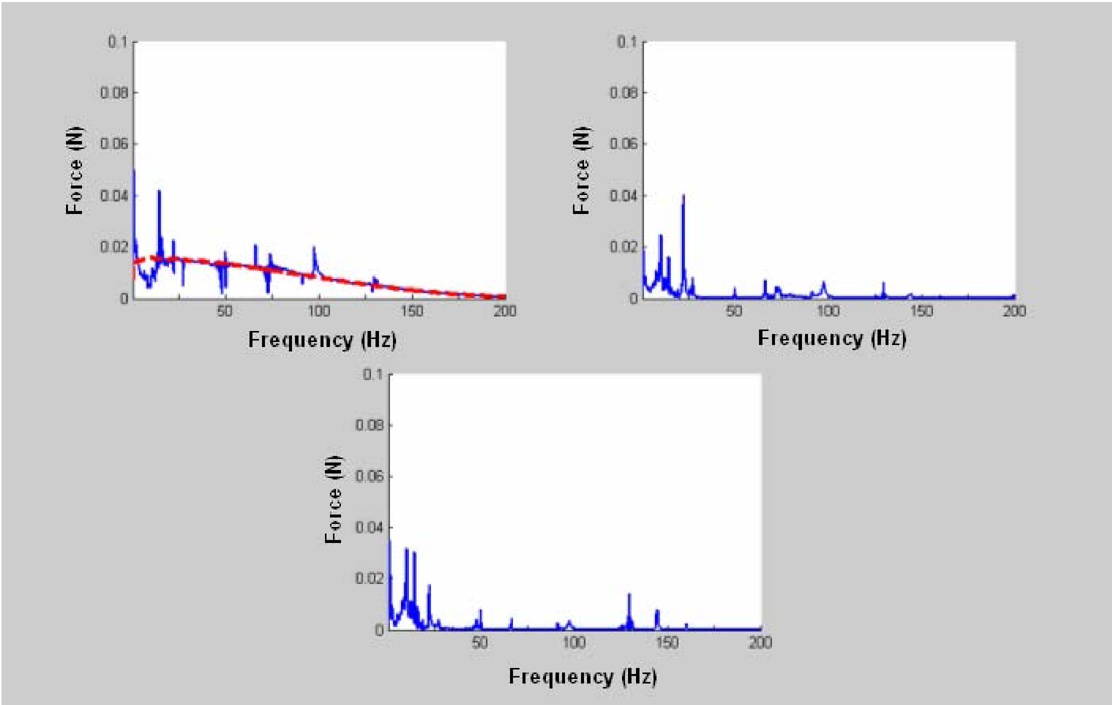


Figure 7: Comparison of predicted (blue) and the actual force spectrums (red) on L-shaped plate

5 CONCLUDING REMARKS

The accuracy and the applicability of the force estimation method is examined using both numerical and experimental case studies. The main limitation of the method studied here seems to be the ill-conditioning property of the FRF matrix during its inversion. It has been shown that careful inspection of the numerical properties of the data is essential for correct interpretation of the predicted forces. It has also been found that determination of the dynamic forces uniquely is still a difficult problem especially if the number of forces to be determined is not obvious.

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