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ELIMINATION OF SUSPENSION EFFECTS FROM MEASURED FREQUENCY RESPONSE FUNCTIONS

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ABSTRACT
Modal testing is a very effective tool in order to determine the dynamic behaviour of structures. However, the measured Frequency Response Functions (FRFs) and the models obtained from modal testing often have errors due to factors inherent in the measurement process. One group of the errors is the systematic errors such as transducer mass-loading effects, effects of suspensions and shaker-structure interaction. This paper deals with improving the quality of measured FRFs by removing the suspension's spring effects and presents a numerical application in order to verify the proposed method. It is shown that the adverse suspension effects can be removed from measured FRFs provided that some additional FRFs concerned with the suspension points are also measured.
Keywords: Modal testing, suspension effect, frequency response function.

1. INTRODUCTION
Measured Frequency Response Functions (FRFs) are used for many purposes including system identification, model verification, model updating and structural modification. In many applications, it is very desirable to have measured FRFs with high quality. However, there are some unavoidable experimental error sources originating from the testing equipments and environment. Mass loading effects of transducers, shaker-structure interaction and support effects are mainly mechanical errors which adversely affect measured FRFs. For a successful experimental modal analysis and reliable applications, it is necessary to eliminate these undesirable and unwanted effects from the measured FRFs [1-3].
In this study, the effects of the suspensions and their removal from measured FRFs are investigated. The support of the structure under test is an important part of the test setup. Free-free boundary condition is most frequently employed for laboratory testing of components or structures. For a structure to be perfectly free, it should be suspended in air or free in space without any support whatsoever. Although satisfying this condition perfectly in a laboratory environment is clearly impossible, free-free condition can be simulated by suspending or supporting the structure using very flexible springs. However, the springs used to simulate free-free conditions can adversely interfere with the results. It is known that if the frequency of the rigid body motion is greater than 10-20% of that of the first flexible modes, other flexible modes can be affected by suspension system [3]. For example, it is presented in [4] that the rigid and flexible modes of an airplane are coupled when it is tested on its tyres. There are a few numbers of studies to eliminate this undesirable effect from FRFs. Ashory [5] presented a method for removing the support effects by measuring the same FRFs with different springs. Authors of this paper presented a series of papers on the elimination of transducer mass effects and suspension effects from FRFs [6-8] where the presented methods are based upon the Sherman-Morrison formula. In [8], the theory and a numerical application which includes removing the suspension effects only at one coordinate were given. However, it is usual in modal testing that the structures may need to be suspended from more than one location. Therefore, this paper investigates removing the suspension effects from FRFs when the structure is suspended from more than one

location and assesses the applicability and the success of the proposed method for this situation. This is done by focussing on a numerical case study which includes a plate suspended at three distinct points by using three springs. Also, the performance of the method is investigated by simulating noisy data by adding white noise to FRFs.

2. THEORY

A suspension system used for supporting a mechanical structure has spring and damping effects and modifies the structure mechanically. According to this, the effects of suspension system on dynamics of the structure can be removed simply by negative spring and negative damping modifications. This implies that structural modification techniques can be used in order to remove these adverse effects of the suspension system. The presented method in this study is based on a structural modification technique describe in the formula known as Sherman-Morrison [6-10]. The theory of the method was already described in early studies of the authors; hence a short summary is included here for brevity.

Let [α] be the receptance FRF matrix of any mechanical system and assume any structural modification including mass, spring or damping on the system is expressed as a product of two vectors $\{u\}\{v\}^T$. The receptance matrix of resultant system after modification, [α*], can be written as follows by using Sherman-Morrison formula [6-10]:

$$[\alpha^*] = [\alpha] - \frac{([\alpha]\{u\})(\{v\}^T[\alpha])}{1-\{v\}^T[\alpha]\{u\}} \tag{1}$$

It should be noted that the [α] matrix in Eq.(1) contains all FRFs of the system, but in real applications the full matrix is rarely available. However, Eq.(1) can be written in the more suitable form as [10]:

$$[\alpha^*_{aa}] = [\alpha_{aa}] - \frac{([\alpha_{aa}]\{u_a\})(\{v_a\}^T[\alpha_{aa}])}{1+\{v_a\}^T[\alpha_{aa}]\{u_a\}} \tag{2}$$

where the subscript (a) indicates active coordinates, i.e. excitation, response and modification coordinates.

Let us suppose that the structure is suspended only at one coordinate and consider that p, q and r represent response, excitation and modification coordinates, respectively. If the modification vectors are expressed as $\{u_r\} = \{0\cdots 0\ 1\ 0\cdots 0\}^T$; $\{v_r\} = \{0\cdots 0\ -k\ 0\cdots 0\}$, then any FRF of the modified system can be written as follows [8]:

$$\alpha^*_{pq} = \frac{\alpha^{(r)}_{pq} - k(\alpha^{(r)}_{rr}\alpha^{(r)}_{pq} - \alpha^{(r)}_{pr}\alpha^{(r)}_{rq})}{1-k\alpha^{(r)}_{rr}} . \tag{3}$$

It is seen in Eq.(3) that removing the effects of the suspension spring from $\alpha^{(r)}_{pq}$ requires the availability of additional three FRFs, i.e, $\alpha^{(r)}_{rr}$, $\alpha^{(r)}_{pr}$ and $\alpha^{(r)}_{rq}$ which are related to modification coordinate r.

In practice the structures are mostly tested by using more than one suspension. In that case the Eq.(3) can be applied sequentially in order to remove the effects of suspensions. It should be noted, however, that this process would require measurement of additional FRFs related to individual modification coordinates including the cross FRFs between the suspensions coordinates.

3. NUMERICAL SIMULATION

In order to simulate an experiment, a rectangular plate suspended by three springs as seen in Fig. 1 was considered. Plate dimensions are 0.50x0.21x0.003 m^3 and mechanical properties were arbitrarily chosen as: Young's modulus E=207E10 N/m^2, mass density ρ=7800 kg/m^3 and Poisson rate ν=0.3.

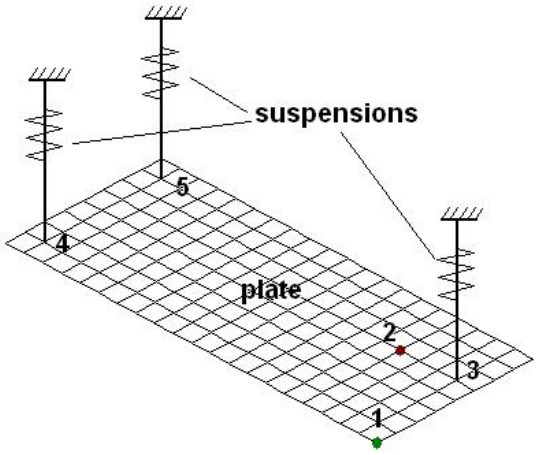

Figure 1. A rectangular plate suspended by three springs

First, the plate with three grounded springs whose spring constants are chosen as 500 N/m, is modelled by Finite Elements and eigenfrequencies and eigenvalues are obtained. Then a transfer FRF $\alpha_{12}^{(3,4,5)}$ is calculated numerically. This FRF simulates the measured FRF affected by three springs at locations 3, 4 and 5, and is named as 'measured' FRF. The aim of this application is to remove the effects of these three springs from the 'measured' FRF $\alpha_{12}^{(3,4,5)}$. For this purpose, the Eq.(3) can be used three times, sequentially. For comparison purposes, the plate is also modelled without any suspension springs so as to establish the 'exact' FRFs. In this context, α_{12} is calculated and named as 'exact' FRF. It should be noted that Eq.(3) needs additional FRFs calculations as mentioned before. For this system the FRFs required can be given in the matrix form as follows:

$$[\alpha]=\begin{bmatrix} X & \alpha_{12}^{3,4,5} & \alpha_{13}^{3,4,5} & \alpha_{14}^{3,4,5} & \alpha_{15}^{3,4,5} \\ & X & \alpha_{23}^{3,4,5} & \alpha_{24}^{3,4,5} & \alpha_{25}^{3,4,5} \\ & & \alpha_{33}^{3,4,5} & \alpha_{34}^{3,4,5} & \alpha_{35}^{3,4,5} \\ & Sym. & & \alpha_{44}^{3,4,5} & \alpha_{45}^{3,4,5} \\ & & & & \alpha_{55}^{3,4,5} \end{bmatrix} \tag{4}$$

where the letter (X) indicates the FRFs which are not needed for the calculations. It can be seen that in addition to the FRF that will be corrected, twelve more FRFs are needed in order to eliminate the effects of the three suspension springs from any measured FRF. The number of needed FRFs for calculations changes according to the number of coordinates involved for the suspension. It is obvious that as the number of suspension coordinates is increased, more FRFs need to be measured.
In Fig. 2, FRFs corresponding to the free-free and suspended systems are presented. As expected, it is seen that the natural frequencies of the plate are shifted upwards due to the suspension springs. Furthermore, due to the flexibility of the springs, plate structure exhibits three rigid body modes with non-zero natural frequencies. After numerically removing springs, the corrected FRF coincides with the exact FRF as can be seen in Fig. 3.
Finally, performance of the proposed method is assessed when the measured data contain noise. This is simulated by adding 1% white noise to the 'measured' FRFs, Fig. 4, and the procedure for removing the suspension effects is repeated. As can be seen in Fig. 5 corrected FRF is again almost identical to exact FRF.

4. CONCLUSION

In modal testing, the suspension systems are often used to simulate free-free boundary conditions. However, it is known that the suspension system can adversely affect the dynamics of the system. This paper deals with the elimination of the suspension's stiffness effects from measured FRFs. It is shown that the proposed method can be effective for removing this undesirable effect from measured

FRFs provided that additional FRFs related to suspension coordinates are also measured. It should also be noted that the damping effects of the suspensions may also be removed by using the proposed method.

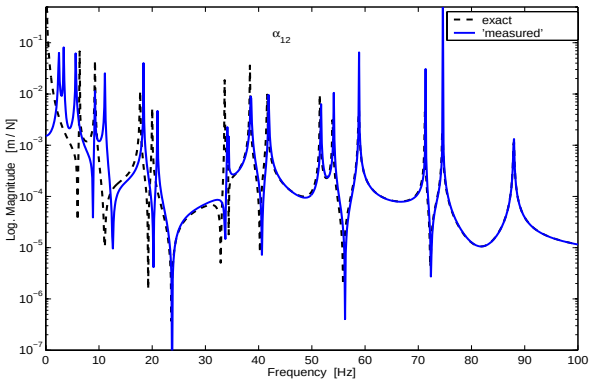

Figure 2. Comparison of the FRFs of free-free (exact) and suspended ('measured') plates.

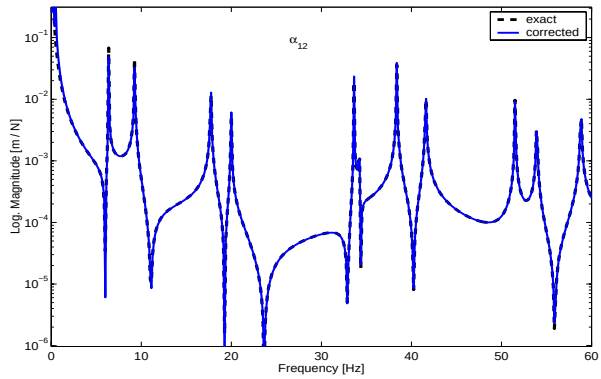

Figure 3. Comparison of the exact and corrected FRFs.

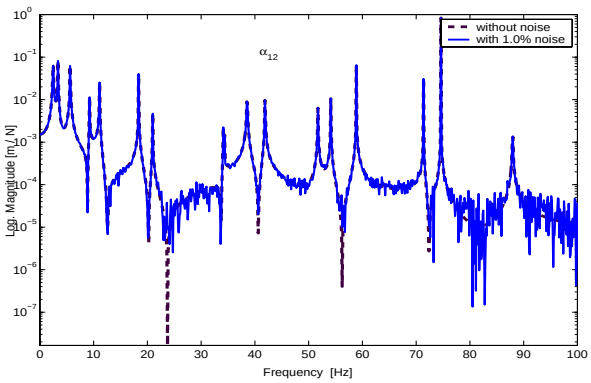

Figure 4. Comparison of the FRFs with and without noise.

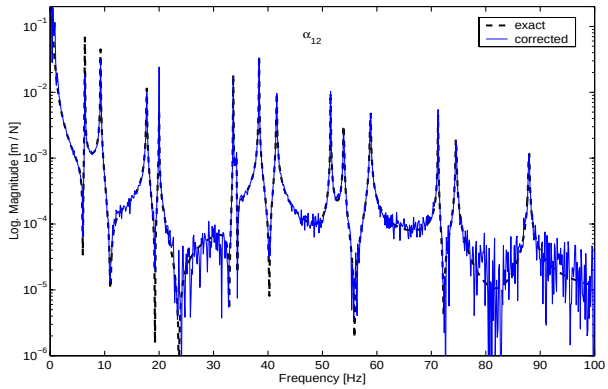

Figure 5. Comparison of the exact and corrected FRFs (noisy case).

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