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ASSESSMENT OF THE ACCURACY OF DAMPING ESTIMATION FOR LIGHTLY DAMPED STRUCTURES

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Estimation of the damping levels in structures is required in many applications and the theoretical damping predictions in many cases are either difficult or not reliable. Therefore, experimental determination of the damping levels is employed quite often. However, there are also difficulties in experimental estimation of damping especially for lightly damped structures. The most widely used methods for damping determination are those of the frequency-based methods. These methods require vibration spectrums or the Frequency Response Function(s), which are obtained by transforming the time-domain data via Fourier Transform. During this process it is often necessary to modify the time-domain data by using an exponential window so as to minimize the leakage effect in the spectrum. It is well-known that the application of an exponential window to time-domain data introduces a known level of damping to individual modes. The so-called numerical damping added by this process can be subtracted later in order to obtain the correct damping level. However, for lightly damped structures, the artificially introduced numerical damping can be much greater than the physical damping level. This inevitably brings the accuracy and the reliability issues. This paper addresses the accuracy of the damping estimation in frequency domain for lightly damped structures when numerical damping has to be introduced during the signal processing phase of the spectrum estimation. Measured data are used to evaluate the accuracy of the damping estimation. Some results are presented showing the relationship between the accuracy of the estimated damping and the level of added numerical damping.

1. Introduction

Damping is probably the most uncertain parameter in structural engineering. Although there are many studies related to theoretical damping modelling and prediction [1-7], it is still very difficult to rely on predictions alone. Therefore, if possible, it is still common practise to determine damping based on experimental approach. However, there are also uncertainties in experimental determination of damping levels in structures, especially for lightly damped structures.

Damping levels of structures can be determined experimentally with a good accuracy if the experiments and the associated signal processing are performed in an appropriate manner. There are a few numbers of methods for determining damping levels using experimental data [3 and 8-9]. These methods are broadly divided in two main groups depending on whether the response of the system is expressed as a function of time or as a function of frequency, i.e. time response methods or frequency-response methods. Logarithmic decrement method, step-response method and hysteretic loop method are time-response methods, whereas the half-power point method, circle-fit method and the line-fit method are frequency-response methods. Logarithmic decrement method is effective when damping of the structure is low while frequency response based methods are more common in many applications. Frequency response methods are applied to spectrums or the Frequency Response Function(s), which are obtained by transforming the time-domain data via Fourier Transform [2-3 and 7]. There are a large number of damping studies in the literature and just a few of them are cited here. Mielczarek et al. [10] measured the logarithmic decrement of the free decaying bending vibrations of a beam by exciting a cantilever beam arrangement by a permanent magnet placed at the end of a coil system. Li et al. [11] in their paper performed a time series analysis method to obtain the relationship between damping and vibration amplitude based on full scale measurements of damping in a tall building. Lamarque et al. [12] introduced a wavelet-based formula similar to the logarithmic decrement formula to estimate damping in multi-degree-of-freedom systems from time-domain responses. Both analytical and numerical approaches are investigated. Huang et al. [13] proposed a new approach for identification of modal damping ratios from free vibration response of a linear structure with viscous damping in their paper.

Although damping is studied extensively in the literature, uncertainties in damping estimation is a topic which has not attracted the attention it deserves. This topic is especially important for lightly damped structures as the error introduced by various sources can be very significant compared to the real level of damping. This paper addresses a particular source of uncertainty during damping estimation, i.e. the uncertainty caused by the use of exponential windowing prior to the spectrum estimation. This is particularly relevant for damping estimation methods using Frequency Response Functions (FRFs) when impact (hammer) testing is used to measure the FRFs. In such FRF measurements, an exponential window often needs to be applied so as to minimise the leakage effects. This is especially true for lightly damped structures as the oscillations created by an impact do not decay to zero within the measurement period. This process, i.e., applying an exponential window to transient signals inevitably introduces so-called numerical damping which may well be much higher than that of the actual damping in the structures, This situation is then raises the question of damping uncertainty due to adding high level of artificial damping

In this paper, it is aimed to present results of an investigation related to uncertainties in damping levels when numerical damping is added to the measured signal due to exponential windowing. A helicopter blade is chosen for case studies.

The outline of this paper is as follows. First of all, a review of the effects of exponential windowing function is summarised. Then, the damping estimation method, Line-Fit method, is very briefly described. After that, experimental setup and FRFs with and without using exponential windowing function are described. During this process, the details of the windowing function are noted so as to calculate the numerical damping added to the system. Also, many repeat measurements are made in order to assess the uncertainty caused by the use of exponential windowing. In the next step, using line-fit method [3 and 9], modal damping levels of the structure are determined for each mode and the numerical damping is removed from the total damping level. Finally, results are processed to evaluate the accuracy or the uncertainty of the damping estimation caused by using exponential windowing for a lightly damped system. Some results are presented showing the relationship between the accuracy of the estimated damping and the level of added numerical damping.

2. Theoretical Background

2.1 Numerical Damping and Its Removal

Windowing functions are often applied to signals in order to avoid leakage. If $x(t)$ is the real response of the structure, $w(t)$ is exponential windowing function the windowed signal, $x_{win}(t)$, is obtained by multiplication of $x(t)$ and $w(t)$, that is,

$$x_{win}(t) = w(t) \cdot x(t)\ . \tag{1}$$

Theoretically, the added numerical damping due to the multiplication of the windowing function with the response signal can be removed. This can be summarised briefly here for the case of single degree of freedom (SDOF) system. The free vibration response of a SDOF system is expressed as

$$x(t) = Xe^{-\omega_0 \xi t} e^{j\left(\omega_0 \sqrt{1-\xi^2}\right)}\ . \tag{2}$$

Here, ω_0 is the undamped natural frequency, ξ is the viscous damping ratio. If the signal $x(t)$ is multiplied by an exponential function ($w(t) = e^{-\tau t}$), the modified response $\acute{x}(t)$ can be written as

$$\acute{x}(t) = Xe^{-\tau t} e^{-\omega_0 \xi t} e^{j\left(\omega_0 \sqrt{1-\xi^2}\right)} = Xe^{-(\tau+\omega_0 \xi)t} e^{j\left(\omega_0 \sqrt{1-\xi^2}\right)}\ . \tag{3}$$

As can be seen from Eq. (3), total damping for the windowed signal can be written as:

$$\bar{\xi} = \frac{\tau+\omega_0 \xi}{\omega_0} = \xi + \frac{\tau}{\omega_0}\ . \tag{4}$$

Therefore, the correct (real) level of damping can be determined by removing the artificial (numerical) damping as:

$$\xi = \bar{\xi} - \frac{\tau}{\omega_0}\ . \tag{5}$$

If the damping level is to be expressed in terms of structural damping (η), Eq. (5) becomes:

$$\eta = \bar{\eta} - \frac{2\tau}{\omega_0}\ . \tag{6}$$

As stated, the method of removing the effect of numerical damping due to exponential windowing is based on SDOF approach. However, according to the principle of superposition, the same approach can be extended for multi-degree of freedom (MDOF) systems; hence the real damping for the r^{th} mode of a MDOF system is given by:

$$\eta_r = \bar{\eta}_r - \frac{2\tau}{\omega_r}\ . \tag{7}$$

where η_r (η_{real}) and $\bar{\eta}_r$ (η_{total}) are the real and the total loss factors of the r^{th} mode respectively, and ω_r is the natural frequency of the r^{th} mode.

It should be noted that this paper aims to investigate the accuracy or uncertainty involved during this process, i.e., during determination of the total damping, numerical damping and their difference.

2.2 Experimental Damping Estimation

In this study, natural frequencies and modal damping values are extracted from measured FRFs using one of the well established frequency based method, that is line-fit method [3, 9 and 14-15].

The line-fit analysis procedure for a MDOF system assumes that around a particular natural frequency ω_r, with modal constant ${}_rA$ and damping η_r, the response includes a constant residual term R_r which represents the contribution of the modes other than the one of the interest [3, 14] as:

$$\alpha_{jk}(\omega)_{\omega \approx \omega_r} \cong \frac{{}_rA_{jk}}{\omega_r^2 - \omega^2 + i\eta_r \omega_r^2} + R_r\ . \tag{8}$$

The method defines a new form of FRF, $\alpha'_{jk}(\omega)$, which is the difference between the actual FRF and the value of the FRF at one fixed frequency in the range of interest ($\alpha_{jk}(\Omega)$) :

$$\alpha'_{jk}(\omega) = \alpha_{jk}(\omega) - \alpha_{jk}(\Omega) \ . \tag{9}$$

The inverse FRF parameter that will be used for the modal analysis is defined as in Eq. (10):

$$\Delta(\omega) = \frac{\left(\omega^2 - \Omega^2\right)}{\alpha'_{jk}(\omega)} \ . \tag{10}$$

This can also be expressed as:

$$\Delta(\omega) = \left(\omega_r^2 - \omega^2 + i\eta_r\omega_r^2\right)\left(\omega_r^2 - \Omega^2 + i\eta_r\omega_r^2\right)\Big/A_r \ . \tag{11}$$

This inverse FRF parameter can be separated into real and imaginary parts (Eq.12) which are supposed to fit on a line when plotted against ω^2. This is the reason why this method is called "Line-Fit" method.

$$\mathrm{Re}(\Delta) = m_R\omega^2 + C_R, \qquad \mathrm{Im}(\Delta) = m_I\omega^2 + C_I \ . \tag{12}$$

Using this method, the modal parameters are identified by determining the constants in Eq.(12) for various values of Ω. Interested readers may refer to references [3, 9, and 14] for further details.

3. FRF Measurements

The damping uncertainty due to exponential windowing will be explored here by studying a practical test structure experimentally. For this purpose, a lightly damped helicopter blade is chosen. In experiments, FRFs are measured using a modal impact hummer, both charge and ICP type accelerometers and an analyser with proper signal conditioning hardware. Repeatability check of the measurements is performed and calibration of all the system is checked both at the beginning and at the end of experiments.

First set of measurements are acquired without the need for exponential windowing. This is achieved by deliberately setting the capture time sufficiently long so that the transient vibration signals decay to almost zero levels at the end of the measurement period. This set of measurement will provide the real (correct, actual) damping without the adverse effect of numerical damping. Then keeping the number of data samples constant, measurement period is decreased each time and an appropriate exponential windowing is applied to transient vibration signal. In other words, during each measurement, the number of data points, say N, is kept constant while the sampling rate and the measurement period (T) are varied. As the measurement period is decreased, more and more numerical damping is to be applied due to exponential windowing. This is precisely the effect this paper aims to address.

During each measurement, exponential windowing parameter is chosen such that, its unit value at the beginning drops to 1 percent of the unit value at the end of the measurement period. Another point to note is that as the sampling rate increases, the frequency span increases and the frequency resolution decreases.

The data acquisition parameters used during the FRF measurements are given in Table 1. As stated before, exponential decay rate (τ) is chosen such that at the end of each measurement period its value is 1 percent of the unit (initial) value.

The structure on which the FRFs are measured is given in Fig. 1. In Fig. 2, a typical time domain signals with and without windowing are given. As can be seen, multiplying the signal with an exponential windowing function forces the signal to decay to zero at the end of the measurement period. Also, a measured FRF without using any windowing function and FRFs obtained using windowed signals are presented in Fig. 3. The effect of the exponential windowing is clearly visible in these FRFs.

Table 1. Data acquisition parameters.

Measurement No	Measurement Period, T [s]	Decay Rate, τ [s^{-1}]	Frequency Span, [Hz]	Frequency Resolution, Δf [Hz]
1	32.0	No Windowing	0.7-200	0.0313
2	32.0	0.14	0.7-200	0.0313
3	16.0	0.29	0.7-400	0.0625
4	8.00	0.58	0.7-800	0.1250
5	6.40	0.72	0.7-1000	0.1563
6	4.00	1.15	0.7-1600	0.2500
7	3.20	1.44	0.7-2000	0.3125
8	2.00	2.30	0.7-3200	0.5000
9	1.28	3.60	0.7-5000	0.7813
10	1.00	4.61	0.7-6400	1.0000

Figure 1. Experimental setup for case study.

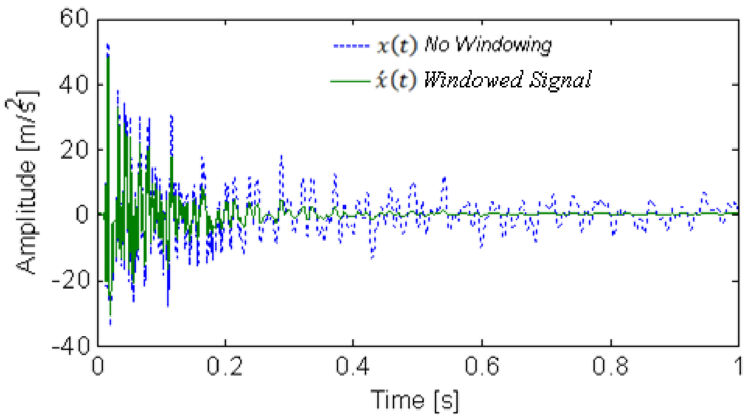

Figure 2. Time signals with and without windowing.

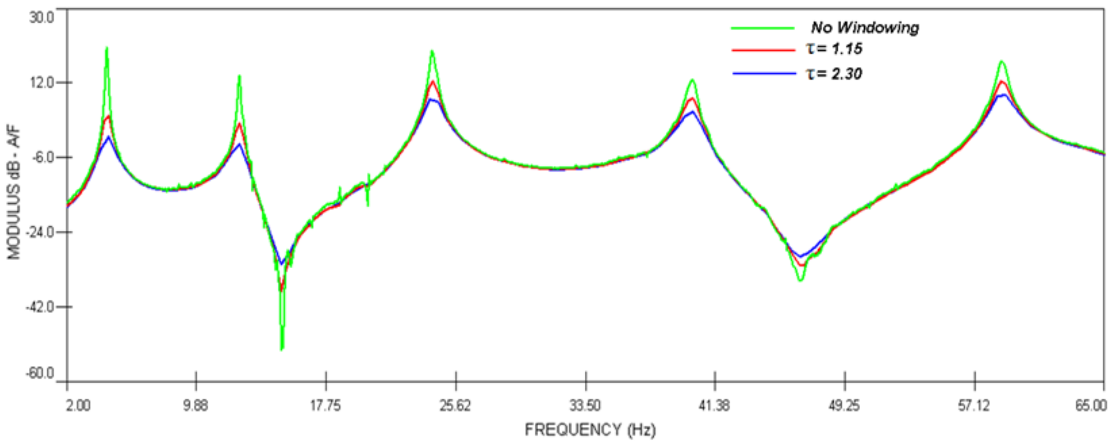

Figure 3. Measured FRFs for various cases.

4. Results

Measured real modal damping values for the first five modes of the helicopter blade (Measurement No: 1) are given in Table 2. These values are determined using measured FRFs in which windowing function is not applied. Therefore, they are free of numerical damping and they are considered as the ‘correct’ damping values.

Similarly, using the parameters summarised in Table 1, measurements with Measurement No 2 to 10 are performed and related modal damping values including the numerical damping are extracted from FRFs [15]. Each set of measurement indicated by "Measurement No" in Table 1 actually consists of many repeated measurements, number of repeat measurements being varied between 10-15. Using the expression (7), the contribution of the numerical damping is removed from the estimated damping values. Ideally, the corrected damping level should be identical to the real damping level. However, this may not be so in practice: if the artificial (numerical) damping is relatively high, this can mask the real damping in the structure and the results may well be different.

Table 2. Real modal damping values (obtained without applying exponential windowing).

Mod No	1	2	3	4	5
Frequency, f [Hz]	4.4	12.5	24.2	40.0	58.8
Modal Damping, η [%]	1.15	0.98	1.20	1.38	0.95

Using these corrected damping values and real damping values in Table 2, damping uncertainties with respect to the level of added numerical damping are explored. It should be noted that the damping uncertainty in this paper is defined below as:

$$\text{Uncertainty}=\frac{1}{2}\,\frac{|\eta_{\max}-\eta_{\text{real}}|+|\eta_{\min}-\eta_{\text{real}}|}{\eta_{\text{real}}}\cdot 100\ . \tag{13}$$

For each decay rate in Table 1, the total damping value, which is the sum of the real damping and numerical damping, is determined using the Line-Fit approach. Then, the real damping values are calculated using the expression in Eq. (7). As shown in Fig. 4, the maximum and the minimum damping values are determined and uncertainty values are calculated for each decay rate using Eq. (13). Fig. 4 is illustrating a typical result for a particular mode. Such calculations are also performed for all other modes within the frequency range of interest.

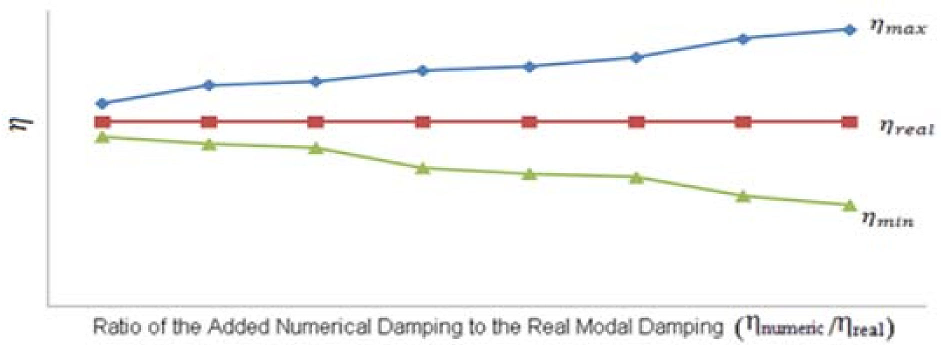

Figure 4. Parameters used in the definition of uncertainty.

Damping uncertainties are presented as a function of decay rate in Fig. 5 for the first four modes. For mode 1 (Fig. 5a), the added numerical damping values is very high. In fact, modal damping for this mode could not be determined for (τ >1.44 s^{-1}). Similar observation can be made for mode number 2 (Fig. 5b), but in this case, the modal damping could not be determined for τ >2.3 s^{-1}. As can be seen in Fig. 5, the uncertainty values in damping increase for all the modes as the decay rate (the level of the added numerical damping) increases.

A normalized parameter is defined in order to make the results independent of the natural frequencies of the test structure used here. This parameter is the ratio of the added numerical damping to the real modal damping ($\eta_{numeric}/\eta_{real}$). When this normalized parameter is used, uncertainty values of the damping levels for different modes (frequencies) can be presented on the same plot as in Fig. 6. As seen, there is a consistence increase in uncertainty as the added numerical damping increases for all modes. These results can be used as a coarse estimation of the possible damping uncertainty when the numerical level and the estimated damping levels are known. However, it should be noted that the results presented here is applicable to the hardware (and software) used in this study, hence the use of different hardware/software may result in different level of uncertainty.

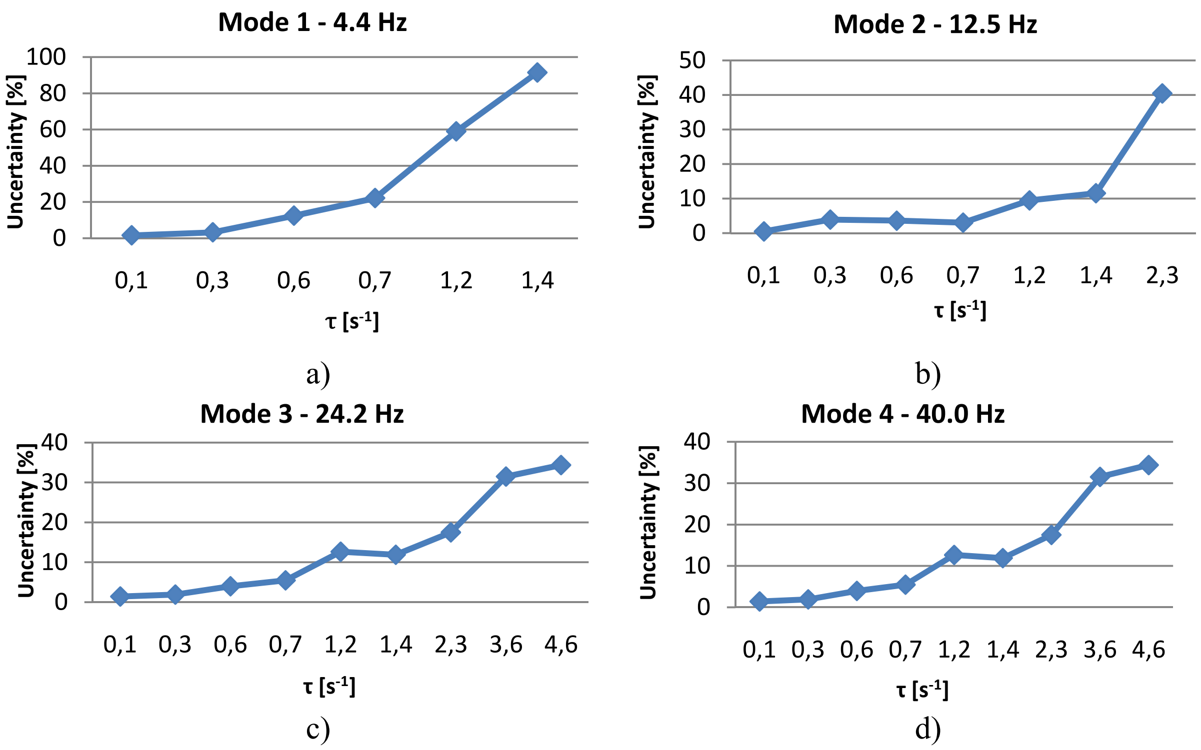

Figure 5. Uncertainties in modal damping values with respect to decay rate, τ for mode 1 (a), mode 2 (b), mode 3 (c) and mode 4 (d).

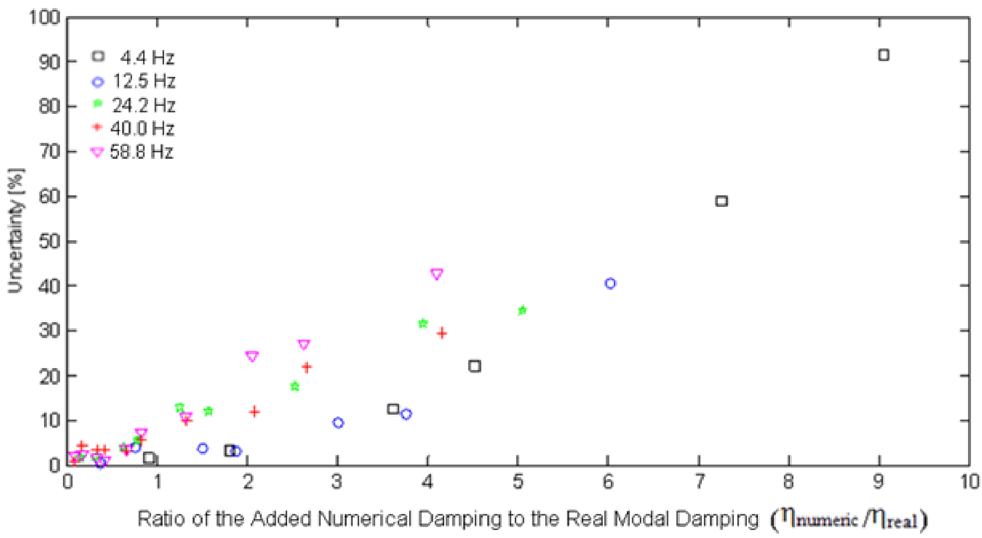

Figure 6. Uncertainties in damping values as a function of the added numerical damping.

5. Concluding Remarks

In this paper, the effect of the added numerical damping due to the use of exponential windowing on the uncertainty of the estimated damping is investigated. This is done for various modes by (i) determining the damping levels without introducing any numerical damping (ii) determining the total damping when there is numerical damping, (iii) removing the numerical damping from the total damping and (iv) comparing the real damping with the estimated damping via removing the effect of the numerical damping.

It is found that the uncertainty of the estimated damping can be very significant when the numerical damping is large compared to the actual damping. The damping uncertainty increases as the numerical damping increases. This uncertainty can reach to about 40% when the numerical damping is about four times greater than that of the actual damping. Also, the results are presented in normalised format so as to make it independent from the test case used in this study. However, it should be noted that the results presented here is applicable to the hardware (and software) used in this particular study, hence the use of different hardware/software may result in somewhat different levels of uncertainty.

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