MODELLING AND VALIDATION OF A MULTILAYERED COMPOSITE FINITE ELEMENT WITH DAMPING CAPABILITY

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Abstract

This paper is devoted to the development and validation of a multilayered isotropic composite finite element with damping capability. The formulation is based on stacking four-noded shell elements where individual layers have different material properties. The damping capability is also included in the formulation by means of complex elemental stiffness matrix representing the elastic and damping properties of the individual layers of the composite element. The formulation is implemented in a Finite Element code and the theoretical predictions are verified using experimental data. Verification process comprised three steps. In the first step, frequency-dependent material properties of the individual layers are measured using a dedicated test rig and these properties are used as input data for the prediction of the dynamic properties of a sample structure with local damping coating. In the second step, the Frequency Response Functions of a sample structure are measured so as to identify the natural frequencies and the modal damping ratios. Finally, predicted and measured results are compared. Results indicate that the composite finite element formulation presented in this paper yields acceptable accuracy provided that the elastic and damping properties of the individual materials forming the composite elements are known.

INTRODUCTION

Composite structures are widely used in industry due to their superior performance compared to conventional constructions. One of the application areas is to provide additional damping to structures so as to reduce excessive vibration and noise levels.

In such applications, load carrying structural members are usually coated with effective damping materials. Sometimes, the construction is made in a multilayered fashion so as to tailor the properties to meet specific needs [1-4]. Although significant amount of work has been done on composite materials and structures [5-7], accurate predictions of the dynamic behaviour of composite structures, especially the damping levels, are still difficult due to various reasons, including the temperature and frequency dependency as well as the non-linear behaviour of the properties of the damping materials [8].

In what follows, a basic shell element - with physical 'drilling' degrees of freedom for the rotation in the direction perpendicular to the element normal direction - is summarised first. Then, the formulation of the multilayered isotropic composite element is presented and the accuracy of the formulation is demonstrated by comparing the predictions with experimental data. Finally, some closing remarks are given.

HOMOGENEOUS SHELL ELEMENT

The composite shell element formulation presented in the next section is based on stacking individual flat shell elements on top of each other. Therefore, any shell element can be used as a building block for the multilayered composite element. However, the particular shell element used as a building block in this paper has an important feature in the sense that it has a physical drilling degree of freedom θ_z in the element normal direction. This particular homogeneous shell element, depicted in Fig.1, is a 4-noded shell element which is obtained by implementing the quadrilateral membrane element with rotational degrees of freedoms [9] and the plate element [10]. It is worth stating that the finite elements proposed in [9, 10] are formulated so as to obtain the element stiffness matrix $[k_s]$. The author of this paper has extended this basic shell element for dynamic analysis by computing the element mass matrix $[m_s]$ and also added damping capability by considering the complex Young's modulus of the material as

$$E^* = E(1 + j\eta) \tag{1}$$

where E and the η are respectively the Young's modulus and the loss factor and j represents $\sqrt{-1}$.

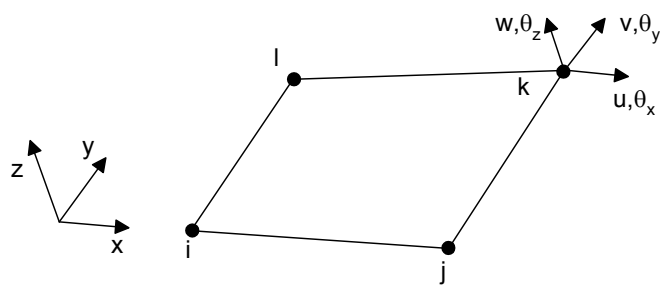


Fig.1 Quadrilateral flat shell element with six nodal degrees of freedoms.

The material damping can also be expressed in terms of loss factor η as:

$$\eta = \frac{D}{2\pi V} \tag{2}$$

where D is the amount of energy dissipated per cycle and the V is the maximum strain energy. The use of complex Young's modulus leads to complex stiffness matrix $[k^*_s]$ for the element and the main aim in this paper is to extend this basic shell element so as to formulate multilayered composite element with damping capability.

FORMULATION OF MULTILAYERED COMPOSITE ELEMENT

As stated before, the formulation presented here is based on stacking the individual layers such that the assembly of layers represents the composite element. Fig.2 shows the connectivity definition as well as the element co-ordinate system, noting that z-axis defines the element normal and the neutral plane is on the x-y plane. In addition to the element co-ordinate system, composite element also requires the definition of the properties of the individual layers including the stacking order, layer thicknesses and the corresponding materials. One way of defining the stacking order and the corresponding thicknesses is to define the distance of the top fiber of each layer from the bottom surface of the composite element, H_i, as schematically illustrated in Fig. 3.

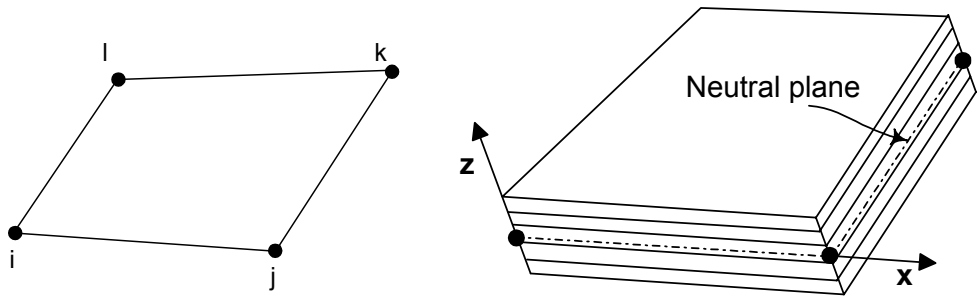


Fig.2 Connectivity and geometry definition of the 4-noded composite element.

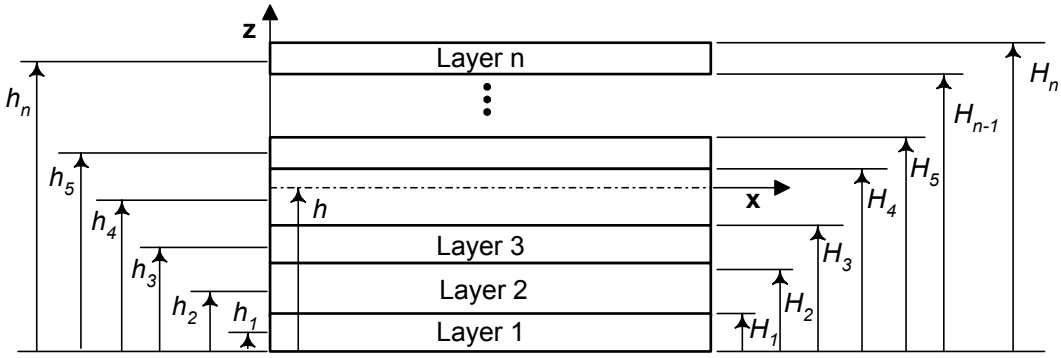


Fig.3 Geometric definition of the layers.

Such a definition allows the determination of the individual thickness t_i as well as the distance of the mid-surface h_i for individual layers as:

$$t_i = H_i - H_{i-1} \;; \quad h_i = 0.5(H_i + H_{i-1}) \qquad i=1,2,\ldots n \tag{3}$$

Referring to Fig.3, layer i is assumed be made of isotropic material i whose properties can be described by E_i, ν_i and η_i, representing the Young's modulus, Poisson's ratio and loss factor, respectively. The composite element also requires the determination of the location of the neutral axis depicted in Figs.2 and 3. Unlike the formulation of the homogeneous shell element outlined in the previous section, it is assumed here - for the purpose of the determination of the neutral axis only - that the plane sections remain plane which allows the use of the basic bending formulation for the determination of the distance of the neutral axis from the bottom surface of the composite element. This leads to:

$$h = \frac{1}{2} \frac{\sum_{i=1}^{n} E_i \left(H_i^2 - H_{i-1}^2\right)}{\sum_{i=1}^{n} E_i \; t_i} \tag{4}$$

Then the offset of the individual layers from the neutral axis can be determined as:

$$z_i = h_i - h \tag{5}$$

Having determined the geometric parameters, the multilayered isotropic composite element is formulated as follows. First, the stiffness and mass matrices for each layer ($[k^*_{si}]$, $[m_{si}]$) are obtained as if each layer is oriented on the neutral plane. Then, the stiffness and mass matrices are transformed so as to take into account that each layer is offset from the neutral position by a distance z_i. This is achieved by using the relationships between the nodal displacements and forces at the neutral plane and those at the offset location for individual layers. This can be expressed in matrix form as:

$$\begin{Bmatrix} u_i \\ v_i \\ w_i \\ \theta_{xi} \\ \theta_{yi} \\ \theta_{zi} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & z_i & 0 \\ 0 & 1 & 0 & -z_i & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} u_o \\ v_o \\ w_o \\ \theta_{xo} \\ \theta_{yo} \\ \theta_{zo} \end{Bmatrix} = [T] \begin{Bmatrix} u_o \\ v_o \\ w_o \\ \theta_{xo} \\ \theta_{yo} \\ \theta_{zo} \end{Bmatrix} \tag{6}$$

where the subscripts i and o denote the locations of the ith layer and the neutral plane, respectively. It should be noted that although the transformation matrix for elemental forces is slightly different than that shown in Eq.(6), it can be shown that the stiffness and mass matrices for individual layers can be expressed with respect to the neutral plane in a conventional manner as:

$$\left[K_{si}^{*}\right]=\begin{bmatrix}[T] & [0] & [0] & [0]\\ [0] & [T] & [0] & [0]\\ [0] & [0] & [T] & [0]\\ [0] & [0] & [0] & [T]\end{bmatrix}^{T}\left[k_{si}^{*}\right]\begin{bmatrix}[T] & [0] & [0] & [0]\\ [0] & [T] & [0] & [0]\\ [0] & [0] & [T] & [0]\\ [0] & [0] & [0] & [T]\end{bmatrix} \tag{7a}$$

$$\left[M_{si}\right]=\begin{bmatrix}[T] & [0] & [0] & [0]\\ [0] & [T] & [0] & [0]\\ [0] & [0] & [T] & [0]\\ [0] & [0] & [0] & [T]\end{bmatrix}^{T}\left[m_{si}\right]\begin{bmatrix}[T] & [0] & [0] & [0]\\ [0] & [T] & [0] & [0]\\ [0] & [0] & [T] & [0]\\ [0] & [0] & [0] & [T]\end{bmatrix} \tag{7b}$$

Finally, the element stiffness and mass matrices for the multilayered composite element is obtained by adding the contributions of individual layers via a simple summation process as:

$$\left[K_{c}^{*}\right]=\sum_{i=1}^{n}\left[K_{si}^{*}\right]; \qquad \left[M_{c}\right]=\sum_{i=1}^{n}\left[M_{si}\right] \tag{8}$$

where the subscripts c above denotes the composite shell. Once the elemental matrices are available with respect to the neutral plane, these matrices are transformed to a common global co-ordinate system and the natural frequencies and modal damping values for a system can be obtained by solving the standard eigenvalue problem:

$$\left(\left[K^{*}\right]-\lambda^{2}\left[M\right]\right)\left\{X\right\}=\left\{0\right\} \tag{9}$$

where $[K^*]$ and $[M]$ are the system matrices, noting that $[K^*]$ is complex, representing non-proportional damping distribution which results in complex mode shapes.

The formulation presented in this paper is implemented in a Finite Element program FINES [11] and it is validated using experimental data as described in the next section.

EXPERIMENTAL VALIDATION

The experimental validation of the multilayered composite element required two experiments. In the first one, frequency dependent material properties were measured

using the Oberst Beam Method outlined in the *ASTM E756 – 93* standard [8]. Measurements were made for bitumen material and results are listed in Table 1.

Table 1. Measured frequency-dependent material properties of the bitumen.

Mode	Frequency [Hz]	Young's Modulus E [N/m^2] x 10^9	Loss Factor η [%]
2	116.4	0.66	55.6
3	323.5	0.62	63.6
4	629.5	0.58	72.2

The second experiment aims to assess the performance of the composite shell element for the prediction of the natural frequencies and damping values for structures having composite construction. First, tests were performed on an uncoated L-shaped plate in Fig.4 which is 3 mm thick and made of mild steel. The modal properties of the plate were determined using a set of Frequency Response Functions. The same L-plate was also used in finite element analysis and the comparison of the measured and predicted results are listed in Table 2. The modal damping values for the steel plate were less than 0.05%.

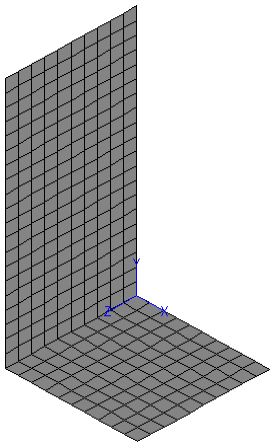


Fig.4 L-plate

Table 2. Measured and predicted natural frequencies of steel L-plate.

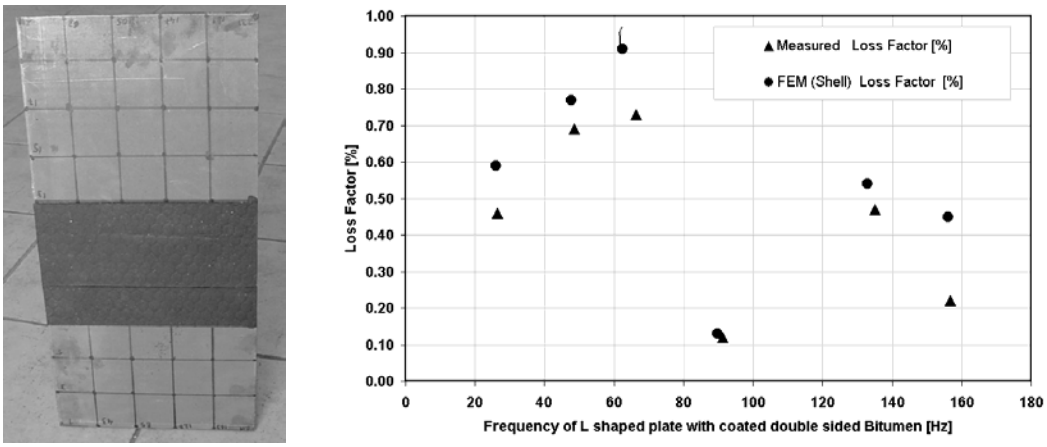
Mode	E steel [N/m^2]x10^9	Measured Frequency [Hz]	Predicted Frequency [Hz]
1	207	26.9	26.2
2	207	49.4	48.4
3	207	66.6	65.8
4	207	91.4	89.8
5	207	144.3	141.7
6	207	161.0	159.6

The L-shaped plate shown in Fig. 4 was then partially coated with 2.2 mm thick bitumen material. The densities of the steel and bitumen were 7800 kg/m^3 and 1943 kg/m^3, respectively. The coating was double sided, hence formed a three layers composite construction. The exact location of the double sided coating is shown in Fig.5a. It should be noted that the Oberst Beam Technique [8] can be used to determine the Young's modulus and loss factor data for a material at specific frequencies. Therefore, the measured material properties in Table 1 for bitumen need to be extrapolated in order to obtain data applicable to the frequency range of interest. For this particular example, Young's modulus and the loss factor for bitumen material were taken as 0.66 x10^9 N/m^2 and %50 respectively within 0-100 Hz. and 0.66 x10^9 N/m^2 and %55 between 100-200 Hz.

Again, Frequency Response Function measurements were made on the coated L-Plate and the finite element predictions using the composite element presented in this paper were made in order to determine the damped natural frequencies and corresponding damping levels. The measured and the predicted results are summarized in Table 3 and plotted in Fig.5b. It is seen that the predictions for the natural frequencies are excellent and qualitative predictions for damping is good. However, predicted damping levels show some deviations from experimental data. It is believed than one of the reasons for the measured damping levels being somewhat less than the predicted values is that the coating on each side was not a single piece. Instead, there were 3 patches which could have caused discontinuities in the strain distribution in the coating material. Nevertheless, considering the difficulty in modelling damping in FE applications, it is believed that the level of accuracy obtained for damping predictions can be considered quite good for practical applications.

Table 3. Natural frequency and the loss factor of L-shaped plate coated with bitumen

Mode	E (Steel) [N/m^2] x10^9	E (Bitumen) [N/m^2] x 10^9	Loss Factor (Bitumen) η [%]	Measured Frequency [Hz]	Measured Loss Factor η [%]	Predicted Frequency [Hz]	Predicted Loss Factor η [%]
1	207	0.66	50	26.5	0.46	26.0	0.59
2	207	0.66	50	48.5	0.69	47.7	0.77
3	207	0.66	50	66.3	0.73	62.5	0.91
4	207	0.66	50	91.2	0.12	89.7	0.13
5	207	0.66	55	135	0.47	133	0.54
6	207	0.66	55	156.7	0.22	156.1	0.45



a) *b)*

Fig.5 a) Double sided coating, b) comparisons of measured and predicted natural frequencies and loss factors.

CONCLUSIONS

The formulation of a multilayered isotropic composite finite element with damping capability is presented. It is based on stacking four-noded shell elements and the damping capability is included in the formulation by means of complex stiffness matrix which represents the elastic and damping properties of individual layers of the composite element. One of the advantages of the formulation is that any shell element can be used as building blocks for developing a similar composite element.

The implementation of the composite element is verified by experimental data. The experiments included measuring the material properties of the coating as well as Frequency Response Functions of a sample structure with and without coating. Comparisons of the measured and the predicted results indicate that the composite element yields acceptable accuracy in predicting the natural frequencies and modal damping levels of composite structures.

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REFERENCES

1. Joseph R. Maly, Kirsten A. Bender, Scott C. Pendleton, "Complex Stiffness Measurement of Vibration Damped Structural Elements", International Modal Analysis Conference, IMAC-XVIII, (2000).
2. B.C. Nakra, "Vibration Control in Machines and Structures Using Viscoelastic Damping" Journal of Sound and Vibration 211, pp. 449-465 (1998).
3. Mohan D. Rao, "Recent Application of Viscoelastic Damping for Noise Control in Automobiles and Commercial Airplanes", Symposium on Emerging Trends in Vibration and Noise Engineering, India – USA (2001).
4. A. D. Nashif, D. I. G. Jones and J.P. Henderson, Vibration Damping. (John Wiley & Sons, New York, 1986).
5. Zhuang Li, Malcolm J. Crocker, "A Study of the Damping in Sandwich Structures", 10th International Congress on Sound and Vibration, pp. 2301-2308, 7-10 July (2003).
6. Mats Dalenbring and Adam Zdunek, "Estimation Of Material Damping For Laminate Structures", 10th International Congress on Sound and Vibration, pp. 2309-2316, (2003).
7. Rajamohan Ganesan, "Dynamic Response Of Laminated Composite Beams Based on Hierarchical and Higherorder Finite Element Formulations", pp. 2351-2358, (2003).
8. "Standard Test Method for Measuring Vibration Damping Properties of Materials", ASTM E 756 – 93.
9. A. Ibrahimbegovic and F. Frey, "Membrane Quadrilateral Finite Elements with Rotational Degrees of Freedom", Engineering Fracture Mechanics, Vol.43, No:1, pp. 13-24 (1992).
10. A. Ibrahimbegovic, "Quadrilateral Finite Elements for Analysis of Thick and Thin Plates", Comp. Methods in Applied Mechanics and Eng., No:110, pp. 195-209 (1993).
11. "FINES: Finite Element for Structures", Version 2004, Istanbul Technical University, Mechanical Engineering Department, Istanbul, Turkey.