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ELIMINATION OF NOISE AND TRANSDUCER EFFECTS

FROM MEASURED RESPONSE DATA

Orhan ÇAKAR E-mail: cakaro@itu.edu.tr Kenan Y. Sanliturk E-mail: sanliturk@itu.edu.tr

Istanbul Technical University Faculty of Mechanical Engineering 80191 Gumussuyu Istanbul, TURKEY

ABSTRACT

This paper deals with improving the quality of measured Frequency Response Functions (FRFs) by removing the undesirable effects of noise and transducer mass. Two distinct methods are employed for the elimination of noise and transducer mass effects from FRFs. One of the methods is based on Singular Value Decomposition (SVD) which is used to remove the noise inherently present in all measured data. The second method, based on so-called the Sherman-Morrison Formula, is utilized to remove a more systematic error from FRFs, also known as mass loading.

The validity of the methods presented here are demonstrated using simulated as well as measured data. Results show that the quality of the data can be improved significantly provided that certain conditions are satisfied.

KEYWORDS: Modal test, FRF, noise, transducer mass effect, SVD, Sherman-Morrison formula.

NOMENCLATURE

j	$\sqrt{-1}$
r	rank
σ_i	<i>i</i> th singular value
3	Numerical precision
ω	Frequency in (rad/s)
$H_{kl}(\omega)$	Frequency Response Function
$\alpha_{kl}(\omega)$	Receptance
$A_{kl}(\omega)$	Accelerance or Inertance
[]; {}	matrix; vector
$\begin{bmatrix} \end{bmatrix}^H$	Hermitian matrix (complex conjugate transpose
	of a matrix)

1 INTRODUCTION

Measured Frequency Response Functions (FRFs) are used for many purposes including system identification, model verification and updating, structural modification, determination of external forces, fault detection as well as solving general vibration and noise problems. In many applications, e.g. structural coupling and structural modification, it is extremely desirable to have high quality FRFs. However, there are some unavoidable experimental error sources originating from the measurement process and experimental set-up. One of the significant error sources in measured FRFs can be classified as 'noise' coming from test environment including electronic devices. Others can be categorized as systematic errors such as mechanical errors including mass loading effects of transducers, shaker-structure interaction and support effects. For a successful experimental modal analysis and other applications, it is necessary to eliminate these undesirable and unwanted effects from the measured FRFs [1-4]. There are a lot of studies on the elimination of noise from electrical signals using Singular Value Decomposition (SVD) technique [8-12] and a few studies on removing the transducer mass loading from FRFs [20-25] in the literature.

This paper deals with improving the quality of measured FRFs, concentrating on removing the undesirable effects of noise and transducers mass from measured data. The first part of the paper presents a method based on Singular Value Decomposition (SVD), aimed at eliminating certain types of experimental errors from FRFs, usually categorized as random errors, or simply noise. After a brief summary of the SVD technique, a method for the elimination of noise in measured test data is presented. The validity of the method is illustrated using a simulated test case where broadband random noise is added to otherwise clean data. It is shown that the method

based on this approach can be successful in elimination of noise from FRFs.

The second part of the paper deals with removing the effect of accelerometer mass from measured FRFs, again aimed at improving the quality of the measured data. The so-called the Sherman-Morrison formula, previously used in the literature for recalculating the inverse of a matrix and for structural modification purposes [23-29], is utilized here to remove the mass loading effect of accelerometer from FRFs. The validity of this approach is demonstrated using simulated as well as experimental data, the experimental data comprising two sets of measured FRFs of a specimen with and without an additional mass in addition to the accelerometer mass itself.

The results obtained so far show that the methods presented here can improve the quality of the measured FRFs significantly. As a result, better data can be provided for modal analysis and other applications.

2 ELIMINATION OF NOISE FROM FRFs

SVD is proved to be a very useful tool in modern linear matrix theory, in particular as a means of estimating the rank of a rectangular matrix [5-7]. The SVD technique has also found a wide range of application areas in engineering. This technique has been used effectively for conditioning the measured signals for a long time, the application areas ranging from medical to telecommunication: e.g., extraction of the foetal electrocardiogram from cutaneous electrode signals, reduction of noise in speech and elimination of noise from electrical signals [8-12].

The use of SVD in structural dynamics is also very common as contaminated (noisy) data are to be used in many applications such as structural coupling and structural modification where reliable inversion of a matrix is required [13], model updating where model parameters are adjusted using modal or frequency response data [14]. In many cases, SVD is used to determine the rank of the data so that optimum solution for over-determined problems can be found, e.g., estimation of modal parameters from measured data [15], optimum test planning for modal testing [16]. Another area of application of the SVD technique in modal testing is for the assessment of the quality and the order of measured data. A good example to this is the work done by Pickrel [17] who used the SVD technique to estimate the effects of frequency band, number of frequencies, number of measurement locations and signal to noise ratio in measured data. The study presented in our paper towards the elimination of noise from measured data is a natural extension of the research done by Pickrel [17]. Application of SVD to a FRF based sub-structuring technique is presented by Lim and Li [18]. SVD is also used for the elimination of noise from FRF data in our previous study [19].

2.1 Theory

SVD of an *MxN* complex matrix [*A*] is given by;

$$[A]_{MxN} = [U]_{MxM} [\Sigma]_{MxN} [V]_{NxN}^{H}$$
(1)

where [U] and [V] are orthogonal matrices, $[\Sigma]$ is the real diagonal matrix and its diagonal elements σ_i are called singular values of [A]. $\sigma_{ij} = \sigma_i$ for i=j; $\sigma_{ij} = 0$ for $i\neq j$. σ_i will be assumed to be arranged in descending order without any loss of generality $\sigma_1 \ge \sigma_2 \ge \cdots \ge \sigma_r \ge 0$, $r = \min\{M,N\}$. More details about the theory of SVD can be found in[5-7].

Let [A] be experimentally measured FRFs matrix arranged in the following form:

$$[A]_{MxN} = [\{H_{11}(\omega)\}_{Mx1} \quad \{H_{21}(\omega)\}_{Mx1} \quad \cdots \quad \{H_{kl}(\omega)\}_{Mx1}] \qquad (2)$$

where $H_{kl}(\omega)$ is a measured FRF relating the response at a given coordinate *k* to excitation force applied at a given coordinate *l* and it is a function of frequency ω . *M* and *N* are number of frequency points and number of measured FRFs respectively. In most experimental studies *M*>*N*, implying that the number of frequency points is greater than the number of measurements, as assumed throughout this paper. When measured FRFs are used to establish the [*A*] matrix, it inevitably contains noise originating from measurement, hence [*A*] can be interpreted as containing the uncontaminated FRFs [\tilde{A}] plus noise, i.e. [*A*]=[\tilde{A}]+[Noise].

The matrix [A] is said to be singular if one or more of the last singular values are zero. This matrix is considered as nearly singular when one or more of its singular values are below a limit of numerical precision, ε . Theoretically, the number of non-zero singular values, r, determine the rank of this matrix, i.e

$$\sigma_i > \varepsilon , \text{ for } i=1 \text{ to } r , \sigma_i \le \varepsilon , \text{ for } i=r+1 \text{ to } N$$
(3)

From modal testing point of view, if enough number of measurement locations are chosen for a given frequency range, i.e. if spatial aliasing is avoided, it can reasonably be assumed that the limit for determination of the rank of [A] will be due to noise and other errors in the measured data. This leads to defining a signal to noise ratio given by [9, 17] as:

$$SNR = \sum_{i=1}^{r} \sigma_i / \sum_{i=r+1}^{N} \sigma_i$$
(4)

This approach leads to identification and elimination of noise from measured data. First, *r* is estimated by carefully examining the relative values of singular values. Then, σ_i are set to zero for *i*>*r* and [*U*], [Σ] and [*V*] are partitioned as

$$[U]_{MxM} = [[U_r]_{Mxr} [U_0]_{Mx(M-r)}]$$
(5a)

$$[V]_{NxN} = [[V_r]_{Nxr} [V_0]_{Nx(N-r)}]$$
(5b)

$$\left[\Sigma\right]_{NxN} = \begin{bmatrix} \left[\Sigma_{r}\right] & 0\\ 0 & \left[\Sigma_{0}\right] \end{bmatrix}$$
(5c)

Finally, an estimate of noise-free matrix $[\tilde{A}]$ can be calculated by using only the first *r* columns of [U], $[\Sigma]$ and [V], i.e.,

$$[A] = [U_r][\Sigma_r][V_r]^H$$
(6)

It should be noted that a new matrix, so-called Principal Response Functions (PRFs) matrix can be created by the multiplication of [U] and $[\Sigma]$ matrices. These PRFs have similar properties to FRFs and posses some additional benefits [1,17].

2.2 Numerical Simulation

A free-free beam, as illustrated in Fig.1, is chosen for a numerical simulation. The beam has dimensions of $0.8 \times 0.01 \times 0.025$ m and the mechanical properties are: Young's modulus E=207·10⁹ N/m⁴, the mass density per unit volume ρ =7800 kg/m³ and Poisson's ratio v=0,3. The beam is modeled by using Finite Element (FE) method and six natural frequencies corresponding to the bending modes of vibration were found in the frequency range from 0 to 2000 Hz (plus rigid body modes). All possible FRFs (a total of 17x17=289) were numerically computed at each of 601 frequencies for the combinations of input and output points, then FRF matrix [A] with dimensions of 601x289 was formed according to Eq.(2).



Figure 1. Free-free beam

The noisy FRFs are simulated by adding white noise to these exact (noise-free) FRFs. The additive white noise is scaled such that its mean deviation is the stated percentage of the mean magnitude of the FRF matrix.

The normalized singular values are shown in Fig.2 for noisy FRFs with 5% additive noise. As can be seen in Fig. 2, the rank of FRF matrix is seven and the noise floor is at the 10⁻¹⁶ level. Also, it is seen in the plots of principal response functions (PRFs) in Fig.3 that seven PRFs have high amplitude levels and others have relatively much lower amplitude levels with a noise-like appearance.

The remaining results illustrate the application of proposed method for noise elimination at different levels of added noise. The exact (error-free) point FRF α_{11} is compared with 5% noisy FRF in Figure 4a and with corrected FRF in Figure 4b. It is seen that the level of noise still remaining in the corrected FRF in Fig. 4b is much lower than that of the noisy FRF in Fig. 4a. For the transfer FRF α_{13} , the comparisons of the exact, noisy



Figure 2. Normalized singular values of FRFs with 5% additive noise



Figure 3. Principal Response Functions (PRFs) with 5% additive noise

and corrected FRFs are given by Fig. 5a and Fig.5b for a frequency range from 600 to 1200 Hz. Another set of results corresponding to 10% noise level are presented in Figs. 6a and 6b. It is clearly seen that the noise elimination method using the SVD technique can significantly eliminate the noise from the FRFs.

It must be noted that it is not practical to measure all possible FRFs in a typical vibration test. The current research activities are focused on the applicability of this method for practical cases where real measurements are used and when only one column of the FRF matrix are measured.



Figure 4a. Comparison of exact and noisy (5%) point FRFs α_{11} .



Figure 4b. Comparison of exact and corrected point FRFs α_{11} .



Figure 5a. Comparison of exact and noisy (5%) transfer FRFs α_{13} .



Figure 5b. Comparison of exact and corrected transfer FRFs $\alpha_{13}. \label{eq:alpha13}$



Figure 6a. Comparison of exact and noisy (10%) transfer FRFs α_{23} .



Figure 6b. Comparison of exact and corrected transfer FRFs α_{23} .

3 CANCELLATION OF THE TRANSDUCER MASS LOADING EFFECT FROM FRFs

Another known problem about the measured FRFs is the mass loading effects of transducer which is mounted on the test structure. Transducer mass causes the natural frequencies of the structure to shift from their correct values, hence introducing a systematic error in the measured FRFs. Generally, this effect is ignored in the analytical and experimental modeling process, based on a usual assumption that the transducer mass is negligible compared to that of the structure under test. However, when light-weighted structures are investigated, this effect can be significant and it can be necessary to eliminate this undesirable side effect before the data are used further analyses. The mass cancellation of transducers from point FRF and removing the stinger mass effects [1,2,20,21] have been studied in the past. However, there are only a few publications [22-24] related to removing these undesirable effects from all measured FRFs and there isn't a practical and effective solution available for use in practice yet.

At the driving point, say l, the transducer mass effect can be removed from the measured accelerance A_{ll} using the approach given by [1,2] as follow:

$$A_{ll}^* = \frac{A_{ll}}{1 - m * A_{ll}} \tag{10}$$

where A_{ll}^* is the corrected accelerance and m^* is the transducer mass. There are a few methods available in the literature for the elimination of transducer mass effects from transfer FRFs. A brief summary of these methods are appropriate here. Decker and Witfeld [22] presented a method based on the process of structural modifications directly from experimental frequency response functions called as SMURF. If a point accelarence A_{ll} is known, the effect of the transducer mass Δm_l can be removed from the transfer accelerance A_{lk} by

$$A_{lk}^{*}(\omega) = A_{lk}(\omega) - \frac{A_{lk}(\omega)A_{ll}(\omega)}{\frac{1}{\Delta m_l} + A_{ll}(\omega)}$$
(11)

where A_{lk}^* is the corrected accelerance. Ashory [23] also aimed at removing the transducer mass loading effect and he considered a measurement with two different accelerometers with different masses. His formulation can be summarized as:

$$\begin{cases} A_{lk}^{*} \\ A_{ll}^{*} \end{cases} = \begin{bmatrix} \frac{1}{A_{lk}} & -m_{1} \\ \frac{1}{\overline{A}_{lk}} & -m_{2} \end{bmatrix}^{-1} \begin{cases} 1 \\ 1 \end{cases}$$
(12)

where m_1 , m_2 are the masses of two different accelerometers and A_{lk} , \overline{A}_{lk} are the measured accelerances at point l with these accelerometers, respectively. Using this procedure, two correct FRFs can be obtained by undertaking two different measurements. Moreover, the driving point FRF A_{ll}^* has to be determined at the co-ordinate l without having to measure it. It must be noted that when m_1 is very close m_2 , A_{lk} will be close \overline{A}_{lk} and therefore the inverse of matrix will be ill-conditioned, although the correction is good when m_1 is quite different m_2 .

3.1 Correction of Transducer Mass Effect via Sherman-Morrison Formula

As discussed above, the transducer mass-loading effect can adversely affect the measured FRFs, especially for delicate structures and the existing methods to remove this effect is still not quite practical. For correction of a transfer FRF, the methods mentioned before need another FRF and there are some limitations when noisy FRFs are to be used and when there are close modes.

An alternative method based on the Sherman-Morrison formula is presented in this paper to remove the adverse effects of transducer mass-loading from FRFs. In what follows, the theory behind this approach is given first. Then, the applicability is demonstrated using both numerical and experimental test cases.

The Sherman-Morrison identity [26] allows a direct inversion of a modified matrix efficiently using the data related to the initial matrix and to the modification. Let $[A]^{-1}$ be the inverse of a non-singular square matrix [A] and consider the modification as a product of two vectors such as $\{u\}\{v\}^T$, so that the modified matrix is $[A'] = [A] + \{u\}\{v\}^T$. The inverse of the modified matrix $[A']^{-1}$ can be calculated by using the Sherman-Morrison formula as follows,

$$[A']^{-1} = [A]^{-1} - \frac{([A]^{-1}\{u\})(\{v\}^{T}[A]^{-1})}{1 + \{v\}^{T}[A]^{-1}\{u\}}$$
(13)

The Sherman-Morrison formula has been used in a wide variety of applications in the past, for example, in the fields of statistics, networks, structural analysis, asymptotic analysis, optimization, and partial differential equations. A more detailed coverage of this approach and various numerical aspects are discussed in [27,28]. For structural dynamic purposes, the main use of the identity given by Eq.(13) is for efficient analysis of structural modification problems. Level et. al. [29] proposed method, using the receptance strategy in conjunction with the Sherman-Morrison formula, to calculate the frequency response of a modified structure. Sanliturk et.al,[30,31], however, extended the application of the basic formulation depicted in Eq.(13) for the analysis of non-linear structures.

Our goal here is somewhat different in the sense that the aim is to remove the transducer mass loading effects from the measured FRFs using the Sherman-Morrison formula. This is achieved by considering the transducer mass as a modification to the original structure and the idea here is to modify the structure again, but this time the modification is to remove the appropriate mass from the structure. In other words, a negative transducer mass is added to the structure as a modification in order to estimate the correct FRFs without the mass loading effect.

Suppose that the dynamic stiffness matrix [Z] of a structure with the transducer mass is given by

$$[Z] = [K] - \omega^2[M] + j\omega[C]$$
(14)

where [K], [M], [C] represent stiffness, mass and damping matrices, ω represent the angular frequency and $j=\sqrt{-1}$. Let $[\Delta Z]$ be the modification to be made to [Z]. Using the well known relationship $[\alpha]=[Z]^{-1}$ and expressing the modification matrix $[\Delta Z] = \{u\}\{v\}^T$, the FRFs of the modified structure can be computed using the Sherman-Morrison formula in Eq.(13) as

$$[\alpha'] = [Z']^{-1} = [\alpha] - \frac{([\alpha]\{u\})(\{v\}^{T}[\alpha])}{1 + \{v\}^{T}[\alpha]\{u\}}$$
(15)

where $[\alpha']$ contains the desired FRFs without the effects of transducer mass loading. It should be noted that the desired FRFs are calculated without the need for any matrix inversion. It is also worth emphasizing that the modifications made here represent removing mass from the structure. Furthermore, only one mass modification is considered here though it is possible to make successive modifications using this approach.

3.2 Numerical Simulation

The same free-free beam that is used in the previous section is also used here to demonstrate the effectiveness of the proposed method in removing the mass loading effects of transducers from FRFs. The beam was modeled by using Finite Element Method and in addition to the rigid body modes six natural frequencies within the frequency range from 0 to 2.0 kHz are calculated. All possible FRFs, a total of 81 corresponding to 9 input and output points, are computed. These FRFs are considered as the correct or the 'exact FRFs' in this numerical simulation.



Figure 7. Beam structure that the accelerometer attached on the point 6.

The mass loading effect of a transducer is also simulated using an FE model. First, a point mass of 20 gr is added to the FE model at an assumed accelerometer location as shown in Fig.7. Then, all the FRFs are calculated again, and these FRFs are considered as the synthesized or "measured" FRFs in this numerical simulation.

A computer program is developed to perform the computations required to remove the mass loading effect via Eq.(15) and the 20 gr transducer mass effect is eliminated from the synthesized ('measured') FRFs, yielding the corrected FRFs. Some of the corrected FRFs are compared to their exact and the 'measured' counterparts in Figs. 8 to 11. As expected, it is clearly seen that the corrected FRFs match perfectly with the exact values.



Figure 8. Comparison of 'measured', corrected and exact point FRFs α_{11} .



Figure 9. Comparison of 'measured', corrected and exact point FRFs α_{66} .



Figure 10. Comparison of 'measured', corrected and exact transfer FRFs α_{23} .



Figure 11. Comparison of 'measured', corrected and exact transfer FRFs α_{67} .

3.3 Experimental Case

Having verified the proposed method using a numerical simulation, it was decided to investigate the applicability and the accuracy of the proposed method in the case of experimental data. A beam with the same physical dimensions as in the numerical case was manufactured and tested. An important feature in these tests was that two sets of FRFs were acquired, corresponding to the configurations depicted in Fig.12. In the fist configuration, Fig.12a, the accelerometer was attached to node 2 and the first set of FRFs were measured. In the second configuration, however, an additional dummy mass (30 gr) was also attached to the point where accelerometer was attached, Fig.12b and the second set of FRFs were measured. In each case, 9 FRFs were measured. It should be noted however that as it stands, the proposed method requires all the FRFs in the $[\alpha]$ matrix. This requirement is satisfied here by generating the unmeasured FRFs using the measured ones by performing a modal analysis first.

Having measured two sets of FRFs, it became possible to eliminate the effect of the dummy mass from the second set of FRFs. The FRFs obtained from such analysis is expected to yield the first set of FRFs measured using the configuration in Fig.12a. The results obtained from this analysis can be used to assess the validity of the method in the case of experimental data.



Figure 12. Experimental test specimen a) without the additional mass, b) with additional mass.

The proposed method is applied first to the second set of FRFs so as to remove the effect of the additional -dummy- mass from FRFs. As mentioned, the outcome of this analysis is expected to yield the measured FRFs in the first set. Results for the point FRF α_{11} are presented in Fig.13 around a natural frequency.



Figure 13. Comparison of measured (with and without dummy mass) and corrected point FRFs α_{11} .

It can be seen that, as expected, the resonance frequency of the system with the dummy mass is lower than that of the system without the dummy mass. However, after eliminating the effect of the dummy mass, both the natural frequency and the FRFs as a whole are in excellent agreement with the correct values. Similar observations can also be made for other FRFs as illustrated in Figs. 14 and 15, clearly validating the proposed approach.

Having verified the method, it can be used to eliminate the transducer effects in the first set of FRFs, a set that has the



Figure 14. Comparison of measured (with and without dummy mass) and corrected transfer FRFs α_{12} .



Figure 15. Comparison of measured (with and without dummy mass) and corrected transfer FRFs α_{23} .

adverse effect of the accelerometer, but not that of the dummy mass. This is a typical situation in practice where only one set of measurements is available. The FRFs obtained from such a filtering process are expected to be the correct FRFs of the system without the mass loading effect. Typical results presented in Figs. 16 and 17 show expected trend in the sense that the natural frequencies of the system without the effect of the transducer mass shift to higher frequencies compared to the values measured in practice using accelerometers.



Figure 16. Comparison of measured (without mass) and corrected point FRFs α_{11} .



Figure 17. Comparison of measured (without mass) and corrected transfer FRFs α_{23} .

4 CONCLUDING REMARKS

Two methods are presented in this paper, both of which aim to improve the quality of the measured data that are to be used for further analyses.

The first method is based on SVD and deals with reducing inevitable noise in measured FRFs. Application of this technique in the case of a numerical simulation indicates that it can successfully reduce the level of noise in FRFs.

The second method is based on the Sherman-Morrison formula and it is used for eliminating the transducer mass effects from measured FRFs. The results obtained from numerical simulations as well as from the experimental study indicate that this method can be very effective in filtering out such undesirable effects from FRFs, especially for lightweight structures.

Although both methods have been found to be quite promising, some aspects of these methods need further research and improvement before they can readily be used in practice. Noise elimination method needs further improvement and need to be verified using experimental data. Moreover, both methods need to be optimized so that the additional measurement requirement is minimized. These will be addressed in a forthcoming paper.

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