Proceedings of ESDA2002:
6th Biennial Conference on Engineering Systems Design and Analysis
Istanbul, Turkey, July 8-11, 2002

ESDA2002/APM-028

AN EFFICIENT METHOD FOR LINEAR AND NONLINEAR STRUCTURAL MODIFICATIONS

Kenan Y. Sanliturk

Istanbul Technical University,
Faculty of Mechanical Engineering
80191, Gumussuyu, Istanbul, TURKEY
E-mail: sanliturk@itu.edu.tr

ABSTRACT

This paper presents a general and efficient method for structural modification purposes and also summarizes various application areas where this method has been used successfully.

The method presented here is based on exact calculation of the Frequency Response Functions (FRF) of the modified structure using the FRF matrix of the initial system and the dynamic stiffness matrix containing the mass, stiffness and damping matrices as modifications. In addition to being an exact method, a distinct feature of the method is that the analysis can be restricted to active co-ordinates only; active co-ordinates being the forcing co-ordinates, co-ordinates where modifications are to be made and those co-ordinates where response levels are needed. This feature makes the method applicable to realistic models with large degrees-of-freedoms (DOFs). Another advantage of the method is that it is ideally suited for the frequency domain analyses of nonlinear structures with localized nonlinearities.

After a brief description of the theoretical basis of the method, the formulations for various types of structural modifications are summarized. These include simple modifications such as discrete mass, stiffness, dashpot and tuned absorber elements as well as a more complex matrix modifications. Various examples are included, which demonstrate the accuracy and the efficiency of the method. These examples are chosen from different application areas, including structural modifications, removing the effect of transducer mass from measured data, analysis of mistuned bladed-disc assemblies and optimization of friction dampers.

1 INTRODUCTION

It is necessary, in many situations, to predict the dynamic properties of structures when some modifications are to be made or when many 'what if' scenarios are to be studied before actual modifications are applied to the structure. Engineers are often faced with this problem when it is realized that the dynamic properties of a structure during design stage, or after a prototype is made, do not meet the design requirements and some changes in the structure are needed to bring about the desired changes by modifying the structure. In these circumstances, it is very desirable to study the effects of many modification scenarios using a mathematical model of the structure so as to avoid costly modification and testing cycles on actual structure.

The first requirement in a theoretical analysis for the prediction of the effects of possible modifications is to have a validated theoretical or experimental model for the unmodified structure. It is assumed here that such a model is already available for further analyses. Once this requirement is met, there are various methods [1] that can be used to predict the changes in the dynamic properties of structures when some modifications are made. An obvious method that can be used for structural modification purposes is to modify the structure and perform the whole analysis again, and repeat this procedure for all possible modification scenarios. This is obviously an expensive method compared to the alternatives, especially when the initial model is large in terms of size. The alternative methods are usually classified into two main groups: (i) those based on modal models and (ii) others based on response models of the components involved in the modification, or coupling, process [2]. Whether it is called 'structural modification' or 'structural coupling' analysis, another classification can be made depending on whether the modifications to the original structure can be accommodated using the existing Degrees-of-Freedoms (DOFs) in the initial model or it is necessary to introduce new DOFs in the analysis, the latter of which could perhaps be a more precise definition of structural coupling analysis. Irrespective of whether modal or response modals are used to describe the components and whether the 'modifications' introduces additional DOFs, almost all the methods available for structural modification/coupling analyses require either the inversion of one or more matrices or a solution of a new eigen value problem to determine the dynamic behaviour of the modified structure [2].

This paper presents an exact method, based on Woodbury-Sherman-Morrison formula [3-4] for the analyses of certain types of modified structures where the modifications could be described without the need for introducing additional DOFs. However, the method brings a distinct advantage in the sense that the Frequency Response Functions (FRFs), hence the response, of the modified structures can be calculated very efficiently, without the need for any matrix inversion or a solution of a new eigenvalue problem [5]. A more detailed coverage of this approach and some numerical aspects are discussed in [6,7]. This paper aims to explore various distinct features that the Woodbury-Sherman-Morrison formula can offer and also to demonstrate these features not only for structural modifications purposes, but also in other application areas. In addition to being an exact method, it offers a possibility of restricting the analysis to active co-ordinates only; active co-ordinates being the forcing co-ordinates, co-ordinates where modifications are made and those co-ordinates where response levels are needed. This feature makes the method applicable to realistic models with large DOFs. Another advantage of the method is that it is ideally suited for the frequency domain analyses of nonlinear structures with localized nonlinearities.

In what follows, the theory behind the so-called Sherman-Morrison-Woodbury formulas are described and the formulation for various types of simple modifications such as mass, stiffness and tuned absorbers are presented. Then, a strategy for nonlinear analysis is presented, combinining structural modification and Harmonic Balance approaches. The rest of the paper presents various application areas demonstrating the applicability and the efficiency of this approach, including (i) modification of a structure by various discrete structural elements so as to obtain the FRFs of the modified structure at desired co-ordinates, (ii) removing the undesired effects of transducer mass loading from measured FRFs, (iii) forced response analyses of mistuned bladed-discs using a realistic tuned model and the mistuning elements as structural modifications and (iv) an optimization of friction dampers via nonlinear analysis.

2 AN EFFICIENT METHOD FOR STRUCTURAL MODIFICATIOINS

2.1 Theory

As mentioned, the analysis method presented in this paper is based on the so-called Sherman-Morrison identity [3] which allows a direct inversion of the modified matrix efficiently using the data related to the initial matrix and to the modification(s). A brief summary of the Sherman-Morrison formula is given below.

Let $[A]^{-1}$ be the inverse of a non-singular square matrix $[A]$. If the inverse of a modified matrix, $[A^*]^{-1}$, is needed where $[A^*]$ is of the form

$$[A^*] = [A] + [\Delta A] = [A] + \{u\}\{v\}^T \tag{1}$$

then it can be calculated using the Sherman-Morrison formula as

$$[A^*]^{-1} = [A]^{-1} - \frac{\left([A]^{-1}\{u\}\right)\left(\{v\}^T [A]^{-1}\right)}{1+\{v\}^T [A]^{-1}\{u\}} \tag{2}$$

The Sherman-Morrison identity is a simplified version of the more general formula of Woodbury-Sherman-Morrison [4], allowing the modification matrix being a multiplication of two matrices, as $[\Delta A]=[U][V]^T$. In this case, the inverse of the modified matrix is given by:

$$[A^*]^{-1} = [A]^{-1} - [A]^{-1}[U]\left([I] + [V]^T[A]^{-1}[U]\right)[V]^T[A]^{-1} \tag{3}$$

This identity is valid for any square matrix $[A]_{NxN}$ and any matrix $[\Delta A]$ expressed as multiplication of two rectangular matrices with appropriate dimensions $[U]_{Nxn}$ and $[V]_{Nxn}$ provided that $[A]$ and $([I]+ [V]^T[A]^{-1}[U])$ matrices are invertible, where $[I]$ is the identity matrix and $n \leq N$.

The similarities between the two identities become apparent when Eqns. (2) and (3) are compared. However, the main advantage of the Sherman-Morrison identity is that it provides the inversion of the modified matrix without any matrix inversion although the modifications are limited to a special form of $\{u\}\{v\}^T$. It is shown later that this is not a real drawback as any matrix modifications can be represented as a sum of a series of modifications in the form of $\{u\}\{v\}^T$.

The formulations presented above are well suited for many applications in structural dynamics. Consider the equations of motion in frequency domain for an unmodified structure given in a customary form as:

$$\left([K] - \omega^2[M] + i[D]\right)\{q\} = [Z]\{q\} = \{f\} \tag{4}$$

where $\{q\}$ is a vector of response amplitudes; $\{f\}$ is a vector of harmonic loads; $[K]$, $[M]$ and $[D]$ are the initial stiffness, mass and structural damping matrices of the system, respectively; $[Z]$ is the frequency (ω) dependent dynamic stiffness matrix and $i=\sqrt{-1}$. A viscous damping matrix can also be included in the expression above. The solution of Eq. (4) provides the response amplitudes as:

$$\{q\}=[Z]^{-1}\{f\}=[\alpha]\{f\} \tag{5}$$

where $[\alpha]$ is the receptance, or the FRF matrix, which is also frequency dependent. The main interest in this paper is to avoid the need for a complete re-analysis of structures when some modifications, represented by $[\Delta Z]$, are to be made to the $[Z]$ matrix and the response amplitudes, or the receptance matrix, need to be calculated again. The Sherman-Morrison identity provides great simplifications during this 're-analysis' stage if the initial receptance matrix is already available. In most cases, the initial receptance matrix is obtained by modal summation after an eigenvalue analysis [2]. Once $[\alpha]$ is available, the Sherman-Morrison identity in Eq.(2) can be written in terms of the receptance matrix as:

$$[\alpha^*]=[\alpha]-\frac{([\alpha]\{u\})\ (\{v\}^T[\alpha])}{1+\{v\}^T[\alpha]\{u\}} \tag{6}$$

When the new receptance matrix for the modified system is available, the response amplitudes corresponding to the modified system can easily be calculated using Eq.(5), but this time using $[\alpha^*]$.

It must be noted that if the modifications affect all the co-ordinates, the formulations here do not provide any advantages over re-analyzing the whole system again. It is obvious, as emphasized in the literature, that the identities in Eqs.(2) and (3) can provide substantial savings in computational time for calculating the receptance (or response) of the modified system when the changes in the original structure are localized. When this is the case, i.e., the changes in the $[Z]$ matrix are localized and can be written as a series of matrices in the form of $\{u\}\{v\}^T$, the new receptance matrix can be obtained efficiently via the Sherman-Morrison identity in Eq.(2) without the need for any matrix inversion. If it is necessary to express the modifications as $[U][V]^T$, then the Woodbury-Sherman-Morrison identity, Eq.(3), can be used. However, this will require a matrix inversion.

It is shown in this paper that the Sherman-Morrison identity can provide substantially more savings in computational time than Eq.(6) implies for certain class of problems where (i) the modifications and the forcing co-ordinates are a small subset of the total co-ordinates and (ii) response amplitudes are to be calculated at selected co-ordinates. It is shown here that when these requirements are met, the analysis is still exact but the calculations are confined to the so-called active co-ordinates. To demonstrate this, suppose that the $\{q\}$ is partitioned such that one of the partitions contains the active co-ordinates (i.e., forcing co-ordinates, modification co-ordinates and those co-ordinates where response levels are needed) and the other partition contains all the other co-ordinates. In accordance with this, the $[\alpha]$ matrix can also be partitioned. This can be written in matrix form as:

$$\{q\}=\begin{Bmatrix}\{q_i\}\\ \{q_a\}\end{Bmatrix},\qquad [\alpha]=\begin{bmatrix}[\alpha_{ii}] & [\alpha_{ia}]\\ [\alpha_{ai}] & [\alpha_{aa}]\end{bmatrix} \tag{7}$$

where the subscripts i and a indicate inactive and active co-ordinates, respectively. If this is inserted in Eq.(6), one can obtain:

$$\begin{bmatrix}\alpha_{ii}^* & \alpha_{ia}^*\\ \alpha_{ai}^* & \alpha_{aa}^*\end{bmatrix}=\begin{bmatrix}\alpha_{ii} & \alpha_{ia}\\ \alpha_{ai} & \alpha_{aa}\end{bmatrix}-\frac{\left(\begin{bmatrix}[\alpha_{ii}] & [\alpha_{ia}]\\ [\alpha_{ai}] & [\alpha_{aa}]\end{bmatrix}\begin{Bmatrix}\{0\}\\ \{u_a\}\end{Bmatrix}\right)\left(\begin{Bmatrix}\{0\}\\ \{v_a\}\end{Bmatrix}^T\begin{bmatrix}[\alpha_{ii}] & [\alpha_{ia}]\\ [\alpha_{ai}] & [\alpha_{aa}]\end{bmatrix}\right)}{1+\begin{Bmatrix}\{0\}\\ \{v_a\}\end{Bmatrix}^T\begin{bmatrix}[\alpha_{ii}] & [\alpha_{ia}]\\ [\alpha_{ai}] & [\alpha_{aa}]\end{bmatrix}\begin{Bmatrix}\{0\}\\ \{u_a\}\end{Bmatrix}} \tag{8}$$

It should be noted that $\{u\}$ and $\{v\}$ vectors are also partitioned and, according to the definition, the modifications are limited to active co-ordinates only. Close inspection of Eq.(8) shows that the Sherman-Morrison identity is also valid at active co-ordinates alone provided that the active co-ordinates include the modification co-ordinates, forcing co-ordinates and those co-ordinates where response levels are needed, i.e.,

$$[\alpha_{aa}^*]=[\alpha_{aa}]-\frac{([\alpha_{aa}]\{u_a\})\ (\{v_a\}^T[\alpha_{aa}])}{1+\{v_a\}^T[\alpha_{aa}]\{u_a\}} \tag{9}$$

The significance of Eq.(9) is that it makes it possible to perform the calculations using active co-ordinates alone, the size of which is much smaller than the total number of degrees of freedoms, N, in many applications. Unless otherwise stated, the rest of the paper deals with calculations using active co-ordinates only and for the sake of brevity the subscript a will be omitted in the equations.

Another issue that is worth mentioning here is that if the total modification matrix, $[\Delta Z]$, cannot be expressed as a multiplication of two vectors ($\{u\}\{v\}^T$), it can be decomposed into several, say p, modifications, such as

$$[\Delta Z]=[\Delta Z_1]+[\Delta Z_2]+[\Delta Z_3]+[\Delta Z_4]+.....+[\Delta Z_p] \tag{9}$$

where $[\Delta Z_j]=\{u_j\}\{v_j\}^T$. This allows the $[\alpha]$ matrix to be calculated in p steps by considering a modification matrix $[\Delta Z_j]$ at a time.

In what follows, how to form the modification matrices for various kinds of simple structural elements are described first. Then, various examples are included to demonstrate the effectiveness this approach.

2.2 Simple Structural Modifications

One of the simplest modifications that can be added to a structure is a spring with a coefficient *k*. If this modification is made between two generalized co-ordinates, then the modification matrix becomes:

$$[\Delta Z_j]=\begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \qquad (10)$$

and such a modification can easily be represented as a product of two vectors as:

$$[\Delta Z_j]=\{u_j\}\{v_j\}^T=\begin{Bmatrix} 1 \\ -1 \end{Bmatrix}\{k \quad -k\} \qquad (11)$$

It should be noted that only the non-zero elements of $[\Delta Z_j]$, $\{u_j\}$ and $\{v_j\}$ are shown in the equations above. If the spring modification is acting along a local axis (with a direction cosines *e*, *f* and *g* with respect to the global co-ordinate system - GCS), the stiffness matrix can be transformed to GCS, noting that the modification matrix in GCS can still be written as:

$$\{u_j\}\{v_j\}^T=\begin{Bmatrix} e \\ f \\ g \\ -e \\ -f \\ -g \end{Bmatrix}\begin{Bmatrix} e.k \\ f.k \\ g.k \\ -e.k \\ -f.k \\ -g.k \end{Bmatrix}^T \qquad (12)$$

A viscous type of damper element with a coefficient *c* can also be formulated in the same way:

$$[\Delta Z_j]=\{u_j\}\{v_j\}^T=\begin{Bmatrix} 1 \\ -1 \end{Bmatrix}\{(i\omega c \quad -i\omega c)\} \qquad (13)$$

Another simple modification is that of mass type. If a mass *m* is to be added to a co-ordinate in the system, the modification matrix has only one element and is given by:

$$[\Delta Z_j]=\{u_j\}\{v_j\}^T=\{1\}\{-\omega^2 m\} \qquad (14)$$

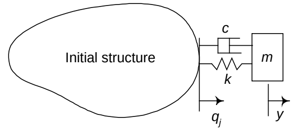

Fig.1 A tuned absorber modification

2.3 Tuned Absorber Modifications

A tuned absorber modification (with parameters *k*, *m* and *c*) is a special type of modification in the sense that the modification introduces a new co-ordinate to the system as shown in Fig.1. However, this type of modification can also be handled by using the same approach if the effect of the tuned absorber at the modification co-ordinate is represented as the impedance 'felt' by the structure. In other words, the response amplitude of the co-ordinate *y* can be expressed as a function of that of q_j, leading to an expression for the impedance modification due to a tuned absorber. This yields a modification matrix with a single element:

$$[\Delta Z_j]=\{u_j\}\{v_j\}^T=\{1\}\{Z_{Tuned}\} \qquad (15)$$

$$Z_{Tuned}=\frac{(-\omega^2 mk - i\omega^3 mc)}{(k-\omega^2 m + i\omega c)} \qquad (16)$$

2.4 Matrix Modifications

Matrix modifications here mean those cases where $[\Delta Z]$ cannot be expressed as $\{u\}\{v\}^T$. Ideally, building up the modification matrix using simple impedance matrices as summarized above has the advantage of performing the calculations very efficiently. However, if this is not feasible, one needs to refer to the Woodbury-Sherman-Morrison identity and this will require a matrix inversion. Another possibility is to use the Sherman-Morrision identity again, but this time considering individual columns, or rows, of the modification matrix at a time until the whole columns, or rows, of the modification matrix are included in the analysis [5, 6].

2.5 Nonlinear Modifications

The method presented here is also extended for the nonlinear analyses of structures by combining this method with the Harmonic Balance approach which can provide equivalent stiffness matrix for nonlinear elements at given response levels [8]. A typical nonlinear modification is illustrated in Fig.2 where a nonlinear element, represented by a crossed box, is connected between two generalized co-ordinates q_p and q_s. If a new variable is defined as

$$y = q_p - q_s \qquad (17)$$

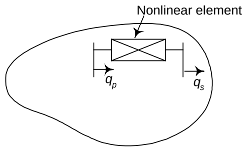

Fig.2 A nonlinear modification.

then, the nonlinear force, say *R*, can be expressed as a function of the relative displacement *y*., i.e., *R*=*R*(*y*). As a first order approximation, the effect of the nonlinear element can be represented as an amplitude-dependent equivalent stiffness as

$$k_{eq}(y) = k_{eq}^{r}(y) + \ i \ k_{eq}^{i}(y) \tag{18}$$

where k_{eq}^{r} and k_{eq}^{i} are the real and imaginary parts of the equivalent stiffness, respectively, as given by

$$k_{eq}^{r} = \frac{1}{\pi |y|}\int_{0}^{2\pi} R(|y|\cos(\theta))\cos(\theta)d\theta \tag{19}$$

$$k_{eq}^{i} = \frac{-1}{\pi |y|}\int_{0}^{2\pi} R(|y|\cos(\theta))\sin(\theta)d\theta \tag{20}$$

The equivalent stiffness can then be considered as a structural modification to the system as:

$$[\Delta Z_j] = \{u_j\}\{v_j\}^T = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}\{k_{eq} \quad -k_{eq}\} \tag{21}$$

It must be noted that this procedure requires iterative solution since the equivalent stiffness depends on the response amplitudes [9,10]. However, this approach, combined with performing the calculations at active co-ordinates, offers the possibility of using realistic models in nonlinear analyses.

3 APPLICATIONS

This section presents various applications of the proposed method demonstrating the applicability and the efficiency of this approach.

3.1 Structural Modifications

As mentioned, the proposed method is ideally suited for structural modification problems, which allows exact calculation of the response of the modified system efficiently without the need for a complete reanalysis. A free-free 'U'-shaped plate subjected to different kinds of modifications is included here as an example.

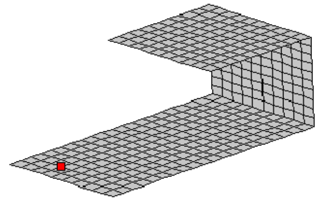

Fig. 3a 'U' plate with a tuned absorber.

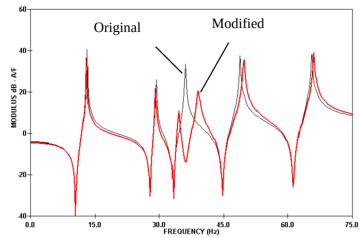

Fig. 3b Point Frequency Response Functions: with and without the tuned absorber.

The structure is illustrated in Fig.3a where the original (unmodified) structure has more than 2500 DOFs. As a first case, the structure is modified with a tuned absorber as indicated by a square (active in the direction perpendicular to the plate), the natural frequency of the tuned absorber being adjusted to the 3rd non-zero natural frequency of the initial structure. The dynamic behaviour of the initial system is described by its natural frequencies and mode shapes hence, the receptance matrix for the initial model at desired co-ordinates can be calculated via modal summation. A point FRF of the system is calculated using the approach presented in this paper and the result is compared in Fig.3b to that of the original system so as to assess the effect of this modification (although not included here, a re-analysis of the modified system yielded the same result). The important point here is that performing such a calculation was trivial; a single co-ordinate is involved in the calculations since the excitation, response and the modification were assumed to be at the same co-ordinate. As far as the calculation of any FRF is concerned, the maximum number of active co-ordinates involved in the calculations will be 3; the excitation, response and the modification co-ordinates. Therefore, irrespective of the original DOFs in the system, any FRF can be calculated almost instantly in this case. Interested readers are referred to [11] for a similar application where a two-dimensional tuned absorber was designed and implemented to reduce excessive vibrations of an industrial tower, the preliminary investigations being carried using the same approach presented here.

The second case comprises additional modifications, i.e., adding two springs and a dashpot in addition to the tuned absorber as illustrated in Fig.4a. In this case, the number of active co-ordinates was 9, 7 of them being modification co-ordinates while the other 2 being excitation and response co-ordinates. Again, the result presented in Fig.4b is exact and the CPU time involved in these calculations is negligible compared to that of a re-analysis.

3.2 Cancellation of Transducer Mass Loading Effects

The second application area presented here is quite different in the sense that fast computation is not the primary objective in this application. Instead, it is shown that the same approach can be used to remove the undesirable effects of transducer mass loading from measured FRFs. It is shown elsewhere that this adverse effect can be significant, especially for lightweight structures, and it may be necessary to eliminate this side effect before the data are used for further analyses [12]. Cancellation of transducer mass loading effect can also be considered as a structural modification problem, but this time the aim is to calculate the FRFs of the unmodified system, the modified system being the original system plus the mass of the accelerometer used in the measurements. In this case, the desired properties of the original system are obtained by removing accelerometer mass from the system (modifying the system with a negative mass).

The applicability and the accuracy of this approach, i.e., removing the mass loading effects of transducers from measured FRFs, is validated using experimental data. A typical set of results is presented in Fig.5. It is shown that the corrected FRF of the system is in excellent agreement with the measurement corresponding to that of the system without the mass. Further details of this subject are available in [13].

3.3 Mistuning Analysis of Bladed Discs

Nominally-identical blades which make up bladed disc assemblies inevitably possess minor differences in their natural frequencies due to manufacturing tolerances and assembly processes: the effect is referred to as 'mistuning'. It is also well known that the dynamic response of such bladed disc assemblies can be significantly different from that of their tuned counterparts so that the response amplitude of individual blades may vary widely within the same assembly [14-16]. Although substantial effort has been devoted to predict and control the effect of mistuning, most of the previous studies on this subject are based on simple models, mainly due to the excessive computational cost [17].

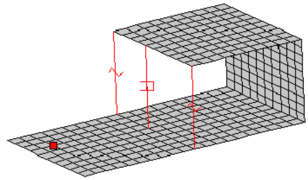

Fig. 4a A ‘U’ plate with a tuned absorber, two springs and a dashpot.

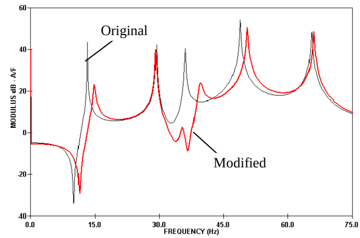

Fig. 4b Cross FRFs: with and without modifications.

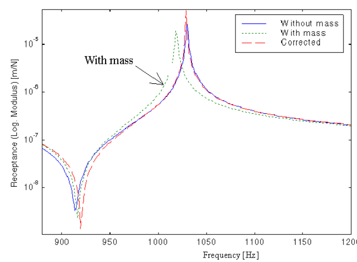

Fig.5 Removing the effects of accelerometer mass loading from measured FRFs.

The formulation based on structural modification approach has been adopted for the mistuning analysis of bladed discs using realistic models by the author and his colleagues [18-19]. The model presented in Fig.6a for a tuned bladed disc assembly (reproduced from Ref.[19] for illustration purposes) has more than 100 thousand DOFs.

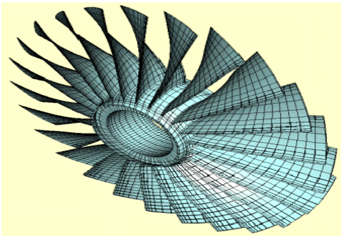

Fig.6a A three-dimensional model of a bladed disc.

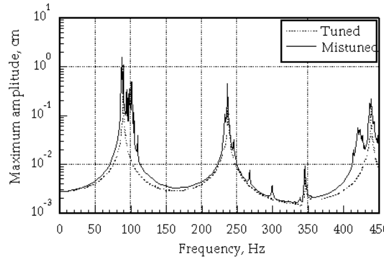

Fig.6b Comparison of response levels for tuned and mistuned bladed discs.

However, using (i) a sector model to represent the tuned system, (ii) the concept of active co-ordinates and (iii) the structural modification approach to model the mistuning, have made it possible to obtain realistic response calculations as shown in Fig.6b [19].

3.4 Nonlinear Analysis: Friction Damper Optimization

Any nonlinear analysis demands more computational power than its linear counterpart and the size of the initial model becomes a critical parameter in such analyses. In many cases, this leads to being unable to analyze representative models due to excessive computational cost. However, the nonlinear analysis methodology based on a combination of the harmonic balance method and a structural modification approach, as briefly summarized in this paper, has made it possible to overcome this problem for certain types of nonlinear analyses where the active co-ordinates can be a fraction of the total number of DOFs in the linear model. Contact modeling and friction damper optimization problems are typical examples of this type since most parts of a structure can be considered as linear in many applications.

Fig.7a shows a realistic model of two turbine blades with a holding block. The behaviour of a friction damper - a wedge-shaped underplatform damper- is analyzed at a given relative response amplitude between the damper connection points and the friction damper is linearized as equivalent complex stiffnesses, representing both restoring and energy dissipation characteristics. The equivalent complex stiffnesses are then added to the otherwise linear system and the response levels of the modified system are calculated again, the procedure being repeated until convergence is achieved. The response levels obtained at current frequency are used as initial guesses for the next frequency increment.

Comparison of the predictions with the experimental data, Fig.7b reveals that analyses method can predict the nonlinear response levels with acceptable accuracy. Further details related to these applications are available in [10,20].

4 CONCLUDING REMARKS

A method for efficient analysis of modified structures is presented. This method is ideally suited for certain kinds of modifications where the changes in the structure can be accommodated using the existing degrees-of-freedoms. However, this limitation can be overcome for certain types of modifications by eliminating the new degrees-of-freedom(s) and by determining the net impedance at the modification co-ordinate(s) as in the case of tuned absorber modifications.

Various application areas, namely; simple structural modifications, removing the undesirable effects of transducer mass loading from measured FRFs, mistuning analysis of realistic bladed disc assemblies and friction damper optimization via nonlinear analysis are presented. In each case, the proposed method provided significant advantages over its alternatives and the results obtained have been found to be very satisfactory.

As a whole, the Sherman-Morrison identity provides an efficient analysis algorithm especially for certain class of problems where the modifications - whether they are linear or not - are localized, permitting the analyst to perform the calculations using a subset of the original degrees-of-freedoms, so-called actives co-ordinates in this paper. As such, it can offer huge savings in computational time and can be well suited for various other applications including design optimizations and probabilistic analyses of structures.

5 ACKNOWLEDGMENTS

Some of the research presented in this paper was carried out while the author was a member of the Centre of Vibration Engineering at Imperial College, London, and the financial support provided by Rolls-Royce Plc during that period is greatly acknowledged.

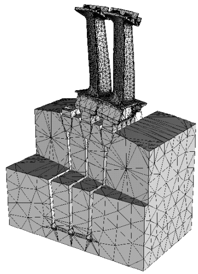

Fig.7a FE model for the linear part of two-blade assembly.

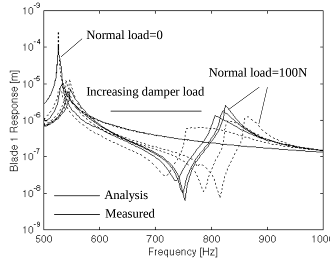

Fig.7b Comparison of measured and predicted response levels at different normal loads.

6 REFERENCES

[1] Skingle, G. M. "Structural Dynamic Modification Using Experimental Data", PhD Thesis, Imperial College, University of London, 1989.

[2] Ewins, D. J., "Modal Testing: Theory, Practice and Application", Research Studies Press, 2nd Edition, 2000.

[3] Sherman, J., Morrison, W.J. "Adjustment of an Inverse Matrix Corresponding to Changes in the Elements of a Given Column or a Given Row of the Original Matrix", Ann. Math. Statist., 20, 1949, p. 621.

[4] Woodbury, M. "Inverting Modified Matrices", Memorandum Report 42, Statistical Research Group, Princeton University, Princeton, NJ, 1950

[5] Level, P., Moraux, D., Drazetic P. and Tison T. "On A Direct Inversion of The Impedance Matrix in Response Reanalysis". Communications in Numerical Methods in Engineering,**12**, pp. 151-159, 1996.

[6] Hager W. G., "Updating the Inverse of a Matrix", SIAM Rev., 31, No.2, pp.221-239, 1989.

[7] Akgün, M.A., Garcelon, J.H., and Haftka, R.T. "Fast Exact Linear and Non-linear Structural Reanalysis and the Sherman-Morrison-Woodbury Formulas". International Journal for Numerical Methods in Engineering, 50, pp. 1587-1606, 2001

[8] Sanliturk K. Y., Imregun M. and Ewins. D.J. "Harmonic Balance Vibration Analysis of Turbine Blades with Friction Dampers", ASME, J. Vibration and Acoustics,119, 96-103, 1997.

[9] Sanliturk K. Y., and Ewins D. J. "Modelling Two-Dimensional Friction Contact and Its Application Using Harmonic Balance Method". J. of Sound and Vibration, 193(2), 511-523, 1996.

[10] Sanliturk K. Y., Ewins D. J., Elliot R. and Green J. S., "Friction Damper Optimization: Simulation of Rainbow Tests", ASME Journal of the Engineering for Gas Turbines and Power, Vol. 123, No.4, pp.930-939. (2001)

[11] Sanliturk K. Y., Belek H. T., "Design and Implementation of a 2-Dimensional Vibration Absorber on a Pre-Heater Tower at a Cement Factory", *Proceedings of the 8th International Congress on Sound and Vibration*, 2-6 July 2001, pp.2259-2264 Hong Kong, China, Edited by C. Cheng et. al.

[12] Ashory, M.R., "Correction of Mass Loading Effects of Transducers and Suspension Effects in Modal Testing". Proc. of the 16th International Modal Analysis Conference,1998 pp. 815-823.

[13] Cakar O. and Sanliturk K. Y., "Elimination of Noise and Transducer Effects from Measured Response Data", submitted for presentation in *ESDA2002: 6th Biennial Conference on Engineering Systems Design and Analysis*, Istanbul, Turkey, July 8-11, 2002.

[14] Ewins D. J. "The Effects of Detuning Upon the Forced Vibrations of Bladed Disks", Journal of Sound and Vibration, Vol.9, pp.65-79, 1969.

[15] Sanliturk K. Y., Imregun M., and Ewins D. J. "Statistical Analysis of Random Mistuning of Bladed Assemblies". 7-10 September 1992, 51-58, C432/110, *IMechE Proc. Int. Conf. on Vib. in Rotating Machinery*, Univ. of Bath, UK.

[16] Sanliturk K. Y. and Imregun M., "Vibration Analysis of Mistuned Bladed-Disk Assemblies-Inverse Approach", AIAA Journal, No 4, 1994. 875-865 Vol.32.

[17] Ewins D. J., "The Effects of Blade Mistuning on Vibration response – A Survey", *IFToMM 4th International Conference of Rotordynamics*, Prague, Czechoslovakia, 1991.

[18] Petrov E., Sanliturk K. Y., Ewins D. J. and Elliott R. "Quantitative Prediction of the Effects of Mistuning Arrangement on Resonant Response of a Practical Turbine Bladed Disc", Processdings of the *5th National Turbine Engine High Cycle Fatique (HCF) Conference*, 7-9 March 2000, Chandler, Arizona, USA.

[19] Petrov E., Sanliturk K. Y., Ewins D. J., "A New Method for Dynamic Analysis of Mistuned Bladed Discs Based on Exact Relationship Between Tuned and Mistuned Systems", scheduled for publication in Proc ASME: Journal of Engineering for Gas Turbines and Power, April 2002, Vol.124.

[20] Sanliturk K. Y., Ewins D. J. and Stanbridge A.B., "Underplatform Dampers for Turbine Blades: Theoretical Modelling, Analysis, and Comparison with Experimental Data", ASME Journal of the Engineering for Gas Turbines and Power, Vol. 123, No.4, 2001. pp.919-929.