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Research Article

Analytical and Numerical Methods for the CMKdV-II Equation

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Hirota's bilinear form for the Complex Modified Korteweg-de Vries-II equation (CMKdV-II) $U_t - 6|U|^2U_x + U_{xxx} = 0$ is derived. We obtain one- and two-soliton solutions analytically for the CMKdV-II. One-soliton solution of the CMKdV-II equation is obtained by using finite difference method by implementing an iterative method.

1. Introduction

It has been known that quasilinear parabolic equations or non-linear reaction-diffusion systems arise in physics, chemistry, biology, and other applied sciences. The following three equations are the examples of this type of partial differential equations (PDEs). First one is called the Korteweg-de Vries equation

$$U_t + cUU_x + U_{xxx} = 0, \tag{1.1}$$

and first encountered in the study of waters, Korteweg [1], denoted by KdV. The other is called the Complex Modified Korteweg-de Vries-I equation (CMKdV-I)

$$U_t + \alpha\left(|U|^2U\right)_x + \beta U_{xxx} = 0, \tag{1.2}$$

which arises both in the asymptotic investigation of electrostatic waves in a magnetized plasma and in the asymptotic investigation of one-dimensional plane-wave propagation in a micropolar medium, Erbay [2]. The last one is the Complex Modified Korteweg-de Vries-II equation (CMKdV-II)

$$U_t - 6|U|^2 U_x + U_{xxx} = 0, \tag{1.3}$$

which is another example for quasilinear parabolic equations or non-linear reaction-diffusion systems, Ablowitz [3]. Equation (1.2) does not hold the Painlevé property but the (1.1) and (1.3) do, Mohammad [4]. The equations which have Painlevé property may be solved by the method of Inverse Scattering Transformations (IST) and hence they are completely integrable [5, 6]. Sometimes it is not easy to solve IST problems [3], such as for CMKdV-II equation. Therefore the need for an easy and useful method which has to give soliton solutions for a given PDE is emerged. An important method is developed by Hirota for finding N-soliton solutions of non-linear PDE [5, 6].

In this paper, the Hirota's method is applied to the CMKdV-II equation. The Hirota's method generally requires the transformation of PDE into homogeneous bilinear forms of degree two. Only specific PDEs can be transformed in this way. This means, when a bilinear (form) equation can be solved, then N-parameter solution can be obtained as a series which self-truncates at finite length. These expansions that self-truncate in this way give automatically exact solutions. Self-truncation, however, does not occur for all bilinear equations; if it does, then the equation in question possesses multiple soliton solutions. The reason for this situation has never been adequately explained. In other words, self-truncation which is equivalent to complete integrability would require a connection with the conserved quantities of the original equation.

In this study, it is proven that the CMKdV-II equation has self-truncated Hirota expansions. It is shown that there is a direct equivalence between the N-soliton solutions of Hirota's bilinear form of CMKdV-II and the Backlund transformations proposed by Weiss, Tabor, and Carnevale [7, 8].

Now, the question here is where the soliton comes from. Firstly, J. S. Russel in 1834 recorded his observations of great solitary wave as a mean of developing the mathematical properties of a large class of solvable non-linear evolution equations. Solitary waves, solitons, Backlund transformations, conserved quantities and integrable evolutions which can be also named as completely integrable Hamiltonian systems are in the class of solvable non-linear evolution equations. The description of John Scott Russel has aroused among mathematicians and physicists one hundred and forty years later, Zabusky [9]. In their paper, they were the first ones who defined the solution for the following KdV equation:

$$U_t + 6UU_x + U_{xxx} = 0. \tag{1.4}$$

For the CMKdV-II Equation, the Hirota's bilinear form is given in Section 2 and the analytical one- and two-soliton solutions are presented in Section 3. The numerical procedure and results for one-soliton solution are outlined in Section 4.

2. Hirota's Bilinear Form of the CMKdV-II Equation

It is known that the equation

$$U_t - 6|U|^2 U_x + U_{xxx} = 0 \tag{2.1}$$

is the complex modified Korteweg de Vries II equation (CMKdV-II). Let g and f be the complex and real valued functions, respectively, satisfying

$$U = \frac{g}{f}, \qquad |g|^2 = -f f_{xx} + f_x^2. \tag{2.2}$$

By using the transformation above, CMKdV-II becomes

$$f g_t - f_t g + 3\left(f_{xx} g_x - f_x g_{xx}\right) + f g_{xxx} - g f_{xxx} = 0. \tag{2.3}$$

Let

$$D_x = \frac{\partial}{\partial x} - \frac{\partial}{\partial x'}, \qquad D_t = \frac{\partial}{\partial t} - \frac{\partial}{\partial t'} \tag{2.4}$$

then (2.3) becomes

$$\left[D_x^3 + D_t\right] g(x,t) f(x',t') \Big|_{x' = x, t' = t} = 0 \Longleftrightarrow \left(D_x^3 + D_t\right) g f = 0, \tag{2.5}$$

which is a homogeneous bilinear form and called Hirota's form of CMKdV-II equation. Let

$$U = \frac{U_0}{\Phi} + U_1 \tag{2.6}$$

be the truncated solution of CMKdV-II. Then

$$\begin{gathered}
|U_0|^2 = \Phi_x^2, \\
\overline{U_0} U_1 + \overline{U_1} U_0 = -\Phi_{xx}, \\
6U_{0x}|U_1|^2 - 6\Phi_{xx} U_{1x} = U_{0xxx} + U_{0t}, \\
6\Phi_x U_0 |U_1|^2 - 6\Phi_x^2 U_{1x} = 3(\Phi_x U_{0xx} - \Phi_{xx} U_{0x}) + U_0(\Phi_{xxx} + \Phi t), \\
U_{1t} - 6|U_1|^2 U_{1x} + U_{1xxx} = 0.
\end{gathered} \tag{2.7}$$

Here U and U_1 are both separated solutions of the CMKdV-II equation. Hence it is an onto Backlund transformation of CMKdV-II equation. The system of these equations are called the Painlevé relations. There is a relation between the soliton of the bilinear (2.5) and the function Φ of the Painlevé relations (2.7). Consider the solitons

$$U = \frac{g}{f}, \qquad U_1 = \frac{g^{(1)}}{f^{(1)}} \tag{2.8}$$

having the properties

$$|U|^2 = (\log f)_{xx}, \qquad |U|^2 = -\frac{\partial^2}{\partial x^2}\log\Phi + |U_1|^2. \tag{2.9}$$

It can be shown that

$$f = \Phi f^{(1)}, \qquad g = U_0 f^{(1)} + \Phi g^{(1)}. \tag{2.10}$$

Theorem 2.1. *If $f^{(n)}$ and $g^{(n)}$ satisfy* (2.5) *for all n, with*

$$\left|g^{(n)}\right| = f^{(n)} f_{xx}^{(n)} - \left(f_x^{(n)}\right)^2, \tag{2.11}$$

and if

$$f^{(n)} = \Phi_{(n-1)} f^{(n-1)}, \qquad g^{(n)} = U_0 f^{(n-1)} + \Phi_{(n-1)} g^{(n-1)}, \tag{2.12}$$

then the resulting equations in Φ_{n-1},U_0 and $U^{(n-1)}$ are satisfied by the Painlevé relations (2.7)*. Furthermore*

$$f^{(n)} = \prod_{i=0}^{n-1}\Phi_i, \tag{2.13}$$

with

$$f^{(0)} = 1. \tag{2.14}$$

Proof. When substituting (2.12) into (2.5) and using the Painlevé relations when necessary yields the claim of the theorem. Using (2.12) succesively we obtain the relation (2.13) that completes the proof of the theorem [4]. □

3. Solitons for the CMKdV-II Equation

By using the usual perturbation method, N-parameter exact solitary wave solutions of (7) can be obtained, Nayfeh [10]. The power series of g and f which are given in Hirota [11] in a small parameter ε are:

$$g = \varepsilon g^{(1)} + \varepsilon^3 g^{(3)} + \varepsilon^5 g^{(5)} + \cdots$$
$$f = 1 + \varepsilon^2 f^{(2)} + \varepsilon^4 f^{(4)} + \cdots , \tag{3.1}$$

where g and f are the solutions of the (2.5). Then considering the increasing powers of ε from (2.5) it is clear that

$$g^{(1)}_{xxx} + g^{(1)}_t = 0, \tag{3.2}$$

$$g^{(3)}_{xxx} + g^{(3)}_t = -\left(D^3_x + D_t\right) f^{(2)} g^{(1)}, \tag{3.3}$$

$$g^{(5)}_{xxx} + g^{(5)}_t = -\left(D^3_x + D_t\right)\left[f^{(4)} g^{(1)} + f^{(2)} g^{(3)}\right], \tag{3.4}$$

$$g^{(5)}_{xxx} + g^{(5)}_t = -\left(D^3_x + D_t\right)\left[f^{(6)} g^{(1)} + f^{(4)} g^{(3)} + f^{(2)} g^{(5)}\right] \tag{3.5}$$

and (2.2) yields

$$f^{(2)}_{xx} = -g^{(1)} g^{(1)*}, \tag{3.6}$$

$$f^{(4)}_{xx} = -g^{(1)} g^{(3)*} - g^{(1)*} g^{(3)} - f^{(2)}_x f^{(2)}_x - f^{(2)} f^{(2)}_{xx}, \tag{3.7}$$

$$f^{(6)}_{xx} = -g^{(3)} g^{(3)*} - g^{(1)*} g^{(5)} - g^{(1)} g^{(5)*} - 2 f^{(2)}_x f^{(4)}_x - f^{(4)} f^{(2)}_{xx} - f^{(2)} f^{(4)}_{xx}, \tag{3.8}$$

where $*$ stands for the complex conjugate.

3.1. One-Soliton Solution for the CMKdV-II Equation

To obtain one-soliton solution of the CMKdV-II equation, let's take

$$g^{(1)} = e^{\Phi + I\Psi}; \quad \Phi + I\Psi = a_1 x + b_1 t + c_1. \tag{3.9}$$

Hence

$$\begin{aligned} a_1 &= \mathrm{re}(\mathrm{a}_1) + I \cdot \mathrm{im}(a_1), \\ b_1 &= \mathrm{re}(b_1) + I \cdot \mathrm{im}(b_1), \\ c_1 &= \mathrm{re}(c_1) + I \cdot \mathrm{im}(c_1), \\ \mathrm{re}(b_1) &= -(\mathrm{re}(a_1))^3 + 3 * \mathrm{re}(a_1) - (\mathrm{im}(a_1))^2, \\ \mathrm{im}(b_1) &= (\mathrm{re}(a_1))^3 - 3 * \mathrm{im}(a_1) - (\mathrm{re}(a_1))^2, \\ f^{(2)} &= \frac{e^{2\Phi}}{4(\mathrm{re}(a_1))^2}, \end{aligned} \tag{3.10}$$

and $f^{(2n)} = 0$, $g^{(2n-1)} = 0$ for all $n \geq 2$ where I stands for the complex number i. Therefore

$$f(t,x) = 1 + \frac{\varepsilon^2}{4(\mathrm{re}(a_1))^2} e^{2\Phi}, \qquad g(x,t) = \varepsilon e^{\Phi + I\Psi} \tag{3.11}$$

is a solution of Hirota bilinear form (2.5) and

$$U = \frac{g(x,t)}{f(x,t)} \tag{3.12}$$

is the corresponding one-soliton solution of the CMKdV-II equation.

3.2. *Two-Soliton Solution for the CMKdV-II Equation*

To find two-soliton solution, let's take

$$g^{(1)} = e^{\Phi_1 + I\Psi_1} + e^{\Phi_2 + I\Psi_2}, \tag{3.13}$$

where

$$\begin{aligned} \Phi_1[t,x] &= \mathrm{re}(a_1) * x + \mathrm{re}(b_1) * t + \mathrm{re}(c_1) \\ \Psi_1[t,x] &= \mathrm{im}(a_1) * x + \mathrm{im}(b_1) * t + \mathrm{im}(c_1) \\ \Phi_2[t,x] &= \mathrm{re}(a_2) * x + \mathrm{re}(b_2) * t + \mathrm{re}(c_2) \\ \Psi_2[t,x] &= \mathrm{im}(a_2) * x + \mathrm{im}(b_2) * t + \mathrm{im}(c_2) \end{aligned} \tag{3.14}$$

with

$$\begin{aligned}
\mathrm{re}(b_1) &= -(\mathrm{re}(a_1))^3 + 3 * \mathrm{re}(a_1) - (\mathrm{im}(a_1))^2 \\
\mathrm{im}(b_1) &= (\mathrm{re}(a_1))^3 - 3 * \mathrm{im}(a_1) - (\mathrm{re}(a_1))^2 \\
\mathrm{re}(b_2) &= -(\mathrm{re}(a_2))^3 + 3 * \mathrm{re}(a_2) - (\mathrm{im}(a_2))^2 \\
\mathrm{im}(b_2) &= (\mathrm{re}(a_2))^3 - 3 * \mathrm{im}(a_2) - (\mathrm{re}(a_2))^2.
\end{aligned} \tag{3.15}$$

Hence from (3.8), it can immediately be found that

$$f^{(2)} = \frac{e^{2\Phi_1}}{4(\mathrm{re}(a_1))^2} + \frac{e^{2\Phi_2}}{4(\mathrm{re}(a_2))^2}. \tag{3.16}$$

Since the right hand side of (3.3) is not zero, a special solution for $g^{(3)}$ can be found by the method of undetermined coefficients. Considering (3.7), $f_{xx}^{(4)}$ is a nonzero real function. The right hand side of (3.4) is not zero, and we can take $g^{(5)} = 0$. From (3.8), it is seen that $f_{xx}^{(6)}$ is zero and that $f^{(6)} = 0$ is taken for convenience. Further computations have shown that

$$f^{(2n)} = 0, \quad g^{(2n-1)} = 0, \quad n \geq 3. \tag{3.17}$$

Hence

$$\begin{aligned}
f[t,x] = {} & 1 - \frac{1}{4(\mathrm{re}(a_1))^2} e^{2\Phi_1[t,x]} - \frac{1}{4(\mathrm{re}(a_2))^2} e^{2\Phi_2[t,x]} \\
& + \frac{\left((\mathrm{im}(a_1) - \mathrm{im}(a_2))^2 + (\mathrm{re}(a_1) - \mathrm{re}(a_2))^2\right)^2}{16(\mathrm{re}(a_1))^2(\mathrm{re}(a_2))^2\left[(\mathrm{im}(a_1) - \mathrm{im}(a_2))^2 + (\mathrm{re}(a_1) - \mathrm{re}(a_2))^2\right]} e^{2(\Phi_1[t,x]+\Phi_2[t,x])} \\
& - \frac{1}{[\mathrm{re}(a_1) + I \cdot (\mathrm{im}(a_1) - \mathrm{im}(a_2)) + \mathrm{re}(a_2)]^2} e^{\Phi_1[t,x]+\Phi_2[t,x]+I\cdot\Psi_1[t,x]-I\cdot\Psi_2[t,x]} \\
& - \frac{1}{[\mathrm{re}(a_1) + I \cdot (\mathrm{im}(a_1) - \mathrm{im}(a_2)) + \mathrm{re}(a_2)]^2} e^{\Phi_1[t,x]+\Phi_2[t,x]-I\cdot\Psi_1[t,x]+I\cdot\Psi_2[t,x]}, \\
g[t,x] = {} & e^{\Phi_1[t,x]+\Phi_2[t,x]+I\cdot\Psi_1[t,x]+I\cdot\Psi_2[t,x]} \\
& + \frac{(-I \cdot \mathrm{im}(a_1) - \mathrm{re}(a_1) + I \cdot \mathrm{im}(a_2) + \mathrm{re}(a_2))^2}{[2\mathrm{re}(a_2)(\mathrm{im}(a_1) - I \cdot \mathrm{re}(a_1) - \mathrm{im}(a_2) + I \cdot \mathrm{re}(a_2))]^2} e^{\Phi_1[t,x]+2\Phi_2[t,x]+I\cdot\Psi_1[t,x]} \\
& + \frac{(-I \cdot \mathrm{im}(a_1) - \mathrm{re}(a_1) + I \cdot \mathrm{im}(a_2) + \mathrm{re}(a_2))^2}{[2\mathrm{re}(a_2)(\mathrm{im}(a_1) - I \cdot \mathrm{re}(a_1) - \mathrm{im}(a_2) + I \cdot \mathrm{re}(a_2))]^2} e^{2\Phi_1[t,x]+\Phi_2[t,x]+I\cdot\Psi_2[t,x]}
\end{aligned} \tag{3.18}$$

Table 1: The difference of two consecutive solution values ($\mathbf{N} = 256$, $-128 \leq \mathbf{x} \leq 128$, $\mathbf{M} = 500$, $0 \leq \mathbf{t} \leq 5$).

Δx	Δt	L_2	L_∞
1.0	0.01	0.1410966549	0.6322912259
1.0	0.01	0.0033087030	0.0105278461
1.0	0.01	0.0000823911	0.0002140195
1.0	0.01	0.0000012444	0.0000028444

is a solution of the Hirota's bilinear equation (2.5) and

$$w = \frac{g(x,t)}{f(x,t)} \tag{3.19}$$

is the corresponding two-soliton solution of the CMKdV-II (1.3). Similarly, it is possible to find N solitary wave solutions by taking

$$g^{(N)} = \sum_{i=1}^{n} e^{a_i x + b_i t + c_i}, \tag{3.20}$$

where

$$b_i = -a_i{}^3 \tag{3.21}$$

but the computations are very tedious for $i > 3$.

4. Numerical results

4.1. Iterative Methods Using Finite Difference Schemes

Previously many researchers have used the finite difference methods to solve the KdV equation, Feng [12]. In the last decade, the CMKdV-II type equations were solved numerically by using split-step Fourier method [13–15]. Also parallel implementation of the split-step Fourier method using Fast Fourier Transform (FFT) has been studied by Taha [16] (see references therein). Here, in this work, the one-soliton solution of the CMKdV-II equation is considered. A finite interval for our numerical purposes is subjected, namely, $[a, b]$. The constants a and b can be chosen sufficiently large so that the boundaries do not affect the propagation of solitons. For the CMKdV-II (1.3), a numerical (finite difference) method of solution using iterative method is introduced. U_t is approximated by using forward time difference scheme, U_x and U_{xxx} by the central-space difference scheme using four-points. Equation (1.3) becomes

$$\left(\frac{z_m^{n+1} - z_m^n}{\Delta t}\right) - 6\left|z_m^{n-1}\right|^2 \left[\frac{-z_{m+2}^{n+1} + 8z_{m+1}^{n+1} - z_{m+1}^{n+1} + z_{m-2}^{n+1}}{12\Delta x}\right] + \left[\frac{z_{m+2}^{n+1} - 2z_{m+1}^{n+1} + 2z_{m-1}^{n+1} - z_{m-2}^{n+1}}{2(\Delta x)^3}\right] = 0, \tag{4.1}$$

where $z_m^n = z(t_n, x_m) = z(nk, mh)$, $k = \Delta t$, $h = \Delta x$.

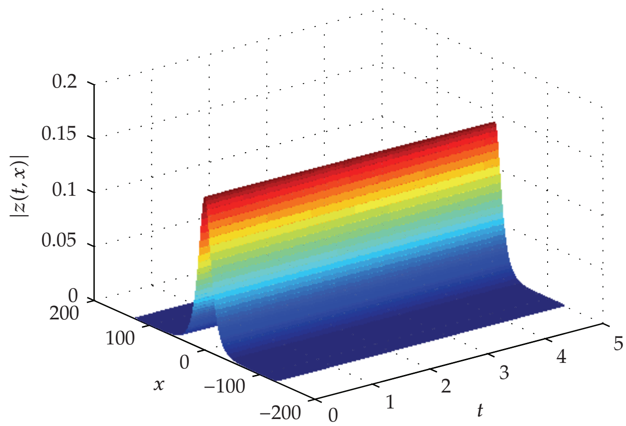

Figure 1: The modulus of the one-soliton numerical solution for the CMKdV-II equation with $N = 256$, $-128 \le x \le 128$, $M = 100$, $0 \le t \le 500$.

Multiplying both sides by $2(\Delta x)^3$ and rearranging the terms we get

$$\begin{aligned} &z_{m+2}^{n+1}\left[1 + 6(\Delta x)^2\left(\left|z_m^{n-1}\right|^2\right)\right] + z_{m+1}^{n+1}\left[-2 - 48(\Delta x)^2\left|z_m^{n-1}\right|^2\right] \\ &\quad + z_m^{n+1}\left[\frac{2(\Delta x)^2}{\Delta t}\right] + z_{m-1}^{n+1}\left[2 + 48(\Delta x)^2\left(\left|z_m^{n-1}\right|^2\right)\right] \\ &\quad + z_{m-2}^{n+1}\left[-1 - 6(\Delta x)^2\left(\left|z_m^{n-1}\right|^2\right)\right] = \frac{2(\Delta x)^3}{\Delta t} z_m^n \end{aligned} \tag{4.2}$$

for $3 \le m \le N-2$. For $m = N-1$ and $m = N$, the backward difference scheme is chosen in U_x and U_{xxx}. Three more equations come from the boundary conditions, namely, $U(a) = 0$, $U(b) = 0$, and $\partial U/\partial x = 0$ at $x = a$. Thus, N unknowns, namely, z_i^{n+1}, $i = 1, \ldots, N$ and N equations are obtained. Since the value of the non-linear term is known here, a system of linear equations is obtained. The initial guess is taken as $U(x,0) = \sqrt{2c/\alpha}\,\operatorname{sech}[\sqrt{c}(x - x_0)]e^{(i\theta_0)}$ which represents a solitary wave initially at x_0 moving to the right with velocity c and θ_0 is the polarization angle. The main idea is to assume that the non-linear term $|z_m^{n-1}|^2$ is zero first and then solve the problem for whole time domain. Afterwards, this solution is taken and substituted for the non-linear term and solved again iteratively. The following two norms, namely, L_∞ and L_2 are used to measure the accuracy of the approximate solutions for stopping criteria. These norms are defined as following:

$$L_\infty = \max_n(\|\tilde{z}_n| - |z_n\|), \qquad L_2 = \sqrt{\sum_n (|\tilde{z}_n| - |z_n|)^2}, \tag{4.3}$$

where $\tilde{z}_n$ and z_n are the two consecutive new and old approximate solutions, respectively, at point $(n\Delta x, T)$ for all n, where T is the final or terminating time. FORTRAN and MATLAB are used to obtain the results and figures, respectively. The graph of one-soliton numerical solution is shown in Figure 1.

5. Conclusions

In this study, Hirota's bilinear form for the complex modified Korteweg-de Vries-II equation is derived. One- and two-soliton solutions of the CMKdV-II equation are obtained analytically. One-soliton solution of the CMKdV-II equation is obtained by using finite difference method by implementing an iterative method. The computational cost is due to only finding the inverse of the matrix. The difference of two consecutive solution values according to the formula which is given in (4.3) is shown in Table 1 result. The convergence rate in the method presented above is quadratic as it can be seen in Table 1 result. It would be interesting to see what happens if this numerical scheme for the interaction of two-soliton waves for the CMKdV-II equation is applied. The numerical scheme deserves further study according to its application to the CMKdV-II equation.

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International Journal of Mathematics and Mathematical Sciences

Special Issue on New Trends in Geometric Function Theory

Call for Papers

Geometric function theory is the branch of complex analysis which deals with the geometric properties of analytic functions. It was founded around the turn of the twenteenth century and has remained one of the active fields of the current research. Moreover, in spite the famous coefficient problem, "Bieberbach conjecture", was solved by Louis de Branges in 1984, it suggests us various approaches and directions for the study of geometric function theory. It is very important for us to find new observational and theoretical results in this field with various applications. The cornerstone of geometric function theory is the theory of univalent functions, but new related topics appeared and developed with many interesting results and applications.

We invite authors to present their original articles as well as review articles that will stimulate the continuing efforts in developing new results in geometric function theory. The special issue will become an international forum for researches to summarize the most recent developments and ideas in this field. The main aim of the special issue of our journal is to invite the authors to present their original articles which not only provide new results or methods but also may have a great impact on other people in their efforts to broaden their knowledge and investigation. Review articles with some open problems are also welcome. The topics to be covered include, but are not limited to:

- Conformal mapping theory
- Differential subordinations and superordinations
- Entire and meromorphic functions
- Fractional calculus with applications
- General theory of univalent and multivalent functions
- Harmonic functions
- Quasiconformal mappings

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Fixed Point Theory and Applications

Special Issue on Takahashi's Legacy in Fixed Point Theory

Call for Papers

On March 31, 2009, Professor Wataru Takahashi will be retiring from the Tokyo Institute of Technology. Among his many fundamental contributions to nonlinear analysis, the existence theorems for noncommutative families of nonexpansive mappings, convergence theorems for generalized nonexpansive mappings, and fixed point theorems in Banach, topological vector, and metric spaces have proved to be of remarkable value. In fact, researchers continue extending his results aggressively. There cannot be any better time than this moment to dedicate this special issue in his honor.

Subjects connected with his theory, such as the following, will be welcome:

- Existence of fixed points for (families of) (generalized) nonexpansive mappings and others
- Approximations to fixed points (or zeros) of (generalized) nonexpansive mappings, monotone mappings, and others
- Fixed point theorems in topological vector spaces
- Fixed point theorems in (convex) metric spaces
- Invariant means and nonlinear analysis
- Theories and applications around the above

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