PROJECT # 1

The two-dimensional Laplace equation on a domain $[0, 1] \times [0, 1]$ is given by

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \tag{1}$$

with the Drichlet boundary conditions:

$$\begin{aligned} \psi(x, 0) &= 0 & (2) \\ \psi(x, 1) &= (x - x^2) & (3) \\ \psi(0, y) &= 0 & (4) \\ \psi(1, y) &= 0 & (5) \end{aligned}$$

Use the second-order accurate finite difference discretization with uniform 41×41, 81×81 and 161×161 Cartesian meshes to solve the above Laplace equation. For this purpose

1. Implement Jacobi, Gauss-Seidel and SOR algorithms and compare their convergence rates with the iteration numbers.
2. Implement the fully implicit solution algorithm and use a direct solver (LU factorization) to solve.
3. Plot Error function versus the mesh space Δx in a log-log scale. Compute the spatial convergence rate.

The error function is given by

$$\text{Error} = \frac{\|u_{i,j} - u_{analytic}\|_2}{\sqrt{i_{max} j_{max}}} \tag{6}$$

Several useful MATLAB commands:
Crate a sparse matrix
i=[];
j=[];
s=[];
m=100;
n=100;
A=sparse(i, j, s, m, n);
To solve a sparse linear system
$x = A\backslash b$;

[0] UUT510E, Return date: 2 April 2015