DIGITAL COMMUNICATIONS - Homework 2 due on May 04, 2017

1- Let a random process be given as $Z(t) = X(t)A \cos(2\pi f_c t)$ where $A$ is a zero-mean unit-variance random variable. $X(t)$ is a (wide-sense) stationary random processes with mean $\mu_x$ autocorrelation $R_x(\tau)$. $A$ and $X(t)$ are independent. (a) Express the mean and autocorrelation functions of $Z(t)$. (b) Is the process $Z(t)$ stationary? Explain.

2- A random process is defined as $Y(t) = X(t) - X(t-T)$ where $X(t)$ is a zero-mean (wide-sense) stationary process with autocorrelation $R_x(\tau)$ and power spectral density $S_x(f)$. (a) Express the output autocorrelation function $R_y(\tau)$ in terms of $R_x(\tau)$ and $R_x(\tau \pm T)$. (b) Express the output power spectral density $S_y(f)$ in terms of $S_x(f)$. (c) Plot $S_y(f)$ when $S_x(f) = 1$ and $T = 10$ msec.

3- The signal $g(t) = A \text{rect}\left(\frac{t-T/2}{T}\right)$ is applied to the linear time-invariant (LTI) system whose impulse response is given as $h(t) = g(T-t)$. (a) Express and plot the output of the LTI system. (b) The input to the LTI system is white noise with spectral constant $N_0 = 10^{-2}$ W/Hz. Moreover, $T = 10^{-2}$ sec. and $A = 2$. Compute the output power (variance) due to white noise input.

4- Consider linear functionals generating random variables of the form $X_i = \int_0^T W(t)\phi_i(t)dt$ $i=1,2$ where $W(t)$ is a white process, and $\phi(t)$’s are orthonormal energy functions, namely, $\int_0^T \phi_i^2(t)dt = 1$ $i=1,2$ and $\int_0^T \phi_i(t)\phi_j(t)dt = 0$. Show that $X_1$ and $X_2$ are uncorrelated.

5- A discrete-time LTI system is governed by the difference equation $Y(n) = b_0 X(n) + b_1 X(n-1) + b_2 X(n-2)$. Write a MATLAB code which realizes this equation. Drive the system by standard white Gaussian noise and estimate the cross-correlation function as $\hat{R}_{xy}(k) = \frac{1}{N+1} \sum_{n=0}^N Y(n+k)X(n)$ for the purpose of determining the system impulse response. Compare the true and estimated impulse responses. Plot (superimposed) the true and estimated frequency response $H(f)$ (magnitude only). Include your comments and Matlab codes in your answers. Select the coefficients $b_0, b_1, b_2$ as the last three digits of your ID number.